From Soft Sets to Information Systems

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Abstract—**This paper discusses the relationship between soft sets and information systems. It is showed that soft sets are a class of special information systems. After soft sets are extended to several classes of general cases, the more general results also show that partition-type soft sets and information systems have the same formal structures, and that fuzzy soft sets and fuzzy information systems are equivalent.**

Index Terms—**Soft set, Information system, Approximation space, Partition-type soft set, Fuzzy soft set.**

I. Introduction

Based on the detail analysis of inherent difficulties of some theories for dealing with uncertainty and incompleteness of information and data such as interval analysis, fuzzy set theory, and so on, Molodtsov [5] proposed soft set theory which contains sufficient parameters such that it is free from the corresponding difficulties, and a series of interesting applications of the theory in stability and regularization, game theory, operations research, probability and statistics are presented. Moreover, Maji et al. [4] proposed several operations on soft sets, and some basic properties of these operations are revealed.

On the other hand, information systems have been intensively studies by many authors from several domains containing knowledge engineering [1,3,6]), rough set theory [7-9,13], granular computing [11], data mining and knowledge discovery [14], and so on.

Through a serious observation one can see that there exist some compact connections between soft sets and information systems. We will clarify connections of the two branches, and we intend to unify them in this paper. The present paper is organized as follows. In the next section, concepts and main properties of soft sets and their operations are briefly reviewed, and some results about the structure of soft sets are presented. Sections three extends the concept of soft sets, and we will introduces several classes of generalized soft sets which will be very useful in some application domains. Then Section four investigates relationships between soft sets and information systems. Our results show that soft sets are a class of special information systems, and both researches of soft sets and information systems can be unified, and furthermore, some new results and methods can be expected. Finally, some concluding remarks are given in the last section.

II. Concepts, properties of soft sets

In this paper, we employ $\mathcal{P}(U)$ to stand for the power set of some set U, i.e., the set of all subsets of U, and $\mathcal{F}(U)$ to stand for the set of all fuzzy subsets of U.

Definition 1. Let U be an nonempty finite set of objects called a universe. An ordered pair (F, E) is called a soft set over U where F is a mapping from E to $\mathcal{P}(U)$.

The set of all soft sets over U is denoted by $\mathcal{S}(U)$.

It has been interpreted that a soft set indeed is a parameterized family of subsets of U, and thus E is referred to as a set of parameters [5].

Sometimes we only consider the so-called standard soft sets over U in which the parameter sets are the same, i.e., E, and the set of all standard soft sets over U is denoted by $\mathcal{S}_0(U)$.

Two trivial soft sets are the null soft set and the total soft set (or, absolute soft set [4]) which are respectively defined as follows:

(i) The null soft set $\Phi = (F, E)$: $F(e) = \emptyset$ for all $e \in E$;

(ii) The total soft set $\Psi = (F, E)$: $F(e) = U$ for all $e \in E$.

Molodtsov [5] showed that fuzzy sets and topological spaces can be seen as special soft sets.

Suppose A is a fuzzy set of the universe U, we take the parameter set $E = [0,1]$, and define the mapping $F: E \to \mathcal{P}(U)$ as follows:

$$F(\alpha) = \{x \in U \mid A(x) \geq \alpha\}, \ \alpha \in E.$$

In other words, $F(\alpha)$ is the α-level set of A.

According to this manner and by using the decomposition theorem of fuzzy sets [2], we see that a fuzzy set [12] can be uniquely represented as a soft set.

For a topological space $(X, \mathcal{T})$, if $F(x)$ is the family of all open neighborhoods of a point x in X, i.e.,

$$F(x) = \{V \in \mathcal{T} \mid x \in V\},$$

then the ordered pair (F, X) indeed is a soft set over X.

Similarly, we can put rough sets [7,8] into the framework of soft sets as follows.

For some given set U, $R \subseteq U \times U$ is called a binary relation on U. R is said to be reflexive if $(x, x) \in R$ for all $x \in U$. R is said to be symmetric if $(x, y) \in R$ implies $(y, x) \in R$ for all $x, y \in U$. R is said to be transitive if $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$ for all $x, y, z \in U$. An equivalence relation on U is a reflexive, symmetric and transitive relation on U.

In Pawlak's sense [7,8], the ordered pair (U, R) is called an approximation space, or a relation information system, where U is a universe, and R is an equivalence relation on U. For each subset A of U, the approximation mapping $\underline{apr}_R$ maps A to the its lower approximation $\underline{apr}_R(A)$, and the approximation mapping $\overline{apr}_R$ maps A to its upper approximation $\overline{apr}_R(A)$, where

$$\begin{aligned} \underline{apr}_R(A) &= \{x \in U \mid [x]_R \subseteq A\} \\ \overline{apr}_R(A) &= \{x \in U \mid [x]_R \cap A \neq \emptyset\} \end{aligned}$$

where $[x]_R$ is the equivalence class of x with respect to R. Therefore, the rough set model (U, R) can be seen as two soft sets $(\underline{apr}_R, \mathcal{P}(U))$ and $(\overline{apr}_R, \mathcal{P}(U))$ over U.

Originally, Pawlak [7,8] assumed that the binary relation R is an equivalence relation on the universe U. Such approximation spaces are called classical approximation spaces, or Pawlak approximation spaces. After then, many scholars devote to generalize Pawlak approximation spaces to more general cases. For example, Yao [10] (also see [14]) extended Pawlak approximation spaces to the cases where the equivalence relations are replaced by arbitrary binary relations on U which are called generalized approximation spaces.

Throughout this paper, we employ the notation $PAS(U)$ to stand for the set of all Pawlak approximation spaces on the universe U.

According to the idea given by Molodtsov [5], Maji et al. [4] introduced the concepts of inclusion, equality and several operations of soft sets. However, in order to discuss the algebraic structure of the set $\mathcal{S}_0(U)$ efficiently, we suitably revise their definitions as follows.

Definition 2. Let $(F, A), (G, B) \in \mathcal{S}(U)$.

(i) (F, A) is said to be a soft subset of (G, B), denoted $(F, A) \subseteq (G, B)$, if $A \subseteq B$ and $F(a) \subseteq G(a)$ for all $a \in A$.

(ii) (F, A) and (G, B) are said to be soft equal, denoted $(F, A) = (G, B)$, if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

Definition 3. Let $(F, E) \in \mathcal{S}(U)$, and $E = \{e_1, \ldots, e_n\}$ be the parameter set with a negation operation $\neg$. For $A \subseteq E$, denote $\neg A = \{\neg e \mid e \in A\}$. The soft set $(F^c, \neg A)$ is called the complement of (F, A), denoted $(F, A)^c$, where $F^c : \neg A \to \mathcal{P}(U)$ is defined as follows

$$F^c(e) = \sim F(e) = U \setminus F(\neg e), \ e \in \neg A.$$

We observe that the complement $(F, A)^c$ of the soft set (F, A) is a soft set over U.

Definition 4. Let $(F, A), (G, B) \in \mathcal{S}(U)$.

(i) The basic intersection of two soft sets (F, A) and (G, B) is defined as the soft set

$$(H, C) = (F, A) \wedge (G, B)$$

over $U \times U$ where $C = A \times B$, and $H(a, b) = F(a) \cap G(b)$ for all $(a, b) \in A \times B$;

(ii) The basic union of two soft sets (F, A) and (G, B) is defined as the soft set

$$(H, C) = (F, A) \vee (G, B)$$

over $U \times U$ where $C = A \times B$, and $H(a, b) = F(a) \cup G(b)$ for all $(a, b) \in A \times B$;

(iii) The union (H, C) of soft sets (F, A) and (G, B), denoted $(F, A) \cup (G, B)$, is defined as $C = A \cup B$, and for all $c \in C$,

$$H(c) = \begin{cases} F(c), & c \in A \setminus B, \\ G(c), & c \in B \setminus A, \\ F(c) \cup G(c), & c \in A \cap B; \end{cases}$$

(iv) The intersection (H, C) of soft sets (F, A) and (G, B), denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $H(c) = F(c) \cap G(c)$ for all $c \in C$.

It is meaningful to notice that for (F, A) and (G, B) of $\mathcal{S}(U)$ we have $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ belong to $\mathcal{S}(U \times U)$. However, $(F, A) \cup (G, B)$ and $(F, A) \cap (G, B)$ belong to $\mathcal{S}(U)$.

In (iv) of the above definition, we have corrected a mistake of [4] with respect to the intersection of soft sets because generally, $H(c)$ and $G(c)$ are not necessarily equal for $c \in A \cap B$.

Proposition 1. Let $(F, A), (G, B) \in \mathcal{S}(U)$. Then

(i) $\vee$ and $\wedge$ satisfy de Morgan Law with respect to ^{c}:

$((F, A) \vee (G, B))^c = (F, A)^c \wedge (G, B)^c$,

$((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$.

(ii) Both $\vee$ and $\wedge$ are associative and distributive:

$(F, A) \vee ((G, B) \vee (H, C)) = ((F, A) \vee (G, B)) \vee (H, C)$,

$(F, A) \wedge (G, B) \wedge (H, C) = ((F, A) \wedge (G, B)) \wedge (H, C)$,

$(F, A) \vee ((G, B) \wedge (H, C)) = ((F, A) \vee (G, B)) \wedge ((F, A) \vee (H, C))$,

$(F, A) \wedge ((G, B) \vee (H, C)) = ((F, A) \wedge (G, B)) \vee ((F, A) \wedge (H, C))$.

(iii) Both $\cup$ and $\cap$ are idempotent, associative and distributive:

$(F, A) \cup (F, A) = (F, A)$,

$(F, A) \cap (F, A) = (F, A)$,

$(F, A) \cup ((G, B) \cup (H, C)) = ((F, A) \cup (G, B)) \cup (H, C)$,

$(F, A) \cap (G, B) \cap (H, C) = ((F, A) \cap (G, B)) \cap (H, C)$,

$(F, A) \cup ((G, B) \cap (H, C)) = ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C))$,

$(F, A) \cap ((G, B) \cup (H, C)) = ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C))$.

(iv) The null soft set $\Phi \in \mathcal{S}_0(U)$ is an unit element of $\cup$, and a null element of $\cap$, i.e., for all $(F, E) \in \mathcal{S}_0(U)$:

$(F, E) \cup \Phi = (F, E)$,

$(F, E) \cap \Phi = \Phi$.

(v) The total soft set $\Psi \in \mathcal{S}_0(U)$ is a null element of $\cup$, and an unit element of $\cap$, i.e., for all $(F, E) \in \mathcal{S}_0(U)$:

$(F, E) \cup \Psi = \Psi$,

$(F, E) \cap \Psi = (F, E)$.

Summarizing the properties of operations of soft sets, we may give the following conclusions.

Proposition 2. The structure $(\mathcal{S}_0(U), \Phi, \Psi, \cup, \cap)$ is a bounded distributive lattice.

However, the structure $(\mathcal{S}_0(U), \Phi, \Psi, ^c, \vee, \wedge)$ is not a bounded distributive lattice because two operations $\vee, \wedge$ are not closed in $\mathcal{S}_0(U)$. However, we can easily obtain the following properties which are similar to the complement law of Boolean algebras.

Proposition 3. For all $(F, E) \in \mathcal{S}_0(U)$, the following conditions hold:

(i) $(F, E) \vee (F, E)^c = \Psi$,

(ii) $(F, E) \wedge (F, E)^c = \Phi$,

where Φ and Ψ are the null and total soft sets over $U \times U$, respectively.

III. GENERALIZED SOFT SETS

Let us see an example of soft sets given by Molodtsov [5] to describe the attractiveness of houses which somebody is going to buy.

Example. Denote

U — the set of houses under consideration;

E={*expensive, cheap, beautiful, modern, in the green surrounding, wooden, in good repair, in bad repair*} — the set of parameters;

F — a mapping from E to $\mathcal{P}(U)$ to point out *expensive* houses, *beautiful* houses, and so on.

The following table comes from [4]:

O	E	B	W	C	G
h1	0	1	0	1	0
h2	1	0	0	0	0
h3	0	1	1	1	0
h4	1	0	1	0	0
h5	0	0	1	1	0
h6	0	0	0	0	0

In the above table, the letter O stands for "object", E stands for "expensive", B stands for "beautiful", W stands for "wooden", C stands for "cheap", and G stands for "in the green surroundings".

From this classical example, we may see the structures of soft sets are very simple in the following two manners: the first is that for every parameter, the mapping only classify the objects into two simple classes (yes or no); and the other is that the image of every parameter under the mapping is a crisp subset of the universe. However, in the theoretical and practical researches of soft sets, the situations are usually very complex. Therefore, a very natural problem is to extend the concept of soft sets to more general cases.

Firstly, from the view of set-theory, we consider the situations in which for every parameter, the given standard (e.g., "*expensive*") contains multiple grades, not only two grades. For example, in the previous example, the expensive degree of houses can be divided into three grades, high, medium and low.

In these situations, every parameter determines a partition of the universe. Hence we extend the classical concept of soft sets to the cases in which the image of a parameter under the mapping is a partition of the universe.

Let $\mathcal{T} \subseteq \mathcal{P}(U)$. $\mathcal{T}$ is called a partition of the universe U, if the following conditions hold:

(i) $\emptyset \notin \mathcal{T}$,

(ii) For all $A, B \in \mathcal{T}$, $A \neq B \Longrightarrow A \cap B = \emptyset$,

(iii) $\bigcup \mathcal{T} = U$.

The set of all partitions of the universe U is denoted by $Par(U)$.

Definition 5. Let U be a universe. An ordered pair (F, E) is called a partition-type soft set over U if F is a mapping from E to the set $Par(U)$ of all partitions of the universe U.

The sets of all partition-type soft sets over U are denoted by $PS(U)$.

We can see that a classical soft set (F, E) over U indeed is a special partition-type soft set $(\widetilde{F}, E)$. In fact, for every parameter $e \in E$, one can define the image $\widetilde{F}(e)$ as the partition $\{F(e), U \setminus F(e)\}$ whenever $F(e) \neq \emptyset$ and $F(e) \neq U$. In addition, in the cases $F(e) = \emptyset$ or $F(e) = U$, we can easily obtain the corresponding partitions of U (they only contain an element U).

By making use of the fact that a partition of some universe must be a covering of the universe, we may more generally consider the situations in which the image of every parameter under the mapping is a covering of the universe.

Let $\mathcal{C} \subseteq \mathcal{P}(U)$. $\mathcal{C}$ is called a covering of the universe U, if the following conditions hold:

(i) $\emptyset \notin \mathcal{C}$,

(ii) $\bigcup \mathcal{C} = U$.

The set of all coverings of the universe U is denoted by $Cov(U)$.

Definition 6. Let U be a universe. An ordered pair (F, E) is called a covering-type soft set over U if F is a mapping from E to the set $Cov(U)$ of all coverings of the universe U.

The sets of all covering-type soft sets over U are denoted by $CS(U)$.

Secondly, by virtue of the view of fuzzy theory, the classical concept of soft sets can be extended to the situations in which the image of every parameter under the mapping is a fuzzy set of the universe. These class of extensions indeed are very natural according to the view of practice. In the previous example, the terms that a house is "*expensive*", "*beautiful*" or "*cheap*" are all fuzzy. One usually gives a real number in the unit interval [0,1] to measure the degree for some index of house quality.

A mapping A from the universe U to the real unit interval [0,1] is called a fuzzy set on U. The set of all fuzzy sets on U is denoted by $\mathcal{F}(U)$ [12] (also see [2]).

Definition 7. Let U be a universe. An ordered pair (F, E) is called a fuzzy soft set over U if F is a mapping from E to the set $\mathcal{F}(U)$ of all fuzzy sets on U.

The set of all fuzzy soft sets over U is denoted by $FS(U)$.

Obviously, a classical soft set (F, E) over a universe U can be seen as a fuzzy soft set $(\widetilde{F}, E)$ according to the following manner, for $e \in E$, the image of e under $\widetilde{F}$ is defined as the characteristic function of the set $F(e)$, i.e.,

$$\widetilde{F}(e) = \chi_{F(e)} = \begin{cases} 1, & a \in F(e) \\ 0, & a \notin F(e) \end{cases}$$

For three classes of generalized soft sets we can introduce similar concepts such as inclusion, equality and operations as Definitions 2-4.

IV. Relationships between soft sets and information systems

From the concept and the example of soft sets given in the previous section it can be seen that a classical soft set indeed is a simple information system in which the attributes only take two values 0 and 1. The following definition gives the precise concept of information systems [6, 8, 9, 11, 13, 14].

Definition 8. The quadruple (U, A, F, V) is called an information system, or a database system, an information table, where $U = \{x_1, \dots, x_n\}$ is a universe containing all interested objects, $A = \{a_1, \dots, a_m\}$ is a set of attributes, $V = \bigcup_{i=1}^{m} V_i$ where V_j is the value set of the attribute a_j, and $F = \{f_1, \dots, f_m\}$ where $f_j : U \to V_j$.

Usually, one assumes that every V_j only contains finite elements (these elements may be and may not be numbers) for every $j \leq m$. Such information systems are called classical information systems. However, if $V_j = [0, 1]$ for every $j \leq m$ then the corresponding information systems are called fuzzy information systems. Furthermore, if $f_j : U \to \mathcal{P}(V_j)$ is a mapping from U to the power set of V_j for all $j \leq m$ then the corresponding information systems are called set-valued information systems. Similarly, many other types information systems are applied to information processing and data analysis.

Throughout this paper, we employ the notations $IS(U)$, $FIS(U)$, and $SIS(U)$ to stand for the sets of all classical information systems, all fuzzy information systems, and all set-valued information systems, respectively.

The following theorem shows that a soft set indeed is a simple information system.

Proposition 4. If (F, E) is a soft set over the universe U with the parameter set $E = \{e_1, \dots, e_m\}$ then (F, E) is an information system.

Proof. In fact, the given soft set (F, E) can be seen as an information system (U, G, A, V) according to the following manner:

$$G = \{g_1, \dots, g_m\}, \ g_i : U \to V_i, \ g_i(x) = \begin{cases} 1, & x \in F(e_i), \\ 0, & \text{otherwise.} \end{cases}$$

$$A = E, \quad V = \bigcup_{i=1}^{m} V_i, \quad V_i = \{0, 1\}, \ 1 \leq i \leq m$$

We notice that an information system (U, G, A, V) can be naturally translated into an approximation space (U, R) where R is the indiscernibility relation induced by the attribute set $A = \{a_1, \dots, a_m\}$ according to the following manner:

$$R = \{(x, y) \in U \times U \mid g_i(x) = g_i(y), 1 \leq i \leq m\}. \qquad (1)$$

Hence, we can obtain the following corollary from the above theorem.

Corollary 1. Let (F, E) is a soft set over the universe U with the parameter set $E = \{e_1, \dots, e_m\}$ then (U, R) is an approximation space where R is defined similarly to (1).

According to the above theorem we see that soft sets are simple information systems. However, it is obvious that information systems are not necessarily soft sets. Based on this consideration, if we discuss partition-type soft sets the situation will be completely different.

Proposition 5. The sets $PS(U)$ and $IS(U)$ are the same, or there exists a bijection between $PS(U)$ and $IS(U)$.

Proof. If $(F, E) \in PS(U)$ is a partition-type soft set over U with the parameter set $E = \{e_1, \dots, e_m\}$ and the mapping $F : E \to Par(U)$, then (F, E) is an information system with the form (U, A, G, V) according to the following manner:

$$G = \{g_1, \dots, g_m\}, \ g_i : U \to V_i, \ g_j(x) = v_j^{(i)}, \ x \in [x_j]_i, \ j \leq n_i.$$

$$A = E, \quad V = \bigcup_{i=1}^{m} V_i, \quad V_i = \{v_1^{(i)}, \dots, v_{n_i}^{(i)}\}, \ i \leq m$$

where $[x_j]_i$ is the block of the partition $F(e_i)$ which containing x_j and $|F(e_i)| = n_i$ for all $i \leq m$.

Conversely, if $(U, A, G, V) \in IS(U)$ is an information system with the attribute set $A = \{a_1, \dots, a_m\}$ and the set of mappings $G = \{g_1, \dots, g_m\}$ with $g_i : U \to V_i$, then (U, A, G, V) is a partition-type soft set (F, E) with the parameter set $E = A$ and the mapping $F : E \to Par(U)$ with $F(e_i) = U/R$ where R is the indiscernibility relation induced by the attribute set A, and U/R is the partition of U consisting of all equivalence classes with respect to R (also called the quotient set of U with respect to R).

The following corollary is a direct consequence of the above theorem and Corollary 1.

Corollary 2. The sets $PS(U)$ of all partition-type soft sets and the set $PAS(U)$ of all Pawlak approximation spaces are the same, or there exists a bijection between $PS(U)$ and $PAS(U)$.

Moreover, about relationship between fuzzy soft sets and fuzzy information systems we have the following conclusion.

Proposition 6. The sets $FS(U)$ of all fuzzy soft sets and the set $FIS(U)$ of all fuzzy information systems are the same, or there exists a bijection between $FS(U)$ and $FIS(U)$.

Proof. If $(F, E) \in FS(U)$ is a fuzzy soft set over U with the parameter set $E = \{e_1, \dots, e_m\}$ and the mapping $F : E \to \mathcal{F}(U)$, then (F, E) is a fuzzy information system with the form (U, A, G, V) according to the following manner:

$$G = \{g_1, \dots, g_m\}, \ g_i : U \to V_i, \ g_j(x) = F(e_j)(x), \ x \in U.$$

$$A = E, \quad V = \bigcup_{i=1}^{m} V_i = [0, 1], \quad V_i = [0, 1], \ i \leq m$$

Conversely, if $(U, A, G, V) \in FIS(U)$ is an fuzzy information system with the attribute set $A = \{a_1, \dots, a_m\}$ and the set of mappings $G = \{g_1, \dots, g_m\}$ with $g_i : U \to V_i = [0, 1]$, $i \leq m$, then (U, A, G, V) is a fuzzy soft set (F, E) with the parameter set $E = A$ and the mapping $F : E \to \mathcal{F}(U)$ where $F(e_i) = g_i$.

By virtue of the above theorems, a class of compact connections have been built between (generalized) soft sets and (generalized) information systems. By making use of these results, we have provided some new perspective for both researches of soft sets and information systems. For example, we can extend the concepts of inclusion, equality and operations to various generalized soft sets and (generalized) information systems. We take the later as examples.

Definition 9. Let $(U, A, F, V), (U, B, G, W) \in IS(U)$.

(i) (U, A, F, V) is said to be a subsystem of (U, B, G, W), denoted

$$(U, A, F, V) \subseteq (U, B, G, W),$$

if $A \subseteq B$, $V \subseteq W$ and $F(a) = G(a)$ for all $a \in A$.

(ii) (U, A, F, V) and (U, B, G, W) are said to be equal, denoted

$$(U, A, F, V) = (U, B, G, W),$$

if $(U, A, F, V) \subseteq (U, B, G, W)$ and $(U, B, G, W) \subseteq (U, A, F, V)$.

Definition 10. Let $(U, A, F, V), (U, B, G, W) \in IS(U)$.

(i) The basic intersection of two information systems (U, A, F, V) and (U, B, G, W) is defined as the information system

$$(U, C, H, X) = (U, A, F, V) \wedge (U, B, G, W)$$

where $C = A \times B$, and $H(a, b) = F(a) \cap G(b)$ for all $(a, b) \in A \times B$;

(ii) The basic union of two information systems (U, A, F, V) and (U, B, G, W) is defined as

$$(U, C, H, X) = (U, A, F, V) \vee (U, B, G, W)$$

where $C = A \times B$, and $H(a, b) = F(a) \cup G(b)$ for all $(a, b) \in A \times B$;

(iii) The union (U, C, H, X) of information systems (U, A, F, V) and (U, B, G, W), denoted

$$(U, A, F, V) \cup (U, B, G, W)$$

is defined as $C = A \cup B$, and for all $c \in C$,

$$H(c) = \begin{cases} F(c), & c \in A \setminus B, \\ G(c), & c \in B \setminus A, \\ F(c) \cup G(c), & c \in A \cap B; \end{cases}$$

(iv) The intersection (U, C, H, X) of information systems (U, A, F, V) and (U, B, G, W), denoted

$$(U, A, F, V) \cap (U, B, G, W)$$

is defined as $C = A \cap B$, and $H(c) = F(c)$ for all $c \in C$.

About operations or transformation of rough approximation spaces, we will give a detail discussion in the other paper.

V. Conclusion

This paper mainly discusses relationship between soft sets and various information systems. Our results show that a soft set is a simple information system in which the attributes only take two values 0 and 1, and partition-type soft sets and information systems are the same formal structures. Based on these results, we investigate operations of information systems which are parallel to those of soft sets.

An interesting topic for research is to discuss relationships among covering-type soft sets, set-valued information systems and generalized approximation spaces.

Another interesting topic for research is to discuss possible applications of ideas and methods developed in this paper.

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