A Knowledge Acquisition Model Based on Formal Concept Analysis in Complex Information Systems

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Abstract. Normally, in some complex information systems, the binary relation on domain of any attribute is just a kind of ordinary binary, which does not meet some common properties such as reflexivity, transitivity or symmetry. In view of the above-mentioned facts this paper attempts to employ FCA(Formal Concept Analysis), proposes a rough set model based on FCA, in which equivalence relations, dominance relations, similarity relations(or tolerance relations) and neighborhood relations on universe are expanded to general binary relations and problems in rough set theory are discussed based on FCA. Particularly, from the above description of complex information systems, we can see that the relation in domain of any attribute may be extremely complex, which often leads to high time complexity and space complexity in the process of knowledge acquisition. For above reason this paper introduces granular computing(GrC), which can effectively reduce the complexity to a certain extent.

Keywords: Rough set · Formal concept analysis · Granular computing

1 Introduction

Rough set theory, introduced by Pawlak in 1982 [6], is a mathematical theory which can be used to deal with vague and uncertain problems. The theory of FCA, proposed by Wille in the same year [10], is a tool for concept discovery from data, in which the relationship of concepts is embodied by concept lattice. As two active relevant research fields in artificial intelligence and information science, they have many common characteristics, such as they have common research backgrounds and aims, and they are both closely related to topology, algebra and logic. Therefore, the study of combination of two theories has fundamental significance, in recent years scholars have done a lot of research in this area [3,5,9].

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An information system is an quadruple $IS = (U, AT, V, f)$, where U is a finite nonempty set of objects, called a universe, and AT is a finite nonempty set of attributes, $V = \bigcup_{m \in AT} V_m$ and V_m is a domain of attribute m; $f : U \times AT \to V$ is a function such that $f(x, m) \in V_m$ for every $x \in U$, $m \in AT$, called an information function.

Pawlak rough set theory is defined using the indiscernibility relation, which implies that values in any V_m are independent of one another. However, in the real world, we may face cases that there are complex internal relationship in any V_m, that is, values describing attributes may exist some special relationship, such as some attribute values are ordinal or similar. Therefore, Pawlak rough set theory is inapplicable in dealing with above information systems. To overcome this insufficiency, scholars have done a lot of extended research work, in which rough set models based on dominance relations or similarity relations are the most common and widely used models [2,4,8].

However, in some complex information systems, the relationship between different values in any V_m is more complex rather than ordinal or similar, meanwhile, the corresponding relation on V_m often does not satisfies common properties such as reflexivity, transitivity or symmetry. Obviously, in above situation, for finding potential, valuable, simple information from chaotic, strong interference and large data, we need to extend the rough set theory further. For all that, on the basis of previous research this paper attempts to introduce advantages of FCA into rough set theory.

Extremely complex relations of domains of attributes often lead to complex lattice structure and huge concepts, and cause high time complexity and space complexity for further calculation and analysis of problems. For above-mentioned reasons this paper introduces GrC which has unique advantages in modeling and analysis of the large and complex data, and it can effectively simplify the complex structure and reduce the scale of concepts to a certain extent.

In general, this paper not only provides an useful method for applying FCA and GrC to complex information systems, but also offers a new idea for the extension of the rough set model. And above result is also the innovation of this thesis.

The rest of the paper is organized: FCA is briefly introduced in Sect. 2; Sect. 3 discusses the classification problem in the domain of any attribute based on GrC; Sect. 4 translates complex information systems into one-valued formal contexts; Sect. 5 discusses algebraic structures in complex information systems; Sect. 6 offers solutions to the problem of reduct, core and dependency etc.; conclusions and the discussion of further work close the paper in Sect. 7.

2 Basic Notions of FCA

A formal context is a triple $K = (G, M, I)$, where G and M are sets, and $I \subseteq G \times M$ is a binary relation. In the case, members of G are called objects and members of M are called attributes, and I is viewed as an incidence relation between objects and attributes. Accordingly, $(g, m) \in I$ denotes "the object g has the attribute m".

Definition 1. *[1] Let $K = (G, M, I)$ be a formal context, for any $A \subseteq G$ and $B \subseteq M$, we define:*

$$A' = \{ m \in M \mid (g, m) \in I, \forall g \in A \}; \quad B' = \{ g \in G \mid (g, m) \in I, \forall m \in B \}$$

If $A' = B$ and $B' = A$, then (A, B) is called a concept. In this case, A is called the extent, B is called the intent. The order "$\leq$" between concepts (A_1, B_1) and (A_2, B_2) is defined as

$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \Leftrightarrow B_1 \supseteq B_2$$

The ordered set $(\mathscr{B}(K), \leq)$ is a complete lattice, where $\mathscr{B}(K)$ is the set of all concepts.

Proposition 1. *[1] If $K = (G, M, I)$ is a formal context, $A, A_1, A_2 \subseteq G$ are sets of objects and $B, B_1, B_2 \subseteq M$ are sets of objects, then*

$$(1)\ A_1 \subseteq A_2 \Rightarrow A_2' \subseteq A_1' \qquad (2)\ B_1 \subseteq B_2 \Rightarrow B_2' \subseteq B_1'$$
$$(3)\ A \subseteq A''; B \subseteq B'' \qquad (4)\ A' = A'''; B' = B'''$$

Proposition 2. *[1] The ordered set $(\mathscr{B}(K), \leq)$ is a complete lattice, its corresponding infimum and supremum are:*

$$(1) \bigwedge_{t \in T} (A_t, B_t) = \left(\bigcup_{t \in T} A_t, \left(\bigcup_{t \in T} B_t \right)'' \right) \quad (2) \bigvee_{t \in T} (A_t, B_t) = \left(\left(\bigcup_{t \in T} A_t \right)'', \bigcup_{t \in T} B_t \right)$$

3 Classification Analysis in Domain of Attribute Based On GrC

Let V_m be the domain of attribute m, R_m be a relation on V_m. If R_m does not meet some common properties such as reflexivity, transitivity or symmetry, that we say R_m is just a general binary relation on V_m. For example, If there exists some type of relationship "$\perp$" between values in V_m, then we can deduce a binary relation $R_m = \{(v, w) \mid v \perp w,\ v, w \in V_m\}$ on V_m. In fact, R_m objectively reflects the relationship between values in V_m, if $(v, w) \notin R_m$, then there exists no relationship "$\perp$" between v and w; if $(v, w) \in R_m$, then there exists relationship "$\perp$" between v and w.

In view of the above-mentioned facts, for any attribute $m \in AT$, if the corresponding relation R_m on V_m is general, then we say $IS = (U, AT, V, f)$ is a complex information system. For convenience IS mentioned above is formalized as $IS = (U, AT, \Re)$, where $\Re = \{R_m \mid m \in AT\}$.

For example, Table 1 is a complex information system about cars, where $U = \{1, 2, \ldots, 8\}$ is the set of various type of cars and $AT = \{a, b, c, d, e\}$ is the set of attributes with a="the evaluation of price", b="the evaluation of size", c="the evaluation of engine", d="the evaluation of maximum speed", and e="the evaluation of performance/ price ratio". $R_a, R_b, \ldots, R_e$ in Table 2

Table 1. A complex information system

	a	b	c	d	e
1	u_1	v_1	w_1	x_1	z_1
2	u_2	v_2	w_2	x_2	z_2
3	u_3	v_3	w_3	x_3	z_3
4	u_4	v_4	w_4	x_4	z_4
5	u_5	v_5	w_5	x_5	z_5
6	u_6	v_6	w_6	x_6	z_6
7	u_7	v_7	w_7	x_7	z_7
8	u_8	v_8	w_8	x_8	z_8

Table 2. The $\Re$ of complex information system in Table 1

(a) R_a

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
u_1	×	×	×	×	×	×		
u_2	×	×	×	×	×	×		
u_3	×	×	×		×	×	×	×
u_4	×	×		×	×	×	×	×
u_5	×	×	×		×	×	×	×
u_6	×	×	×	×	×		×	×
u_7			×	×	×	×	×	×
u_8			×	×	×	×	×	×

(b) R_b

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	×	×	×	×				
v_2	×	×	×	×				
v_3	×	×	×	×	×	×	×	
v_4	×	×		×	×	×		×
v_5			×	×	×	×	×	×
v_6			×	×	×	×	×	×
v_7			×		×	×	×	×
v_8			×		×	×	×	×

(c) R_c

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
w_1	×		×		×		×	
w_2		×	×	×		×	×	
w_3		×	×	×		×		×
w_4		×	×		×	×	×	
w_5	×	×	×	×		×		×
w_6	×			×	×		×	×
w_7		×	×	×	×	×	×	×
w_8	×		×					

(d) R_d

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	×		×				×	×
x_2		×		×	×	×	×	×
x_3	×		×		×			
x_4		×						×
x_5	×	×	×					×
x_6		×			×	×	×	×
x_7	×	×			×	×	×	
x_8	×			×	×		×	×

(e) R_e

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
z_1	×	×			×	×	×	×
z_2	×	×					×	×
z_3	×	×	×	×	×			
z_4	×	×	×	×		×		
z_5	×		×	×		×	×	×
z_6		×	×	×	×	×	×	×
z_7			×	×			×	×
z_8			×	×			×	×

are corresponding binary relations on $V_a, V_b, \ldots, V_e$ in Table 1. In fact, for any $m \in AT$, R_m in Table 2 reflects relationships among various evaluations in V_m(in Table 1).

From the above description of complex information systems, we can see the relation in domain of any attribute may be extremely complex, that often leads to high time complexity and space complexity in the process of knowledge acquisition. For above mentioned reasons this paper introduces GrC which has unique advantage in modeling and analysis of the large and complex data, and discusses the classification problem of domain in the different granulation, it can effectively reduce the complexity of knowledge acquisition to a certain extent. In recent years, GrC plays an important role in knowledge discovery, data mining and soft computing, which helps to solve the problem more scientific, rational and easy. For example, when the problem is too complex or costly, in order to better understand and solve problems rather than submerging in unnecessary details, larger granulations help to identify useful information and hide some specific details, and the problem can be solved from the overall picture. It can be said GrC has the unique advantage in large, complex data modeling and analysis.

Typically, attribute values can simply be classified as 'numeric' and 'non-numeric'. For any attribute $m \in AT$, if the type of values in V_m is 'non-numeric', we need to determine whether any two values are similar and how to measure the degree of the similarity. For example, We can use the method similar to the AHP theory [7], which employs integers $1, \ldots, 9$ as the metric values. In particular, human's subjective judgments usually conclude following levels: no similarity, weak similarity, similarity, strong similarity and complete similarity, which were denoted as 1,3,5,7,9 correspondingly. and then another four levels between above levels are denoted as 2,4,6,8 respectively.

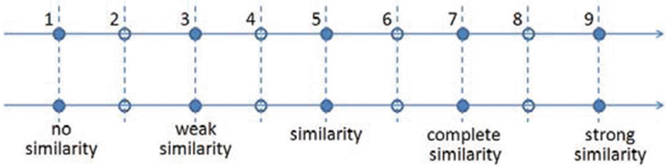

Fig. 1. Similarities at different levels.

If the type of values in V_m is 'non-numeric', then based on human's subjective judgment the degree of similarity between $v_i, v_j \in V_m$ is:

$$\tilde{r}_{ij} = \frac{1}{8} \cdot (s_{ij} - 1)\,, where\ s_{ij} \in \{1, 2, \ldots, 9\}$$

If the type of values in V_m is 'numeric', then the degree of the similarity between $v_i, v_j \in V_m$ is defined as follows:

$$\tilde{r}_{ij} = 1 - \frac{1}{\max\{v_1, v_2, \ldots, v_n\}} |v_i - v_j|$$

Based on above discussion, we can define a similarity relation matrix $\tilde{R}_m = (\tilde{r}_{ij})_{n \times n}$. Obviously, the fuzzy clustering algorithm can be used to calculate the fuzzy equivalence relation matrix R_m of $\tilde{R}_m$. And further by introducing parameter $\lambda \in [0, 1]$, we can obtain the λ-cut equivalence relation matrix $R_m^\lambda = (r_{ij})_{n \times n}$. In fact R_m^λ is a equivalence relation on V_m, and V_m/R_m^λ is a partition of V_m. Let $V_m^\lambda = V_m/R_m^\lambda = \{X_1, X_2, \cdots, X_l\}$, then the knowledge granulation of R_m^λ is defined as:

$$\nu(R_m^\lambda) = \frac{1}{\mathrm{n}^2}\sum_{i=1}^{l}|X_i|^2$$

$X_i \in V_m^\lambda$ is called a granule or a class.

The process of the classification of domain of any attribute is not the keystone of the paper, so we only give a brief account. In the following, we assume the classification result is known, and overlook the calculating process. In this paper, the classification results of $V_a \ldots V_e$ in Table 1 are shown as follows separately.

- $V_a^\lambda : u_1,\ u_2,\ u_3u_6,\ u_4u_5,\ u_7,\ u_8;$
- $V_b^\lambda : v_1,\ v_2,\ v_3v_4,\ v_5,\ v_6,\ v_7,\ v_8;$
- $V_c^\lambda : w_1w_2,\ w_3w_4w_5w_6,\ w_7w_8;$
- $V_d^\lambda : x_1x_2,\ x_3,\ x_4,\ x_5, x_6x_8,\ x_7;$
- $V_e^\lambda : z_1,\ z_2,\ z_3,\ z_4,\ z_5z_6,\ z_7,\ z_8$

4 One-Valued Formal Contexts

As we know that operators in Definition 1 only can be used for one-valued contexts. Based on this, before applying operators in Definition 1 we need to translate complex information systems into one-valued formal contexts.

Definition 2. *For every attribute $m \in AT$, we can further expand R_m to R_m^λ, where R_m^λ is described as follows: for any $v, w \in V_m$, if*

$$\frac{1}{|X| \times |Y|} \times |(X \times Y) \cap R_m| \geq \delta,\ \textit{where}\ v \in X \in V_m^\lambda\ \textit{and}\ w \in Y \in V_m^\lambda$$

then $(v, w) \in R_m^\lambda$. In the following, we say R_m^λ is a variable-precision relation of R_m.

For example, let $\delta = 0.6$, then base on Definition 3 and the classification result in Sect. 3, variable-precision relations shown in Table 3 can be obtained from binary relations in Table 2. In addition, we can easily discover from Fig. 2 and Fig. 3 that R_c^λ is simpler and more intuitive than R_c.

From above discussion, some conclusions can be inferred immediately as follows:

- when λ is invariable, if $\delta \downarrow$, then $|R_m^\lambda| \uparrow$.
- if $\lambda \uparrow$, then $|V_m^\lambda| \uparrow$.

In fact, in $IS = (U, AT, \Re)$, for any $v \in V_m$, this paper only care about the relationship between it and other value in V_m rather than the size of it. Based on this we try to translate IS into an one-valued formal context, which has filtered out some redundant information(we do not care about in IS). There are some differences between the above procedure with the classic scaling procedure in FCA, such as compared to classic scaling procedure the above procedure removes some redundant information, which we do not care about.

Table 3. The $\Re$ of complex information system in Table 1

(a) R_a^λ

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
u_1	×	×	×	×	×	×		
u_2	×	×	×	×	×	×		
u_3	×	×	×	×	×	×	×	×
u_4	×	×	×	×	×	×	×	×
u_5	×	×	×	×	×	×	×	×
u_6	×	×	×	×	×	×	×	×
u_7			×	×	×	×	×	×
u_8			×	×	×	×	×	×

(b) R_b^λ

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	×	×	×	×				
v_2	×	×	×	×				
v_3	×	×	×	×	×	×		
v_4	×	×	×	×	×	×		
v_5			×	×	×	×	×	×
v_6			×	×	×	×	×	×
v_7					×	×	×	×
v_8					×	×	×	×

(c) R_c^λ

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
w_1			×	×	×	×		
w_2			×	×	×	×		
w_3	×	×	×	×	×	×	×	×
w_4	×	×	×	×	×	×	×	×
w_5	×	×	×	×	×	×	×	×
w_6	×		×	×	×	×	×	×
w_7			×	×	×	×		
w_8			×	×	×	×		

(d) R_d^λ

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1					×	×	×	×
x_2					×	×	×	×
x_3								
x_4								
x_5	×	×			×	×	×	×
x_6	×	×			×	×	×	×
x_7	×	×			×	×	×	×
x_8	×	×			×	×	×	×

(e) R_e^λ

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
z_1	×	×			×	×	×	×
z_2	×	×			×	×	×	×
z_3	×	×	×	×				
z_4	×	×	×	×				
z_5			×	×	×	×	×	×
z_6			×	×	×	×	×	×
z_7			×	×			×	×
z_8			×	×			×	×

Definition 3. *Let* $IS = (U, AT, \Re)$ *be a complex information system,* $0 \leq \lambda \leq 1$, $K_\lambda = (U \times U, AT, J_\lambda)$ *is called an one-valued context deduced from* IS*, where* J_λ *is described as: for any* $x, y \in U$ *and* $m \in AT$*, there exists*

$$(x, y)J_\lambda m \Leftrightarrow (v, w) \in R_m^\lambda, \textit{where } f(x, m) = v,\ f(y, m) = w$$

By the above translation rule the one-valued formal context can be deduced from Table 1, which is shown in Table 4.

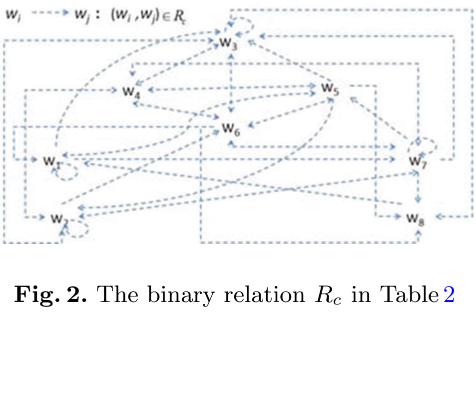

Fig. 2. The binary relation R_c in Table 2

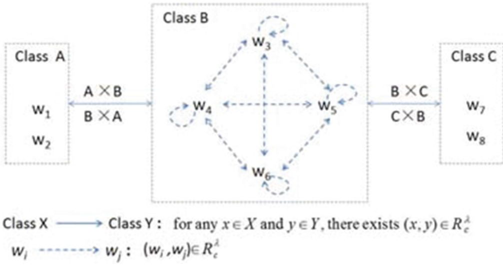

Fig. 3. The variable-precision relation R_c^λ of R_c in Table 2

Table 4. The one-valued context deduced from Table 1

	a	b	c	d	e
(1, 1)	×	×			×
(1, 2)	×	×			×
(1, 3)	×	×	×		
(1, 4)	×	×	×		
(1, 5)	×		×	×	×
(1, 6)	×		×	×	×
(1, 7)				×	×
⋮	⋮	⋮	⋮	⋮	⋮
(8, 6)	×	×	×		×
(8, 7)	×	×		×	×
(8, 8)	×	×		×	×

5 Algebraic Structure in the Complex Information System

In recent years many scholars discussed the granularity model based on equivalence relation, dominance relation, similarity relation, tolerance relation, neighborhood relation and other complex relations. Based on above discussions this paper expands above common binary relations to the more general binary relation

$$R_B^\lambda = \{(x, y) \in U \times U \mid \forall m \in B,\ f(x, m) = v, f(y, m) = w,\ (v,\ w) \in R_m^\lambda\}$$

In K_λ, it's obvious that operators in Definition 1 are described as: for any $R \subseteq U \times U$,

$$R' = \{m \in M \mid ((x, y), m\,) \in J_\lambda, \forall (x, y) \in R\}$$

and for any $B \subseteq AT$,

$$B' = \{(x, y) \in U \times U \mid ((x, y),\ m) \in J_\lambda,\ \forall m \in B\,\}$$

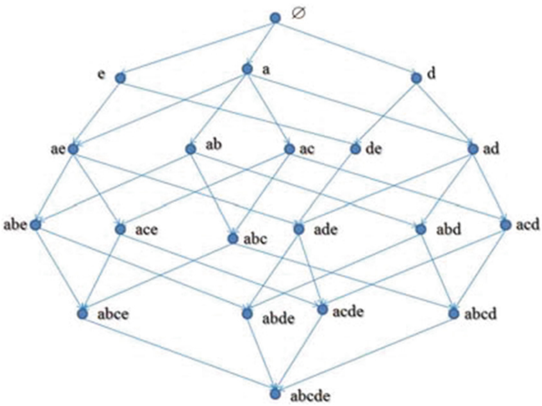

Fig. 4. A λ relational concept lattice with respect to Table 4

if $R' = B$ and $B' = R$, then we say (R, B) is a λ-relational concept. Let (R_1, B_1) and (R_2, B_2) be λ-relational concepts, we define

$$(R_1, B_1) \leq_\lambda (R_2, B_2) \Leftrightarrow R_1 \subseteq R_2 \Leftrightarrow B_2 \subseteq B_1$$

Obviously, the ordered set $(\mathscr{B}(K_\lambda), \leq_\lambda)$ is a complete lattice, which is called a λ relational concept lattice. For example, from Table 4 we can obtain a lattice shown in Fig. 4 (the extent is not easy to be expressed, so only the intent of concept is given).

Theorem 1. *In $K_\lambda = (U \times U, AT,\ J_\lambda)$, the following statements are true*

(1) *Let $B \subseteq AT$, then $B' = R_B^\lambda$;*
(2) *If $(R, B) \in \mathscr{B}(K_\lambda)$, then $R = R_B^\lambda$;*
(3) *Let $B, D \in AT$, if $B' = D'$, then $R_B^\lambda = R_D^\lambda$.*

Proof. The conclusion (1) can be immediately inferred by the definition of operators in K_λ. (2) If $(R, B) \in \mathscr{B}(K_\lambda)$, then we can obtain $R = B'$. In addition, based on conclusion (1) there exists $B' = R_B^\lambda$. Together with $R = B'$ we can see $R = R_B^\lambda$. Therefore (2) is true. (3) We can see from conclusion (1) that $R_B^\lambda = R_D^\lambda$ is true.

In summary, $(\mathscr{B}(K_\lambda), \leq_\lambda)$ can organize all the general relations on U in the form of lattice, which can be viewed as the algebraic structure contained in the complex information system IS.

6 Knowledge Acquisition in Complex Information Systems

We know that solutions to problems of reduct, core and dependency etc. in FCA occupy important position. Based on this, in complex information systems, we propose a solution based on operators in K_λ.

Table 5. A set of dependencies in Table 1

$bc \to abc$	$bcde \to abcde$	$cde \to acde$	$bde \to abde$
$b \to ab$	$c \to ac$	$bd \to abd$	$ce \to ace$
$cd \to acd$	$be \to abe$	$bce \to abce$	$bcd \to abcd$

Definition 4. *In $IS = (U, AT, \Re)$, let $B \subseteq AT$, if $m \in B$ and $R^{\lambda}_{B} \neq R^{\lambda}_{B-m}$, we say m is indispensable in B; if any $m \in B$ is indispensable, we say B is independent, the set of all independent sets is denoted by IND_{λ}; let $C \subseteq B$, if C is independent and satisfies $R^{\lambda}_{B} = R^{\lambda}_{C}$, then C is called a reduct of B, the set of all reducts in B is denoted as $RED_{\lambda}(B)$; let$B \subseteq AT$, the set of all indispensable attributes in B is called the core of B denoted as $CORE_{\lambda}(B)$; let$B, D \subseteq AT$, for $\forall x, y \in U$, if*

$$(\,\forall m \in B,\ (f(x,m), f(y,m)\,) \in R^{\lambda}_{m}\,) \Rightarrow (\,\forall n \in D,\ (f(x,n), f(y,n)\,) \in R^{\lambda}_{n}\,)$$

we say D is dependent on B, and $B \to D$ is a dependency. If D is not dependent on B, then the corresponding degree of dependence is defined as follows:

$$\gamma_B(D) = \frac{1}{|B'|} \times |B' \cap D'|$$

For example, when $B = \{a, b\}$ and $D = \{c, d, e\}$, it's not hard to calculate D is not dependent on B, and $\gamma_B(D) = 0.4$.

Theorem 2. *In above Definition there exist following conclusions*

(1) $IND_{\lambda} = \{B \subseteq AT \mid \forall a \in B, (B - a)' \neq B'\}$;
(2) $CORE_{\lambda}(B) = \{a \in B \mid (B - a)' \neq B'\}$;
(3) *if C is the minimal set in B satisfying $C' = B'$, then $C \in RED_{\lambda}(B)$.*
(4) $B \to D \Leftrightarrow B' \subseteq D'$.

Proof. Conclusions can be immediately inferred by Definition 4 and Theorem 1.

Theorem 3. *Let $B, D \subseteq AT$, $B \to D$. If $B \subseteq B_1$ and $D_1 \subseteq D$, then$B_1 \to D_1$.*

Proof. $B' \subseteq D'$ can be inferred from $B \to D$. In addition, $B'{}_1 \subseteq B'$ and $D' \subseteq D'_1$ can be deduced from $B \subseteq B_1$ and $D_1 \subseteq D$. Hence $B'_1 \subseteq D'_1$, that is, $B_1 \to D_1$ is true.

Definition 5. *If $B \to D$ can be inferred from $B_1 \to D_1$ by some inference rule ξ, we say $B \to D$ can be ξ-inferred from $B_1 \to D_1$. In this case, we call $B \to D$ is relatively redundant.In addition, if $B = D$, we say $B \to D$ is absolutely redundant.*

Normally, the number of dependencies is quite large. In the following, to find valuable dependencies we need to remove some redundant dependencies. In fact, from Theorem 3 we can define a inference as follows:

$$\frac{B \subseteq B_1,\ D_1 \subseteq D,\ B \to D}{B_1 \to D_1}$$

and then based on above inference, we can remove all relatively redundant dependencies. For example, in Table 1, if we remove all relatively redundant dependencies based on above inference and absolutely redundant dependencies, and then can obtain a smaller set of dependencies shown in Table 5.

7 Conclusions

As a tool for data analysis, FCA possesses good mathematical properties, and has attracted great concerns in recent years. With the research background of complex information systems this paper proposes a rough set model based on FCA, in which common relations on universe such as equivalence relation, dominance relation, similarity relation, tolerance relation and neighborhood relation are expanded to a general binary relation and problems of algebraic structure, reduct, core, dependency are discussed. In fact, the algebraic structure in the complex information system discussed in this paper is a lattice structure that can organize binary relations on universe in the form of lattice organically. Finally, how to eliminate redundant dependencies is studied. Particularly, since the relation on domain of any attribute may be extremely complex, often cause the high time complexity and space complexity to solve above problems. For above reason this paper introduces granular computing(GrC), which can effectively reduce the complexity to a certain extent. In general, this paper not only provides a useful method for applying the formal concept analysis to the complex information system, but also offers a new idea for the extension of the rough set model. Exploration of wider combinations of FCA and rough set theory will also be one focus of our future research.

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