This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

Three-way Group Conflict Analysis Based on Pythagorean Fuzzy Set Theory

Guangming Lang, Duoqian Miao and Hamido Fujita

***Abstract*—In some real-world situations, Pythagorean fuzzy sets are more powerful and effective than intuitionistic fuzzy sets to describe vague and uncertain information, and there are many Pythagorean fuzzy information systems for conflicts in which attitudes of agents on issues are depicted by Pythagorean fuzzy numbers. In this paper, we first provide the concepts of positive, neutral and negative alliances with two thresholds and employ examples to illustrate how to compute positive, neutral and negative alliances in Pythagorean fuzzy information systems for conflicts. Then we focus on three-way conflict analysis based on Bayesian minimum risk theory and explore examples to show how to compute the positive, neutral and negative alliances with a Pythagorean fuzzy loss function given by an expert. Finally, we study how to calculate positive, neutral and negative alliances with group decision theory and take examples to demonstrate how to construct the positive, neutral and negative alliances with a group of Pythagorean fuzzy loss functions given by more experts.**

***Index Terms*—Bayesian Minimum Risk Theory, Conflict Analysis, Pythagorean Fuzzy Sets, Pythagorean Fuzzy Information System, Pythagorean Fuzzy Loss Function.**

I. Introduction

CONFLICTS are undoubtedly one of the most essential characteristics of Human Society, and the study of which is of utmost significance both theoretically and practically. Especially, conflict analysis [1]–[21], which plays an important role in many fields such as business, political and legal disputes, investigates conflict structures with conflict, neutrality and alliance relations and gives some guidance to conflict resolution. For example, Pawlak [1] initially considered the auxiliary functions and distance functions and offered deeper insight into the structure of conflicts. Cholvy et al. [4] proposed a method for estimating the relative reliability of information sources. Deja [7] transformed conflict analysis problems and conflict-resolving problems into Boolean-reasoning problems with the rough sets and Boolean reasoning methods. Jabbour et al. [9] provided the notion of conflicting variable and investigated quantifying conflicts in propositional logic through prime implicates. Ramanna et al. [13] studied how to model a combination of complex situations among agents where there are disagreements leading to a conflict situation. Silva and Almeida-Filho [14] presented a multicriteria approach for analysis of conflicts in evidence theory. Skowron and Deja [15] explained the nature of conflict and defined the conflict situation model in a way to encapsulate the conflict components in a clear manner. Sun et al. [18] proposed a conflict analysis decision model and developed a matrix approach for conflict analysis based on rough set theory over two universes. Yang et al. [19] investigated evidence conflict and belief convergence based on the analysis of the degree of coherence between two sources of evidence and illustrated the stochastic interpretation for basic probability assignments. Yu et al. [20] provided the supporting probability distance to characterize the differences among bodies of evidence and gave a new combination rule for the combination of the conflicting evidence. Zhu and Wang [21] studied the problems of conflicts of interest in database access security using granular computing based on covering rough set theory.

Three-way decision theory, proposed by Yao [22] for decision making with less risks, promotes thinking and problem solving in threes such as using three regions, three elements, three views, three levels and three stages. Many scholars [23]–[38] have developed three-way decision theory in theoretical and practical aspects, which has become a new mathematical tool to deal with uncertain information and problems. For instance, Chen et al. [23] focused on three-way decision support for diagnosis on focal liver lesions. Feng et al. [24] studied uncertainty and reduction of variable precision multigranulation fuzzy rough sets based on three-way decisions. Hu et al. [25] provided two types of three-way decisions in three-way decision spaces and discussed properties of the three-way decisions. Khan et al. [26] introduced a three-way approach for learning rules in automatic knowledge-based topic models. Li et al. [30] presented cost-sensitive sequential three-way decision modeling using a deep neural network. Qian et al. [31] investigated attribute reduction for sequential three-way decisions under dynamic granulation. Sun et al. [32] studied three-way group decision making based on multigranulation fuzzy decision-theoretic rough sets over two universes. Xu et al. [33] provided a three-way decision model with probabilistic rough sets for stream computing. Yang et al. [37] proposed a unified model of sequential three-way decisions and multilevel incremental processing.

Pythagorean fuzzy sets (PFSs), introduced by Yager [39] for describing uncertainty, are considered as a generalization

Manuscript received September 28, 2017. This work is supported by the National Natural Science Foundation of China (Nos.61603063, 61273304,11526039), Specialized Research Fund for the Doctoral Program of Higher Education of China (No.201300721004), China Postdoctoral Science Foundation (NO.2015M580353), China Postdoctoral Science special Foundation (NO.2016T90383)(Corresponding authors: Duoqian Miao).

Guangming Lang is with the School of Mathematics and Statistics, Changsha University of Science and Technology, Changsha, Hunan 410114, P.R. China; Duoqian Miao is with the Department of Computer Science and Technology and the Key Laboratory of Embedded System and Service Computing, Ministry of Education of China, Tongji University, Shanghai, 201804, P.R. China; Hamido Fujita is with the Faculty of Software and Information Science, Iwate Prefectural University, Iwate 020-0693, Japan, (e-mail: langguangming1984@tongji.edu.cn, dqmiao@tongji.edu.cn, HFujita-799@acm.org).

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

of intuitionistic fuzzy sets (IFSs) and characterized by a membership degree and a non-membership degree satisfying the condition that the square sum of its membership degree and non-membership degree is equal to or less than 1. Many investigations [39]–[58] have focused on Pythagorean fuzzy sets, which have more powerful ability than IFSs to model the uncertain information in decision making problems. For example, Beliakov and James [41] provided the averaging aggregation functions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs. Bustince et al. [42] investigated a historical account of types of fuzzy sets and discussed their relationships. Peng and Yang [45] developed a Pythagorean fuzzy superiority and inferiority ranking method to solve uncertainty multiple attribute group decision making problem. Peng and Selvachandran [46] presented an overview on Pythagorean fuzzy sets with aim of offering a clear perspective on the different concepts, tools and trends related to their extension and provided two novel algorithms in decision making problems under Pythagorean fuzzy environment. Reformat and Yager [50] proposed a novel collaborative-based recommender system that provides a user with the ability to control a process of constructing a list of suggested items using Pythagorean fuzzy sets. Ren et al. [51] extended the TODIM approach to solve the MCDM problems with Pythagorean fuzzy information and analyzed how the risk attitudes of the decision makers exert the influence on the results of MCDM under uncertainty. Wu and Liu [52] proposed a knowledge-augmented logical analysis framework for policy conflicts in order to make services collaboration possible and smooth. Zhang [56] presented a hierarchical QUALIFLEX approach with the closeness index-based ranking methods for multi-criteria Pythagorean fuzzy decision analysis. Zhang et al. [58] introduced the models of Pythagorean fuzzy rough sets over two universes and Pythagorean fuzzy multi-granulation rough sets over two universes.

In conflict analysis, we are mainly interested in finding the relationship among agents taking part in the dispute, and study what measures can be taken for solving the conflict. In this study, we investigate how to compute positive, neutral and negative alliances based on Pythagorean fuzzy set theory. The motivations and innovations of this study are given by answering the following three questions:

(1) Why study conflicts based on Pythagorean fuzzy set theory? In practice, Pythagorean fuzzy sets are more suitable than intuitionistic fuzzy sets for describing attitudes of agents in conflicts. For instance, when a person expresses his preference about the degree of an issue, he gives the degree to support this issue as $\frac{\sqrt{3}}{2}$, and the degree to against this issue as $\frac{1}{2}$, and we have $(\frac{\sqrt{3}}{2})^2+(\frac{1}{2})^2 = 1$ and $\frac{\sqrt{3}}{2}+\frac{1}{2} > 1$, and intuitionistic fuzzy sets can not work in this situation. Furthermore, Pythagorean fuzzy loss functions are more accurate than intuitionistic fuzzy loss functions for measuring losses and risks in decision making problems, which helps people make decisions with less losses and risks.

(2) Why investigate conflicts with three-way decision theory and group decision theory? In conflict situations, we ask the agents to specify their views from disagreement, neutral and agreement and classify all agents into conflict set, neutral set and alliance set of an agent with conflict, neutral and alliance relations, respectively. We also find that three-way decision theory partitions all agents into positive, boundary and negative regions based on Bayesian minimum risk theory, which is consistent with the thought of conflict analysis. Moreover, we see that Pythagorean fuzzy loss functions are given by experts, and different experts have different opinions for the same problem and give different Pythagorean fuzzy loss functions. We employ a group of Pythagorean fuzzy loss functions given by many famous experts to calculate positive, neutral and negative alliances in conflict analysis so as to make decisions with less losses and risks.

(3) What are innovations of this study? We have not observed studies on Pythagorean fuzzy information systems for conflicts, where attitudes of agents are Pythagorean fuzzy numbers. The innovations of this study mainly include: (1) construct positive, neutral and negative alliances with three-way decision theory; (2) classify all agents into positive, neutral and negative alliances based on Bayesian minimum risk theory; (3) employ a group of Pythagorean fuzzy loss functions to compute positive, neutral and negative alliances with the minimum risk.

The contributions of this paper are shown as follows. Firstly, we provide the concept of Pythagorean fuzzy information system and employ an example to illustrate the difference between Pawlak information systems and Pythagorean fuzzy information systems. We provide the concepts of positive, neutral and negative alliances with two thresholds and employ several examples to illustrate how to compute the positive, neutral and negative alliances in Pythagorean fuzzy information systems for conflicts. Secondly, we provide the concept of Pythagorean fuzzy loss function for conflict analysis of Pythagorean fuzzy information systems, and illustrate mechanisms of computing the positive, neutral and negative alliances based on Bayesian minimum risk theory. We also employ several examples to illustrate how to compute the positive, neutral and negative alliances with a Pythagorean fuzzy loss function given by an expert. Thirdly, we demonstrate mechanisms of calculating the positive, neutral, and negative alliances for conflict analysis with group decision theory, and employ several examples to illustrate how to construct the positive, neutral and negative alliances with a group of Pythagorean fuzzy loss functions given by more experts.

The rest of this paper is organized as follows. Section II reviews the basic concepts of Pythagorean fuzzy sets and conflict analysis. Section III proposes the concepts of positive, neutral and negative alliances with two thresholds. Section IV focuses on three-way conflict analysis based on Bayesian minimum risk theory. Section V provides three-way group conflict analysis based on group decision theory. The conclusion is given in Section VI.

II. Preliminaries

In this section, we review the related concepts of Pythagorean fuzzy sets and conflict analysis.

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

A. Pythagorean Fuzzy Sets

Definition 2.1: [53] Let U be an arbitrary non-empty set, and a Pythagorean fuzzy set (PFS) P is a mathematical object of the form as follows:

$$P = \{< x, P(\mu_P(x), \nu_P(x)) > | x \in U\},$$

where $\mu_P(x), \nu_P(x) : U \to [0,1]$ such as $\mu_P^2(x) + \nu_P^2(x) \leq 1$, for every $x \in U$, $\mu_P(x)$ and $\nu_P(x)$ denote the membership degree and the non-membership degree of the element $x \in U$ in P, respectively.

For convenience, we denote the Pythagorean fuzzy number (PFN) and the hesitant degree as $\gamma = P(\mu_\gamma, \nu_\gamma)$ and $\pi_\gamma = \sqrt{1 - \mu_\gamma^2 - \nu_\gamma^2}$, respectively. Moreover, if $\gamma = P(\mu_\gamma, \nu_\gamma)$ satisfying $\mu_\gamma + \nu_\gamma \leq 1$, then γ is an intuitionistic fuzzy number, and the relationship between a Pythagorean fuzzy number and an intuitionistic fuzzy number is illustrated by Fig. 1. Therefore, Pythagorean fuzzy sets, as a generalization of intuitionistic fuzzy sets, are powerful for describing imprecise information.

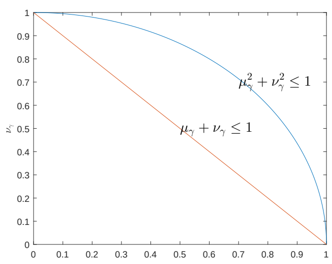


Fig. 1. The relationship between a Pythagorean fuzzy number and an intuitionistic fuzzy number.

Definition 2.2: [53] Let $\gamma_1 = P(\mu_{\gamma_1}, \nu_{\gamma_1})$ and $\gamma_2 = P(\mu_{\gamma_2}, \nu_{\gamma_2})$ be Pythagorean fuzzy numbers. Then a nature quasi-ordering on PFNs is defined as follows:

$$\gamma_1 \geq \gamma_2 \text{ if and only if } \mu_{\gamma_1} \geq \mu_{\gamma_2} \text{ and } \nu_{\gamma_1} \leq \nu_{\gamma_2}.$$

For convenience, we define the function $I(\gamma) = \gamma$ for any Pythagorean fuzzy number γ. So $\gamma_1 \geq \gamma_2 \Leftrightarrow I(\gamma_1) \geq I(\gamma_2)$. Moreover, Yager provided multiplication and summation operations for Pythagorean fuzzy numbers as follows:

(1) $k\gamma = P(\sqrt{1 - (1 - \mu_\gamma^2)^k}, \nu_\gamma^k)$;

(2) $\gamma_1 \oplus \gamma_2 = P(\sqrt{\mu_{\gamma_1}^2 + \mu_{\gamma_2}^2 - \mu_{\gamma_1}^2 * \mu_{\gamma_2}^2}, \nu_{\gamma_1} * \nu_{\gamma_2})$.

Definition 2.3: [57] Let $\gamma = P(\mu_\gamma, \nu_\gamma)$ be a Pythagorean fuzzy number. Then the score function S for γ is defined as follows:

$$S(\gamma) = \mu_\gamma^2 - \nu_\gamma^2.$$

We have that $-1 \leq S(\gamma) \leq 1$ for the Pythagorean fuzzy number γ. Especially, the score function is effective to discern Pythagorean fuzzy numbers.

Definition 2.4: [57] Let $\gamma_1 = P(\mu_{\gamma_1}, \nu_{\gamma_1})$ and $\gamma_2 = P(\mu_{\gamma_2}, \nu_{\gamma_2})$ be Pythagorean fuzzy numbers. Then the Euclidean distance d between γ_1 and γ_2 is defined as follows:

$$d(\gamma_1, \gamma_2) = \frac{1}{2}(|\mu_{\gamma_1}^2 - \mu_{\gamma_2}^2| + |\nu_{\gamma_1}^2 - \nu_{\gamma_2}^2| + |\pi_{\gamma_1}^2 - \pi_{\gamma_2}^2|).$$

Specially, we obtain the Euclidean distance between the Pythagorean fuzzy number $P(\mu_\gamma, \nu_\gamma)$ and the positive ideal PFN $\gamma^+ = P(1,0)$ as follows:

$$d(\gamma, \gamma^+) = \frac{1}{2}(1 - \mu_\gamma^2 + \nu_\gamma^2 + \pi_\gamma^2) = 1 - \mu_\gamma^2,$$

and the Euclidean distance between the Pythagorean fuzzy number $P(\mu_\gamma, \nu_\gamma)$ and the negative ideal PFN $\gamma^- = P(0,1)$ as follows:

$$d(\gamma, \gamma^-) = \frac{1}{2}(1 - \nu_\gamma^2 + \mu_\gamma^2 + \pi_\gamma^2) = 1 - \nu_\gamma^2.$$

Definition 2.5: [56] Let $\gamma = P(\mu_\gamma, \nu_\gamma)$ be a Pythagorean fuzzy number, $\gamma^+ = P(1,0)$ and $\gamma^- = P(0,1)$. Then the closeness index $\mathscr{P}$ for γ is defined as follows:

$$\mathscr{P}(\gamma) = \frac{d(\gamma, \gamma^-)}{d(\gamma, \gamma^+) + d(\gamma, \gamma^-)} = \frac{1 - \nu_\gamma^2}{2 - \mu_\gamma^2 - \nu_\gamma^2}.$$

We see that the closeness index $\mathscr{P}(\gamma)$ of γ is constructed based on the Euclidean distance between the Pythagorean fuzzy number $P(\mu_\gamma, \nu_\gamma)$ and the positive ideal PFN γ^+ and the Euclidean distance between the Pythagorean fuzzy number $P(\mu_\gamma, \nu_\gamma)$ and the negative ideal PFN γ^-. Especially, we have $0 \leq \mathscr{P}(\gamma) \leq 1$ for the Pythagorean fuzzy number γ.

Definition 2.6: [39] Let $\gamma = P(\mu_\gamma, \nu_\gamma)$ be a Pythagorean fuzzy number. Then the function F for γ is defined as follows:

$$F(\gamma) = \frac{1}{2} + \sqrt{\mu_\gamma^2 + \nu_\gamma^2} * (\frac{1}{2} - \frac{2\arccos(\frac{\mu_\gamma}{\sqrt{\mu_\gamma^2 + \nu_\gamma^2}})}{\pi}).$$

The function F provides an effective approach to comparing Pythagorean fuzzy numbers. Moreover, by Definitions 2.2-2.6, we provide the comparison law for discerning Pythagorean fuzzy numbers as follows.

Definition 2.7: Let $\gamma_1 = P(\mu_{\gamma_1}, \nu_{\gamma_1})$ and $\gamma_2 = P(\mu_{\gamma_2}, \nu_{\gamma_2})$ be Pythagorean fuzzy numbers, and $\bullet = I, S, \mathscr{P}, F$. Then

(1) If $\bullet(\gamma_1) > \bullet(\gamma_2)$, then γ_1 is bigger than γ_2, denoted by $\gamma_1 \succ_\bullet \gamma_2$;

(2) If $\bullet(\gamma_1) < \bullet(\gamma_2)$, then γ_1 is smaller than γ_2, denoted by $\gamma_1 \prec_\bullet \gamma_2$;

(3) If $\bullet(\gamma_1) = \bullet(\gamma_2)$, then γ_1 is equal to γ_2, denoted by $\gamma_1 \sim_\bullet \gamma_2$.

We employ the following example to illustrate how to discern Pythagorean fuzzy numbers with Definition 2.7.

Example 2.8: (1) Taking $\gamma_1 = P(\frac{\sqrt{5}}{3}, \frac{1}{3})$ and $\gamma_2 = P(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3})$, we have $I(\gamma_1) = P(\frac{\sqrt{5}}{3}, \frac{1}{3})$ and $I(\gamma_2) = P(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3})$. Since $\frac{\sqrt{5}}{3} > \frac{\sqrt{2}}{3}$ and $\frac{1}{3} < \frac{\sqrt{2}}{3}$, then we have $\gamma_1 \succ_I \gamma_2$.

Sometimes, we can not discern Pythagorean fuzzy numbers with Definition 2.2. For example, it does not work for $\gamma_1 = P(\frac{\sqrt{5}}{3}, \frac{\sqrt{2}}{3})$ and $\gamma_2 = P(\frac{\sqrt{2}}{3}, \frac{1}{3})$.

(2) Taking $\gamma_1 = P(\frac{\sqrt{5}}{3}, \frac{\sqrt{2}}{3})$ and $\gamma_2 = P(\frac{\sqrt{2}}{3}, \frac{1}{3})$, by Definition 2.3, we have $S(\gamma_1) = S(P(\frac{\sqrt{5}}{3}, \frac{\sqrt{2}}{3})) = \frac{3}{9}$ and $S(\gamma_2) = S(P(\frac{\sqrt{2}}{3}, \frac{1}{3})) = \frac{1}{9}$. Therefore, we have $\gamma_1 \succ_S \gamma_2$.

We see that the score function fails to discern some Pythagorean fuzzy numbers. For example, for $\gamma_1 = P(\frac{\sqrt{5}}{3}, \frac{2}{3})$ and $\gamma_2 = P(\frac{2}{3}, \frac{\sqrt{3}}{3})$, we have

$$S(\gamma_1) = (\frac{\sqrt{5}}{3})^2 - (\frac{2}{3})^2 = \frac{1}{9} \text{ and } S(\gamma_2) = (\frac{2}{3})^2 - (\frac{\sqrt{3}}{3})^2 = \frac{1}{9}.$$

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

(3) Taking $\gamma_1 = P(\frac{\sqrt{5}}{3}, \frac{2}{3})$ and $\gamma_2 = P(\frac{2}{3}, \frac{\sqrt{3}}{3})$, by Definition 2.5, we have

$$\mathscr{P}(\gamma_1) = \frac{1-\frac{4}{9}}{2-\frac{5}{9}-\frac{4}{9}} = \frac{5}{9} \text{ and } \mathscr{P}(\gamma_2) = \frac{1-\frac{3}{9}}{2-\frac{4}{9}-\frac{3}{9}} = \frac{6}{11}.$$

Therefore, we get $\gamma_1 \succ_{\mathscr{P}} \gamma_2$.

(4) Taking $\gamma_1 = P(\frac{\sqrt{5}}{3}, \frac{2}{3})$ and $\gamma_2 = P(\frac{2}{3}, \frac{\sqrt{3}}{3})$, by Definition 2.6, we have

$$\begin{aligned} F(\gamma_1) &= \frac{1}{2} + \sqrt{\mu_{\gamma_1}^2 + \nu_{\gamma_1}^2} * (\frac{1}{2} - \frac{2\arccos(\frac{\mu_{\gamma_1}}{\sqrt{\mu_{\gamma_1}^2+\nu_{\gamma_1}^2}})}{\pi}); \\ &\approx 0.5355; \\ F(\gamma_2) &= \frac{1}{2} + \sqrt{\mu_{\gamma_2}^2 + \nu_{\gamma_2}^2} * (\frac{1}{2} - \frac{2\arccos(\frac{\mu_{\gamma_2}}{\sqrt{\mu_{\gamma_2}^2+\nu_{\gamma_2}^2}})}{\pi}); \\ &\approx 0.5402. \end{aligned}$$

Therefore, we get $\gamma_2 \succ_F \gamma_1$.□.

We observe that there are four types of functions for comparing Pythagorean fuzzy numbers. If we choose one of them to discern Pythagorean fuzzy numbers, and it does not work, then the other functions can be applied in practical situations.

B. Conflict Analysis

Definition 2.9: [1] An information system is a 4-tuple $S = (U, A, V, f)$, where $U = \{x_1, x_2, ..., x_n\}$ is a finite set of agents, A is a finite set of issues, $V = \{V_c \mid c \in A\}$, where V_c is the set of values of issue c, and $card(V_c) > 1$, f is a function from $U \times A$ into V.

The classical information system given by Definition 2.9 is called Pawlak information system, and information systems mentioned in this section are Pawlak information systems.

Definition 2.10: [2] Let $S = (U, A, V, f)$ be an information system. Then the auxiliary function $\phi_c(x, y)$ for any $c \in A$ is defined as follows:

$$\phi_c(x,y) = \begin{cases} 1, & \text{if } \quad c(x)\cdot c(y) = 1 \vee x = y; \\ 0, & \text{if } \quad c(x)\cdot c(y) = 0 \wedge x \neq y; \\ -1, & \text{if } \quad c(x)\cdot c(y) = -1, \end{cases}$$

where $c(x)$ and $c(y)$ denote issue values of x and y on c.

If $\phi_c(x, y) = 1$, then x and y have the same opinion about issue c; if $\phi_c(x, y) = 0$, then it means that x or y has a neutral opinion about issue c; and if $\phi_c(x, y) = -1$, then x and y have different opinions about issue c.

Example 2.11: [2] TABLE I shows the information system for the Middle East conflict, where x_1, x_2, x_3, x_4, x_5 and x_6 denote six countries, c_1, c_2, c_3, c_4, c_5 and c_6 denote six issues. For example, $c_1(x_1) = -1$ denotes the agent x_1 is against the issue c_1, and $c_1(x_2) = +1$ denotes the agent x_2 supports the issue c_1, and $c_1(x_4) = 0$ denotes the agent x_4 is neutral to the issue c_1.

TABLE I
INFORMATION SYSTEM FOR THE MIDDLE EAST CONFLICT.

U	c_1	c_2	c_3	c_4	c_5
x_1	−1	+1	+1	+1	+1
x_2	+1	0	−1	−1	−1
x_3	+1	−1	−1	−1	0
x_4	0	−1	−1	0	−1
x_5	+1	−1	−1	−1	−1
x_6	0	+1	−1	0	+1

Remarks: x_1, x_2, x_3, x_4, x_5 and x_6 denote Israel, Egypt, Palestine, Jordan, Syria and Saudi Arabia, respectively. Moreover, c_1 means Autonomous Palestinian state on the West Bank and Gaza; c_2 denotes Israeli military outpost along the Jordan River; c_3 stands for Israel retains East Jerusalem; c_4 is Israeli military outposts on the Golan Heights; c_5 denotes Arab countries grant citizenship to Palestinians who choose to remain within their borders.

Definition 2.12: [2] Let $S = (U, A, V, f)$ be an information system. Then the distance function $\rho_A(x, y)$ for $x, y \in U$ is defined as follows:

$$\rho_A(x,y) = \frac{\sum_{c\in A}\phi_c^*(x,y)}{|A|},$$

where

$$\phi_c^*(x,y) = \frac{1-\phi_c(x,y)}{2} = \begin{cases} 0, & \text{if } \quad c(x)\cdot c(y) = 1 \vee x = y; \\ 0.5, & \text{if } \quad c(x)\cdot c(y) = 0 \wedge x \neq y; \\ 1, & \text{if } \quad c(x)\cdot c(y) = -1. \end{cases}$$

After that, Pawlak provided the conflict, neutral and allied relations for conflict analysis with Definition 2.12 as follows.

Definition 2.13: [2] Let $S = (U, A, V, f)$ be an information system, and the distance function $\rho_A(x, y)$ for $x, y \in U$. Then a pair x and y is said to be

(1) conflict if $\rho_A(x, y) > 0.5$;

(2) neutral if $\rho_A(x, y) = 0.5$;

(3) allied if $\rho_A(x, y) < 0.5$.

Pawlak also proposed the allied, conflict and neutral sets as follows.

Definition 2.14: [2] Let $S = (U, A, V, f)$ be an information system. Then the conflict, neutral and allied sets of $x \in U$ are defined as follows:

(1) $CO(x) = \{y \in U \mid \rho_A(x, y) > 0.5\}$;

(2) $NE(x) = \{y \in U \mid \rho_A(x, y) = 0.5\}$;

(3) $AL(x) = \{y \in U \mid \rho_A(x, y) < 0.5\}$.

We classify all agents with respect to x into three parts: $CO(x), NE(x)$ and $AL(x)$. Since decision-theoretic rough set theory is a powerful mathematical tool for depicting ambiguous information, Lang et al. [27] investigated conflict analysis using decision-theoretic rough set theory, which actually provides constructive advice for decision making with less loss.

Definition 2.15: [27] Let $S = (U, A, V, f)$ be an information system, and $0 \leq \beta \leq \alpha \leq 1$. For any $x \in U$, the probabilistic conflict, neutral and allied sets $CO_\beta^\alpha(x)$, $NE_\beta^\alpha(x)$ and $AL_\beta^\alpha(x)$ of x are defined as follows:

(1) $CO_\beta^\alpha(x) = \{y \in U \mid \rho_A(x, y) > \alpha\}$;

(2) $NE_\beta^\alpha(x) = \{y \in U \mid \alpha \geq \rho_A(x, y) \geq \beta\}$;

(3) $AL_\beta^\alpha(x) = \{y \in U \mid \rho_A(x, y) < \beta\}$.

In some practical situations, Pythagorean fuzzy sets are effective for describing uncertain information, and there are

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

some Pythagorean fuzzy information systems for conflicts, where all issue values are Pythagorean fuzzy numbers, and there has been relatively little progress in developing effective methods for studying Pythagorean fuzzy information systems for conflicts.

III. Conflict analysis of Pythagorean fuzzy information systems

In this section, we investigate Pythagorean fuzzy information systems for conflicts.

Definition 3.1: A Pythagorean fuzzy information system is a 4-tuple $S = (U, A, V, f)$, where $U = \{x_1, x_2, ..., x_n\}$ is a finite set of agents, $A = \{c_1, c_2, ..., c_l\}$ is a finite set of issues, $V = \{V_c \mid c \in A\}$, where V_c is the set of issue values on c, all issue values are Pythagorean fuzzy numbers, and f is a function from $U \times A$ into V.

We see that Pythagorean fuzzy information systems, as a generalization of Pawlak information systems, represent all available information and knowledge, where agents are measured by using a finite number of issues and issue values are PFNs, which provides more information than intuitionistic fuzzy information systems. Furthermore, we provide matrix representation $M(S)$ of the Pythagorean fuzzy information system S for conflict analysis as follows:

$$M(S) = \begin{bmatrix} P(\mu_{11},\nu_{11}) & P(\mu_{12},\nu_{12}) & \cdots & P(\mu_{1l},\nu_{1l}) \\ P(\mu_{21},\nu_{21}) & P(\mu_{22},\nu_{22}) & \cdots & P(\mu_{2l},\nu_{2l}) \\ \vdots & \vdots & \ddots & \vdots \\ P(\mu_{n1},\nu_{n1}) & P(\mu_{n2},\nu_{n2}) & \cdots & P(\mu_{nl},\nu_{nl}) \end{bmatrix},$$

where n and l are the numbers of agents and issues, respectively.

Example 3.2: (1) We employ a Pythagorean fuzzy information system depicted by TABLE II to show the Middle East conflict, where x_1, x_2, x_3, x_4, x_5 and x_6 denote six agents, c_1, c_2, c_3, c_4, c_5 and c_6 denote six issues. For example, we have $c_1(x_1) = P(\mu_P(x_1), \nu_P(x_1)) = P(1.0, 0.0)$, where $\mu_P(x_1) = 1.0$ denotes the support degree of the agent x_1 to the issue c_1, and $\nu_P(x_1) = 0.0$ denotes the against degree of the agent x_1 to the issue c_1; we have $c_5(x_6) = P(\mu_P(x_6), \nu_P(x_6)) = P(0.8, 0.4)$, where $\mu_P(x_6) = 0.8$ denotes the support degree of the agent x_6 to the issue c_5, and $\nu_P(x_6) = 0.4$ denotes the against degree of the agent x_6 to the issue c_5.

TABLE II
The Pythagorean fuzzy information system for the Middle East conflict.

U	c_1	c_2	c_3	c_4	c_5
x_1	$P(1.0, 0.0)$	$P(0.9, 0.3)$	$P(0.8, 0.2)$	$P(0.9, 0.1)$	$P(0.9, 0.2)$
x_2	$P(0.9, 0.1)$	$P(0.5, 0.5)$	$P(0.1, 0.9)$	$P(0.3, 0.8)$	$P(0.1, 0.9)$
x_3	$P(0.1, 0.9)$	$P(0.1, 0.9)$	$P(0.2, 0.8)$	$P(0.1, 0.9)$	$P(0.5, 0.5)$
x_4	$P(0.5, 0.5)$	$P(0.1, 0.9)$	$P(0.3, 0.7)$	$P(0.5, 0.5)$	$P(0.1, 0.9)$
x_5	$P(0.9, 0.2)$	$P(0.4, 0.6)$	$P(0.1, 0.9)$	$P(0.1, 0.9)$	$P(0.3, 0.9)$
x_6	$P(0.0, 1.0)$	$P(0.9, 0.1)$	$P(0.2, 0.9)$	$P(0.5, 0.5)$	$P(0.8, 0.4)$

(2) From TABLE II, we have the Pythagorean matrix $M(S)$ of the Pythagorean fuzzy information system S in Example 3.2(1) as follows:

$$M(S) = \begin{bmatrix} P(1.0,0.0) & P(0.9,0.3) & P(0.8,0.2) & P(0.9,0.1) & P(0.9,0.2) \\ P(0.9,0.1) & P(0.5,0.5) & P(0.1,0.9) & P(0.3,0.8) & P(0.1,0.9) \\ P(0.1,0.9) & P(0.1,0.9) & P(0.2,0.8) & P(0.1,0.9) & P(0.5,0.5) \\ P(0.5,0.5) & P(0.1,0.9) & P(0.3,0.7) & P(0.5,0.5) & P(0.1,0.9) \\ P(0.9,0.2) & P(0.4,0.6) & P(0.1,0.9) & P(0.1,0.9) & P(0.3,0.9) \\ P(0.0,1.0) & P(0.9,0.1) & P(0.2,0.9) & P(0.5,0.5) & P(0.8,0.4) \end{bmatrix}.$$

Remark: We denote Israel, Egypt, Palestinians, Jordan, Syria and Saudi Arabia as x_1, x_2, x_3, x_4, x_5 and x_6, respectively; c_1 means Autonomous Palestinian state on the West Bank and Gaza; c_2 denotes Israeli military outpost along the Jordan River; c_3 stands for Israeli retains East Jerusalem; c_4 is Israeli military outposts on the Golan Heights; c_5 notes Arab countries grant citizenship to Palestinians who choose to remain with their borders. Furthermore, we employ TABLE I and TABLE II to depict the Middle East conflict, and there is no relationship among issue values of agents. We also employ Pythagorean matrix $M(S)$ to represent the Pythagorean fuzzy information system S, which provides an effective tool for studying Pythagorean fuzzy information systems for conflicts.

Definition 3.3: [53] Let $\mathscr{P} = \{\gamma_i | \gamma_i = P(\mu_{\gamma_i}, \nu_{\gamma_i}), i = 1, 2, ..., l\}$ be a collection of Pythagorean fuzzy numbers, and $\mathscr{K} = \{k_1, k_2, ..., k_l\}$ be the weight vector of γ_i $(i = 1, 2, ..., l)$, where k_i indicates the importance degree of γ_i, and satisfies $k_i \geq 0$ $(i = 1, 2, ..., l)$ and $\Sigma_{i=1}^{l} k_i = 1$. Then the Pythagorean fuzzy weighted averaging operator $\mathscr{R}$: $\Theta^l \to \Theta$ is defined as follows: $\mathscr{R}(\gamma_1, \gamma_2, ..., \gamma_l) = P(\Sigma_{i=1}^{l} k_i \mu_{\gamma_i}, \Sigma_{i=1}^{l} k_i \nu_{\gamma_i})$.

By Definition 3.3, we aggregate a collection of Pythagorean fuzzy numbers $\{\gamma_i | \gamma_i = P(\mu_{\gamma_i}, \nu_{\gamma_i}), i = 1, 2, ..., l\}$ into a Pythagorean fuzzy number $\mathscr{R}(\gamma_1, \gamma_2, ..., \gamma_l)$ with the weight vector. For simplicity, we denote $\mathscr{R}(c_1(x), c_2(x), ..., c_l(x))$ as $\mathscr{R}(x)$ in the following discussion. Moreover, we provide the positive, neutral and negative alliances with two thresholds as follows.

Definition 3.4: Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, α and β are two thresholds, and $\bullet$ denotes a function for Pythagorean fuzzy numbers. Then the positive, neutral and negative alliances are defined as follows:

$$\begin{aligned} POA_{(\bullet,\alpha,\beta)}(U) &= \{x \in U \mid \bullet(\mathscr{R}(x)) \geq \alpha\}; \\ CTA_{(\bullet,\alpha,\beta)}(U) &= \{x \in U \mid \beta < \bullet(\mathscr{R}(x)) < \alpha\}; \\ NEA_{(\bullet,\alpha,\beta)}(U) &= \{x \in U \mid \bullet(\mathscr{R}(x)) \leq \beta\}. \end{aligned}$$

By Definition 3.4, we get the positive, neutral and negative alliances $POA_{(\bullet,\alpha,\beta)}(U), CTA_{(\bullet,\alpha,\beta)}(U)$ and $NEA_{(\bullet,\alpha,\beta)}(U)$ with two thresholds α and β, and we have $POA_{(\bullet,\alpha,\beta)}(U) \cup CTA_{(\bullet,\alpha,\beta)}(U) \cup NEA_{(\bullet,\alpha,\beta)}(U) \subseteq U$. Furthermore, we provide four types of positive, neutral and negative alliances when $\bullet = I, S, \mathscr{P}, F$ in Definition 3.4 as follows.

Definition 3.5: Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system.

(1) If α and β are Pythagorean fuzzy numbers, and $P(0, 1) \leq \beta \leq \alpha \leq P(1, 0)$, then we define the first positive, neutral and negative alliances as follows:

$$\begin{aligned} POA_{(I,\alpha,\beta)}(U) &= \{x \in U \mid I(\mathscr{R}(x)) \geq \alpha\}; \\ CTA_{(I,\alpha,\beta)}(U) &= \{x \in U \mid \beta < I(\mathscr{R}(x)) < \alpha\}; \\ NEA_{(I,\alpha,\beta)}(U) &= \{x \in U \mid I(\mathscr{R}(x)) \leq \beta\}. \end{aligned}$$

(2) If $-1 \leq \beta \leq \alpha \leq 1$, then we define the second positive, neutral and negative alliances as follows:

$$\begin{aligned} POA_{(S,\alpha,\beta)}(U) &= \{x \in U \mid S(\mathscr{R}(x)) \geq \alpha\}; \\ CTA_{(S,\alpha,\beta)}(U) &= \{x \in U \mid \beta < S(\mathscr{R}(x)) < \alpha\}; \\ NEA_{(S,\alpha,\beta)}(U) &= \{x \in U \mid S(\mathscr{R}(x)) \leq \beta\}. \end{aligned}$$

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

(3) If $0 \leq \beta \leq \alpha \leq 1$, then we define the third positive, neutral and negative alliances as follows:

$$\begin{aligned} POA_{(\mathscr{P},\alpha,\beta)}(U) &= \{x \in U \mid \mathscr{P}(\mathscr{R}(x)) \geq \alpha\}; \\ CTA_{(\mathscr{P},\alpha,\beta)}(U) &= \{x \in U \mid \beta < \mathscr{P}(\mathscr{R}(x)) < \alpha\}; \\ NEA_{(\mathscr{P},\alpha,\beta)}(U) &= \{x \in U \mid \mathscr{P}(\mathscr{R}(x)) \leq \beta\}. \end{aligned}$$

(4) If $0 \leq \beta \leq \alpha \leq 1$, then we define the fourth positive, neutral and negative alliances as follows:

$$\begin{aligned} POA_{(F,\alpha,\beta)}(U) &= \{x \in U \mid F(\mathscr{R}(x)) \geq \alpha\}; \\ CTA_{(F,\alpha,\beta)}(U) &= \{x \in U \mid \beta < F(\mathscr{R}(x)) < \alpha\}; \\ NEA_{(F,\alpha,\beta)}(U) &= \{x \in U \mid F(\mathscr{R}(x)) \leq \beta\}. \end{aligned}$$

where $\mathscr{R}(x) = (\mu(x), \nu(x))$ and $F(\mathscr{R}(x)) = \frac{1}{2} + \sqrt{\mu(x)^2 + \nu(x)^2} * (\frac{1}{2} - \frac{2\arccos(\frac{\mu(x)}{\sqrt{\mu(x)^2+\nu(x)^2}})}{\pi})$ for $x \in U$.

Example 3.6: (Continuation from Example 3.2) (1) Taking $k_1 = k_2 = k_3 = k_4 = k_5 = \frac{1}{5}$, $\alpha = P(0.7, 0.4)$ and $\beta = P(0.25, 0.85)$. By Definition 3.3, we have the Pythagorean fuzzy weighted averaging closeness index of x_1, x_2, x_3, x_4, x_5 and x_6 on A as follows:

$$\begin{aligned} I(\mathscr{R}(x_1)) &= P(\Sigma_{i=1}^{5}\frac{1}{5} * \mu_{\gamma_{1i}}, \Sigma_{i=1}^{5}\frac{1}{5} * \nu_{\gamma_{1i}}) = P(0.90, 0.16); \\ I(\mathscr{R}(x_2)) &= P(\Sigma_{i=1}^{5}\frac{1}{5} * \mu_{\gamma_{2i}}, \Sigma_{i=1}^{5}\frac{1}{5} * \nu_{\gamma_{2i}}) = P(0.38, 0.64); \\ I(\mathscr{R}(x_3)) &= P(\Sigma_{i=1}^{5}\frac{1}{5} * \mu_{\gamma_{3i}}, \Sigma_{i=1}^{5}\frac{1}{5} * \nu_{\gamma_{3i}}) = P(0.20, 0.80); \\ I(\mathscr{R}(x_4)) &= P(\Sigma_{i=1}^{5}\frac{1}{5} * \mu_{\gamma_{4i}}, \Sigma_{i=1}^{5}\frac{1}{5} * \nu_{\gamma_{4i}}) = P(0.30, 0.70); \\ I(\mathscr{R}(x_5)) &= P(\Sigma_{i=1}^{5}\frac{1}{5} * \mu_{\gamma_{5i}}, \Sigma_{i=1}^{5}\frac{1}{5} * \nu_{\gamma_{5i}}) = P(0.36, 0.70); \\ I(\mathscr{R}(x_6)) &= P(\Sigma_{i=1}^{5}\frac{1}{5} * \mu_{\gamma_{6i}}, \Sigma_{i=1}^{5}\frac{1}{5} * \nu_{\gamma_{6i}}) = P(0.48, 0.58). \end{aligned}$$

By Definition 3.5(1), we have $POA_{(I,\alpha,\beta)}(U) = \{x_1\}$, $CTA_{(I,\alpha,\beta)}(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA_{(I,\alpha,\beta)}(U) = \emptyset$. So we classify $\{x_1, x_2, x_4, x_5, x_6\}$ into $POA_{(I,\alpha,\beta)}(U), CTA_{(I,\alpha,\beta)}(U)$ and $NEA_{(I,\alpha,\beta)}(U)$. But it does not work for x_3. Furthermore, we see that $\{x_1\}$ and $\{x_2, x_4, x_5, x_6\}$ are difference alliances, but x_3 does not belong to any alliance.

(2) Taking $\alpha = 0.5$ and $\beta = -0.5$, we have

$$\begin{aligned} S(\mathscr{R}(x_1)) &= 0.90^2 - 0.16^2 = +0.7844; \\ S(\mathscr{R}(x_2)) &= 0.38^2 - 0.64^2 = -0.2652; \\ S(\mathscr{R}(x_3)) &= 0.20^2 - 0.80^2 = -0.6000; \\ S(\mathscr{R}(x_4)) &= 0.30^2 - 0.70^2 = -0.4000; \\ S(\mathscr{R}(x_5)) &= 0.36^2 - 0.70^2 = -0.3604; \\ S(\mathscr{R}(x_6)) &= 0.48^2 - 0.58^2 = -0.1060. \end{aligned}$$

By Definition 3.5(2), we have $POA_{(S,\alpha,\beta)}(U) = \{x_1\}, CTA_{(S,\alpha,\beta)}(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA_{(S,\alpha,\beta)}(U) = \{x_3\}$. Furthermore, we see that $\{x_1\}$, $\{x_3\}$ and $\{x_2, x_4, x_5, x_6\}$ are difference alliances. Especially, $\{x_1\}$ and $\{x_3\}$ has different opinions on issues, and $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance.

(3) Taking $\alpha = 0.75$ and $\beta = 0.3$, we have

$$\begin{aligned} \mathscr{P}(\mathscr{R}(x_1)) &= \frac{1 - 0.16^2}{2 - 0.90^2 - 0.16^2} = 0.8368; \\ \mathscr{P}(\mathscr{R}(x_2)) &= \frac{1 - 0.64^2}{2 - 0.38^2 - 0.64^2} = 0.4083; \\ \mathscr{P}(\mathscr{R}(x_3)) &= \frac{1 - 0.80^2}{2 - 0.20^2 - 0.80^2} = 0.2727; \\ \mathscr{P}(\mathscr{R}(x_4)) &= \frac{1 - 0.70^2}{2 - 0.70^2 - 0.30^2} = 0.3592; \\ \mathscr{P}(\mathscr{R}(x_5)) &= \frac{1 - 0.70^2}{2 - 0.70^2 - 0.36^2} = 0.3695; \\ \mathscr{P}(\mathscr{R}(x_6)) &= \frac{1 - 0.58^2}{2 - 0.58^2 - 0.48^2} = 0.4584. \end{aligned}$$

By Definition 3.5(3), we have $POA_{(\mathscr{P},\alpha,\beta)}(U) = \{x_1\}, CTA_{(\mathscr{P},\alpha,\beta)}(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA_{(\mathscr{P},\alpha,\beta)}(U) = \{x_3\}$. Moreover, we observe that $\{x_1\}$, $\{x_3\}$ and $\{x_2, x_4, x_5, x_6\}$ are difference alliances. Specially, $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance, $\{x_1\}$ and $\{x_3\}$ are opposite alliances.

(4) Taking $\alpha = 0.75$ and $\beta = 0.3$, we have

$$\begin{aligned} & F(\mathscr{R}(x_1)) \\ &= \frac{1}{2} + \sqrt{0.90^2 + 0.16^2} * (\frac{1}{2} - \frac{2\arccos(\frac{0.90}{\sqrt{0.90^2+0.16^2}})}{\pi}) \\ &= 0.8547; \\ & F(\mathscr{R}(x_2)) \\ &= \frac{1}{2} + \sqrt{0.38^2 + 0.64^2} * (\frac{1}{2} - \frac{2\arccos(\frac{0.38}{\sqrt{0.38^2+0.64^2}})}{\pi}) \\ &= 0.3817; \\ & F(\mathscr{R}(x_3)) \\ &= \frac{1}{2} + \sqrt{0.20^2 + 0.80^2} * (\frac{1}{2} - \frac{2\arccos(\frac{0.20}{\sqrt{0.20^2+0.80^2}})}{\pi}) \\ &= 0.2163; \\ & F(\mathscr{R}(x_4)) \\ &= \frac{1}{2} + \sqrt{0.30^2 + 0.70^2} * (\frac{1}{2} - \frac{2\arccos(\frac{0.30}{\sqrt{0.30^2+0.70^2}})}{\pi}) \\ &= 0.3155; \\ & F(\mathscr{R}(x_5)) \\ &= \frac{1}{2} + \sqrt{0.36^2 + 0.70^2} * (\frac{1}{2} - \frac{2\arccos(\frac{0.36}{\sqrt{0.36^2+0.70^2}})}{\pi}) \\ &= 0.3445; \\ & F(\mathscr{R}(x_6)) \\ &= \frac{1}{2} + \sqrt{0.48^2 + 0.58^2} * (\frac{1}{2} - \frac{2\arccos(\frac{0.48}{\sqrt{0.48^2+0.58^2}})}{\pi}) \\ &= 0.4549. \end{aligned}$$

By Definition 3.5(4), we have $POA_{(F,\alpha,\beta)}(U) = \{x_1\}, CTA_{(F,\alpha,\beta)}(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA_{(F,\alpha,\beta)}(U) = \{x_3\}$. Moreover, we find that $\{x_1\}$, $\{x_3\}$ and $\{x_2, x_4, x_5, x_6\}$ are difference alliances. Specially, $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance, $\{x_1\}$ and $\{x_3\}$ are opposite alliances.□

By Definition 3.5, we partition the universe into three regions: positive, neutral and negative alliances with different operators, and denote the positive, neutral and negative alliances of U as $POA(U), CTA(U)$ and $NEA(U)$ for simplicity.

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

Furthermore, we classify all agents into three regions by Definition 3.5(1) when they are depicted by Pythagorean fuzzy numbers. If Definition 3.5(1) does not work, then we choose Definition 3.5(2) to partition these agents. Specially, if Definitions 3.5(1) and 3.5(2) do not work, we apply Definition 3.5(3) to classify these agents.

IV. Three-way conflict analysis of Pythagorean fuzzy information systems

In this section, we study Pythagorean fuzzy information systems for conflicts based on three-way decision theory and Bayesian minimum risk theory.

Definition 4.1: A Pythagorean fuzzy loss function given by an expert is a 3-tuple $\lambda = (\Omega, \mathscr{A}, \mathscr{L})$ shown as TABLE III, where $\Omega = \{X, \neg X\}$ is a set of 2 states, $\mathscr{A} = \{a_P, a_B, a_N\}$ is a set of 3 actions for each state, $\mathscr{L} = \{\lambda_{PP}, \lambda_{BP}, \lambda_{NP}, \lambda_{PN}, \lambda_{BN}, \lambda_{NN}\}$, $X(\subseteq U)$ and $\neg X(\subseteq U)$ indicate that an agent is in X and not in X, respectively; a_P, a_B and a_N denote three actions in classifying an agent x into $POA(U)$, $CTA(U)$ and $NEA(U)$, respectively; $\lambda_{PP}, \lambda_{BP}$ and λ_{NP} stand for the losses of taking actions a_P, a_B and a_N, respectively, when an agent belongs to X; $\lambda_{PN}, \lambda_{BN}$ and λ_{NN} mean the losses of taking actions a_P, a_B and a_N, respectively, when an agent belongs to $\neg X$, where $\lambda_{PP}, \lambda_{BP}$, λ_{NP}, $\lambda_{PN}, \lambda_{BN}$ and λ_{NN} are Pythagorean fuzzy numbers.

TABLE III
A Pythagorean Fuzzy Loss Function given by an expert.

Action	X	$\neg X$
a_P	$\lambda_{PP} = P(\mu_{\lambda_{PP}}, \nu_{\lambda_{PP}})$	$\lambda_{PN} = P(\mu_{\lambda_{PN}}, \nu_{\lambda_{PN}})$
a_B	$\lambda_{BP} = P(\mu_{\lambda_{BP}}, \nu_{\lambda_{BP}})$	$\lambda_{BN} = P(\mu_{\lambda_{BN}}, \nu_{\lambda_{BN}})$
a_N	$\lambda_{NP} = P(\mu_{\lambda_{NP}}, \nu_{\lambda_{NP}})$	$\lambda_{NN} = P(\mu_{\lambda_{NN}}, \nu_{\lambda_{NN}})$

For simplicity, we employ the same symbol to denote both the set C and the corresponding state; we also denote both the set $\neg C$ and the corresponding state as the same symbol. Furthermore, we assume that the loss of assigning an object into the boundary region is between an incorrect classification and a correct classification. That is, the loss of right decision is less than that of deferred decision, and the loss of deferred decision is less than that of the wrong decision in practice. So $\lambda_{PN}, \lambda_{BN}$, λ_{NN} $\lambda_{PN}, \lambda_{BN}$ and λ_{NN} should satisfy the above relations. Furthermore, there are four types of Pythagorean fuzzy loss functions as follows: (1) the Pythagorean fuzzy loss function satisfying $\lambda_{PP} \leq \lambda_{BP} \leq \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} \leq \lambda_{PN}$; (2) the Pythagorean fuzzy loss function satisfying $S(\lambda_{PP}) \leq S(\lambda_{BP}) \leq S(\lambda_{NP})$ and $S(\lambda_{NN}) \leq S(\lambda_{BN}) \leq S(\lambda_{PN})$; (3) the Pythagorean fuzzy loss function satisfying $\mathscr{P}(\lambda_{PP}) \leq \mathscr{P}(\lambda_{BP}) \leq \mathscr{P}(\lambda_{NP})$ and $\mathscr{P}(\lambda_{NN}) \leq \mathscr{P}(\lambda_{BN}) \leq \mathscr{P}(\lambda_{PN})$; (4) the Pythagorean fuzzy loss function satisfying $F(\lambda_{PP}) \leq F(\lambda_{BP}) \leq F(\lambda_{NP})$ and $F(\lambda_{NN}) \leq F(\lambda_{BN}) \leq F(\lambda_{PN})$. In practice, loss functions are very important for conflict analysis of Pythagorean fuzzy information systems, there are many methods of deriving loss functions such as practical experience and given by famous experts. Although there are plenty of loss functions besides the above four types, we only discuss the Pythagorean fuzzy loss functions given by experts satisfying $\lambda_{PP} \leq \lambda_{BP} \leq \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} \leq \lambda_{PN}$ in this section.

Example 4.2: TABLE IV depicts a Pythagorean fuzzy loss function given by an expert, and $\lambda_{PP}, \lambda_{BP}, \lambda_{NP}, \lambda_{NN}, \lambda_{BN}$ and λ_{PN} are Pythagorean fuzzy numbers. Especially, we have $\lambda_{PP} \leq \lambda_{BP} \leq \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} \leq \lambda_{PN}$.

TABLE IV
A Pythagorean Fuzzy Loss Function.

Action	X	$\neg X$
a_P	$\lambda_{PP} = P(0.1, 0.8)$	$\lambda_{PN} = P(0.9, 0.2)$
a_B	$\lambda_{BP} = P(0.6, 0.5)$	$\lambda_{BN} = P(0.5, 0.6)$
a_N	$\lambda_{NP} = P(0.9, 0.3)$	$\lambda_{NN} = P(0.2, 0.8)$

Suppose $\lambda_{PP}, \lambda_{BP}$, λ_{NP}, $\lambda_{PN}, \lambda_{BN}$ and λ_{NN} are Pythagorean fuzzy numbers, which satisfy $\lambda_{PP} \leq \lambda_{BP} \leq \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} \leq \lambda_{PN}$. For the object $x \in U$, the expected losses $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ under the actions a_P, a_B and a_N, respectively, are shown as follows:

$$\begin{aligned} R(a_P|x) &= \mathscr{P}(\mathscr{R}(x)) * \lambda_{PP} \oplus [1 - \mathscr{P}(\mathscr{R}(x))] * \lambda_{PN}; \\ R(a_B|x) &= \mathscr{P}(\mathscr{R}(x)) * \lambda_{BP} \oplus [1 - \mathscr{P}(\mathscr{R}(x))] * \lambda_{BN}; \\ R(a_N|x) &= \mathscr{P}(\mathscr{R}(x)) * \lambda_{NP} \oplus [1 - \mathscr{P}(\mathscr{R}(x))] * \lambda_{NN}. \end{aligned}$$

We see that the expected loss functions $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ are constructed on the closeness index function $\mathscr{P}(\mathscr{R}(x))$, which are different from the expected loss functions of reference [27]. According to Definition 4.1, we have the expected losses $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ as follows:

$$\begin{aligned} R(a_P|x) &= P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))}\right) \\ &\quad \oplus P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}\right); \\ R(a_B|x) &= P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x))}\right) \\ &\quad \oplus P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}\right); \\ R(a_N|x) &= P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x))}\right) \\ &\quad \oplus P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}\right). \end{aligned}$$

Theorem 4.3: Let $R(a_\bullet|x)$ be the expected loss under the action $a_\bullet$ for the object $x \in U$, where $\bullet = P, B, N$. Then

$$\begin{aligned} R(a_P|x) &= P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (1 - \mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}},\right. \\ &\quad \left.(\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}\right); \\ R(a_B|x) &= P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (1 - \mu^2_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}},\right. \\ &\quad \left.(\nu_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}\right); \\ R(a_N|x) &= P\left(\sqrt{1 - (1 - \mu^2_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (1 - \mu^2_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}},\right. \\ &\quad \left.(\nu_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}\right). \end{aligned}$$

Proof: We assume $t_1 = (1 - \mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))}, t_2 = (1 - \mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}, y_1 = (\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))}$ and $y_2 = (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}$. By

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

Definition 2.1, we have

$$\begin{aligned} R(a_P|x) &= P(\sqrt{1-(1-\mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))}) \\ &\quad \oplus P(\sqrt{1-(1-\mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}) \\ &= P(\sqrt{1-t_1}, y_1) \oplus P(\sqrt{1-t_2}, y_2) \\ &= P(\sqrt{1-t_1+1-t_2-(1-t_1)(1-t_2)}, y_1y_2) \\ &= P(\sqrt{1-t_1t_2}, y_1y_2) \\ &= P(\sqrt{1-(1-\mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (1-\mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \\ &\quad (\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}). \end{aligned}$$

Furthermore, we also prove

$$\begin{aligned} R(a_B|x) &= P(\sqrt{1-(1-\mu^2_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (1-\mu^2_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \\ &\quad (\nu_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}); \\ R(a_N|x) &= P(\sqrt{1-(1-\mu^2_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (1-\mu^2_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \\ &\quad (\nu_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}). \square \end{aligned}$$

We observe that Theorem 4.3 illustrates that the expected losses $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ are Pythagorean fuzzy numbers. Especially, it implies that how to compute the expected losses $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ with the closeness index function $\mathscr{P}(\mathscr{R}(x))$.

Definition 4.4: Let $S=(U,A,V,f)$ be a Pythagorean fuzzy information system, $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ are the expected losses under the actions a_P, a_B and a_N, respectively, for the object $x \in U$. Then the Expected Loss Table $IR(S)$, Score Table $SR(S)$, Closeness Table $PR(S)$ and Preferred Table $FR(S)$ are defined as TABLE V, TABLE VI, TABLE VII and TABLE VIII, respectively.

TABLE V
The Expected Loss Table $IR(S)$.

Action	P	B	N
x_1	$I(R(a_P\|x_1))$	$I(R(a_B\|x_1))$	$I(R(a_N\|x_1))$
x_2	$I(R(a_P\|x_2))$	$I(R(a_B\|x_2))$	$I(R(a_N\|x_2))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$I(R(a_P\|x_n))$	$I(R(a_B\|x_n))$	$I(R(a_N\|x_n))$

TABLE VI
The Score Table $SR(S)$.

Action	P	B	N
x_1	$S(R(a_P\|x_1))$	$S(R(a_B\|x_1))$	$S(R(a_N\|x_1))$
x_2	$S(R(a_P\|x_2))$	$S(R(a_B\|x_2))$	$S(R(a_N\|x_2))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$S(R(a_P\|x_n))$	$S(R(a_B\|x_n))$	$S(R(a_N\|x_n))$

TABLE VII
The Closeness Table $PR(S)$.

Action	P	B	N
x_1	$\mathscr{P}(R(a_P\|x_1))$	$\mathscr{P}(R(a_B\|x_1))$	$\mathscr{P}(R(a_N\|x_1))$
x_2	$\mathscr{P}(R(a_P\|x_2))$	$\mathscr{P}(R(a_B\|x_2))$	$\mathscr{P}(R(a_N\|x_2))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$\mathscr{P}(R(a_P\|x_n))$	$\mathscr{P}(R(a_B\|x_n))$	$\mathscr{P}(R(a_N\|x_n))$

TABLE VIII
The Preferred Table $FR(S)$.

Action	P	B	N
x_1	$F(R(a_P\|x_1))$	$F(R(a_B\|x_1))$	$F(R(a_N\|x_1))$
x_2	$F(R(a_P\|x_2))$	$F(R(a_B\|x_2))$	$F(R(a_N\|x_2))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$F(R(a_P\|x_n))$	$F(R(a_B\|x_n))$	$F(R(a_N\|x_n))$

Theorem 4.5: Let $S=(U,A,V,f)$ be a Pythagorean fuzzy information system, $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ are the expected losses under the actions a_P, a_B and a_N, respectively, for the object $x \in U$, and $\bullet = I, S, \mathscr{P}, F$.

P: If $\bullet(R(a_P|x)) \leq \bullet(R(a_B|x))$ and $\bullet(R(a_P|x)) \leq \bullet(R(a_N|x))$, then we have $x \in POA(U)$;

B: If $\bullet(R(a_B|x)) \leq \bullet(R(a_P|x))$ and $\bullet(R(a_B|x)) \leq \bullet(R(a_N|x))$, then we have $x \in CTA(U)$;

N: If $\bullet(R(a_N|x)) \leq \bullet(R(a_P|x))$ and $\bullet(R(a_N|x)) \leq \bullet(R(a_B|x))$, then we have $x \in NEA(U)$.

Proof: It is straightforward by Bayesian minimum risk theory.□

Example 4.6: (Continuation from Examples 3.2 and 4.2) We compute $POA(U), CTA(U)$ and $NEA(U)$ based on Definition 3.5 and Bayesian minimum risk theory as follows:

(1) First, by TABLE III and Theorem 4.3, for $x_i \in U, 1 \leq i \leq 6$, we have

$$\begin{aligned} R(a_P|x_i) &= P(\sqrt{1-(1-\mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}}, \\ &\quad (\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}) \\ &= P(\sqrt{1-(1-0.1^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.9^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}}, \\ &\quad 0.8^{\mathscr{P}(\mathscr{R}(x_i))} * 0.2^{1-\mathscr{P}(\mathscr{R}(x_i))}); \\ R(a_B|x_i) &= P(\sqrt{1-(1-\mu^2_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}}, \\ &\quad (\nu_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}) \\ &= P(\sqrt{1-(1-0.6^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.5^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}}, \\ &\quad 0.5^{\mathscr{P}(\mathscr{R}(x_i))} * 0.6^{1-\mathscr{P}(\mathscr{R}(x_i))}); \\ R(a_N|x_i) &= P(\sqrt{1-(1-\mu^2_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}}, \\ &\quad (\nu_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}) \\ &= P(\sqrt{1-(1-0.9^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.2^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}}, \\ &\quad 0.3^{\mathscr{P}(\mathscr{R}(x_i))} * 0.8^{1-\mathscr{P}(\mathscr{R}(x_i))}). \end{aligned}$$

Second, by Definition 4.4, we have the Expected Loss Table $IR(S)$ shown by TABLE IX as follows:

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

TABLE IX
THE EXPECTED LOSS TABLE $IR(S)$.

Action	P	B	N
x_1	P(0.4937,0.6380)	P(0.5859,0.5151)	P(0.8675,0.3521)
x_2	P(0.7920,0.3522)	P(0.5450,0.5570)	P(0.7103,0.5360)
x_3	P(0.8378,0.2919)	P(0.5308,0.5709)	P(0.6187,0.6122)
x_4	P(0.8101,0.3291)	P(0.5399,0.5620)	P(0.6808,0.5625)
x_5	P(0.8065,0.3338)	P(0.5410,0.5609)	P(0.6873,0.5568)
x_6	P(0.7694,0.3800)	P(0.5505,0.5514)	P(0.7393,0.5080)

Third, by Theorem 4.5, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_5, x_6\}$ and $NEA(U) = \emptyset$. Obviously, it is difficult to compare the expected losses under the actions a_P, a_B and a_N for the agents x_3 and x_4. So it fails to put the agents x_3 and x_4 into $POA(U)$, $CTA(U)$ and $NEA(U)$. Therefore, $\{x_1\}$ and $\{x_2, x_5, x_6\}$ are different alliances, but we can not identify x_3 and x_4.

(2) First, by Definition 2.3 and TABLE V, for $x_i \in U, 1 \leq i \leq 6$, we have

$$\begin{aligned}
&S(R(a_P|x_i))\\
&= S(P(\sqrt{1-(1-\mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&(\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}))\\
&= S(P(\sqrt{1-(1-0.1^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.9^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&0.8^{\mathscr{P}(\mathscr{R}(x_i))} * 0.2^{1-\mathscr{P}(\mathscr{R}(x_i))}));\\
&S(R(a_B|x_i))\\
&= S(P(\sqrt{1-(1-\mu^2_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&(\nu_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}))\\
&= S(P(\sqrt{1-(1-0.6^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.5^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&0.5^{\mathscr{P}(\mathscr{R}(x_i))} * 0.6^{1-\mathscr{P}(\mathscr{R}(x_i))}));\\
&S(R(a_N|x_i))\\
&= S(P(\sqrt{1-(1-\mu^2_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&(\nu_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}))\\
&= S(P(\sqrt{1-(1-0.9^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.2^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&0.3^{\mathscr{P}(\mathscr{R}(x_i))} * 0.8^{1-\mathscr{P}(\mathscr{R}(x_i))})).
\end{aligned}$$

Second, by Definition 4.4, we have the Score Table $SR(S)$ shown by TABLE X as follows.

TABLE X
THE SCORE TABLE $SR(S)$.

Action	P	B	N
x_1	-0.1633	0.0779	0.6286
x_2	0.5031	-0.0132	0.2172
x_3	0.6168	-0.0442	0.0080
x_4	0.5480	-0.0243	0.1471
x_5	0.5390	-0.0219	0.1623
x_6	0.4476	-0.0010	0.2885

Third, by Theorem 4.5, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_3, x_4, x_5, x_6\}$ and $NEA(U) = \emptyset$. Moreover, we find that $\{x_1\}$ and $\{x_2, x_3, x_4, x_5, x_6\}$ are difference alliances. Specially, $\{x_1\}$ is a positive alliance, and $\{x_2, x_3, x_4, x_5, x_6\}$ is a neutral alliance.

(3) First, by Definition 2.5, for $x_i \in U, 1 \leq i \leq 6$, we have

$$\begin{aligned}
&\mathscr{P}(R(a_P|x_i)\\
&= \mathscr{P}(P(\sqrt{1-(1-\mu^2_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&(\nu_{\lambda_{PP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}))\\
&= \mathscr{P}(P(\sqrt{1-(1-0.1^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.9^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&0.8^{\mathscr{P}(\mathscr{R}(x_i))} * 0.2^{1-\mathscr{P}(\mathscr{R}(x_i))}));\\
&\mathscr{P}(R(a_B|x_i))\\
&= \mathscr{P}(P(\sqrt{1-(1-\mu^2_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&(\nu_{\lambda_{BP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}))\\
&= \mathscr{P}(P(\sqrt{1-(1-0.6^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.5^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&0.5^{\mathscr{P}(\mathscr{R}(x_i))} * 0.6^{1-\mathscr{P}(\mathscr{R}(x_i))}));\\
&\mathscr{P}(R(a_N|x_i))\\
&= \mathscr{P}(P(\sqrt{1-(1-\mu^2_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (1-\mu^2_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&(\nu_{\lambda_{NP}})^{\mathscr{P}(\mathscr{R}(x_i))} * (\nu_{\lambda_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_i))}))\\
&= \mathscr{P}(P(\sqrt{1-(1-0.9^2)^{\mathscr{P}(\mathscr{R}(x_i))} * (1-0.2^2)^{1-\mathscr{P}(\mathscr{R}(x_i))}},\\
&0.3^{\mathscr{P}(\mathscr{R}(x_i))} * 0.8^{1-\mathscr{P}(\mathscr{R}(x_i))})).
\end{aligned}$$

Second, by Definition 4.4, we have the Closeness Table $PR(S)$ shown by TABLE XI as follows.

TABLE XI
THE CLOSENESS TABLE $PR(S)$.

Action	P	B	N
x_1	0.4395	0.5280	0.7797
x_2	0.7015	0.4953	0.5899
x_3	0.7543	0.4841	0.5032
x_4	0.7218	0.4913	0.5603
x_5	0.7176	0.4921	0.5666
x_6	0.6771	0.4997	0.6207

Third, by Theorem 4.5, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_3, x_4, x_5, x_6\}$ and $NEA(U) = \emptyset$. Furthermore, we find that $\{x_1\}$ and $\{x_2, x_3, x_4, x_5, x_6\}$ are difference alliances. Specially, $\{x_1\}$ is a positive alliance, and $\{x_2, x_3, x_4, x_5, x_6\}$ is a neutral alliance.

(4) First, by Definition 4.4, we have the Preferred Table $FR(S)$ shown by TABLE XII as follows:

TABLE XII
THE PREFERRED TABLE $FR(S)$.

Action	P	B	N
x_1	0.4349	0.5319	0.7383
x_2	0.7025	0.4946	0.5787
x_3	0.7543	0.4819	0.5029
x_4	0.7224	0.4901	0.5533
x_5	0.7184	0.4910	0.5588
x_6	0.6784	0.4996	0.6047

Then, by Theorem 4.5, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_3, x_4, x_5, x_6\}$ and $NEA(U) = \emptyset$. Moreover, we see that $\{x_1\}$ and $\{x_2, x_3, x_4, x_5, x_6\}$ are difference alliances. Specially, $\{x_1\}$ is a positive alliance, and $\{x_2, x_3, x_4, x_5, x_6\}$ is a neutral alliance.

(5) First, we list the alliances computed using $IR(S), SR(S), PR(S)$ and $FR(S)$ in TABLE XIII. Concretely, we have the same alliances with $SR(S), PR(S)$ and $FR(S)$,

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

which are different from the results with $IR(S)$; all agents are classified into the positive, neutral and negative alliances with $SR(S), PR(S)$ and $FR(S)$, but we can not put x_3 into any alliance with $IR(S)$; almost all agents are classified into the neutral alliances with $IR(S), SR(S), PR(S)$ and $FR(S)$, and no agents are put into the negative alliances. Second, we have the positive alliance $\{x_1\}$ and the neutral alliance $\{x_2, x_5, x_6\}$ by $IR(S)$; we get the positive alliance $\{x_1\}$ and the neutral alliance $\{x_2, x_3, x_4, x_5, x_6\}$ by $SR(S), PR(S)$ and $FR(S)$. It is obvious that most of countries belong to the neutral alliance, and x_1 holds different opinions on some issues, so it is a single alliance. If x_1 wants to get supports from other countries, then it must change opinions on some issues. Third, there are so many agents in the neutral alliance, so we should choose the appropriate thresholds and provide more effective approaches for studying Pythagorean fuzzy information systems for conflicts.

TABLE XIII
Three alliances based on $IR(S), SR(S), PR(S)$ and $FR(S)$.

Methods	$POA(U)$	$CTA(U)$	$NEA(U)$
$IR(S)$	$\{x_1\}$	$\{x_2, x_5, x_6\}$	$\emptyset$
$SR(S)$	$\{x_1\}$	$\{x_2, x_3, x_4, x_5, x_6\}$	$\emptyset$
$PR(S)$	$\{x_1\}$	$\{x_2, x_3, x_4, x_5, x_6\}$	$\emptyset$
$FR(S)$	$\{x_1\}$	$\{x_2, x_3, x_4, x_5, x_6\}$	$\emptyset$

V. Three-way group conflict analysis of Pythagorean fuzzy information systems

In this section, we investigate Pythagorean fuzzy information systems for conflicts with group decision theory.

Suppose there are m experts $\{E_1, E_2, ..., E_m\}$, who give the loss functions $\{\lambda^{(1)}, \lambda^{(2)}, ..., \lambda^{(m)}\}$ shown by TABLE XIV, where $\lambda^{(i)} = (\Omega, \mathscr{A}, \mathscr{L}^{(i)})$, $\Omega = \{X, \neg X\}$ is a set of 2 states, $\mathscr{A} = \{a_P, a_B, a_N\}$, $\mathscr{L}^{(i)} = \{\lambda^{(i)}_{PP}, \lambda^{(i)}_{BP}, \lambda^{(i)}_{NP}, \lambda^{(i)}_{PN}, \lambda^{(i)}_{BN}, \lambda^{(i)}_{NN}\}$, $X(\subseteq U)$ and $\neg X(\subseteq U)$ indicate that an agent is in X and not in X, respectively. For simplicity, we take the same symbol to denote both the set C and the corresponding state. We also employ both the set $\neg C$ and the corresponding state as the same symbol. Furthermore, a_P, a_B and a_N denote three actions in classifying an agent x into $POA(U)$, $CTA(U)$ and $NEA(U)$, respectively; $\lambda^{(i)}_{PP}, \lambda^{(i)}_{BP}$ and $\lambda^{(i)}_{NP}$ stand for the losses of taking actions a_P, a_B and a_N, respectively, when an agent belongs to X; $\lambda^{(i)}_{PN}, \lambda^{(i)}_{BN}$ and $\lambda^{(i)}_{NN}$ mean the losses of taking actions a_P, a_B and a_N, respectively, when an agent belongs to $\neg X$, where $\lambda^{(i)}_{PP}, \lambda^{(i)}_{BP}$, $\lambda^{(i)}_{NP}$, $\lambda^{(i)}_{PN}, \lambda^{(i)}_{BN}$ and $\lambda^{(i)}_{NN}$ are Pythagorean fuzzy numbers, which satisfy $\lambda^{(i)}_{PP} \leq \lambda^{(i)}_{BP} \leq \lambda^{(i)}_{NP}$ and $\lambda^{(i)}_{NN} \leq \lambda^{(i)}_{BN} \leq \lambda^{(i)}_{PN}$. For the agent $x \in U$, the expected losses $R^{(i)}(a_P|x)$, $R^{(i)}(a_B|x)$ and $R^{(i)}(a_N|x)$ under the actions a_P, a_B, and a_N with respect to the loss given by the expert E_i, respectively, as follows:

$$\begin{aligned} R^{(i)}(a_P|x) &= \mathscr{P}(\mathscr{R}(x)) * \lambda^{(i)}_{PP} \oplus [1 - \mathscr{P}(\mathscr{R}(x))] * \lambda^{(i)}_{PN}; \\ R^{(i)}(a_B|x) &= \mathscr{P}(\mathscr{R}(x)) * \lambda^{(i)}_{BP} \oplus [1 - \mathscr{P}(\mathscr{R}(x))] * \lambda^{(i)}_{BN}; \\ R^{(i)}(a_N|x) &= \mathscr{P}(\mathscr{R}(x)) * \lambda^{(i)}_{NP} \oplus [1 - \mathscr{P}(\mathscr{R}(x))] * \lambda^{(i)}_{NN}. \end{aligned}$$

TABLE XIV
Pythagorean Fuzzy Loss Functions $\{\lambda^{(i)}|i = 1, 2, ..., m\}$.

λ	Action	X	$\neg X$
$\lambda^{(1)}$	a_P	$\lambda^{(1)}_{PP} = P(\mu_{\lambda^{(1)}_{PP}}, \nu_{\lambda^{(1)}_{PP}})$	$\lambda^{(1)}_{PN} = P(\mu_{\lambda^{(1)}_{PN}}, \nu_{\lambda^{(1)}_{PN}})$
	a_B	$\lambda^{(1)}_{BP} = P(\mu_{\lambda^{(1)}_{BP}}, \nu_{\lambda^{(1)}_{BP}})$	$\lambda^{(1)}_{BN} = P(\mu_{\lambda^{(1)}_{BN}}, \nu_{\lambda^{(1)}_{BN}})$
	a_N	$\lambda^{(1)}_{NP} = P(\mu_{\lambda^{(1)}_{NP}}, \nu_{\lambda^{(1)}_{NP}})$	$\lambda^{(1)}_{NN} = P(\mu_{\lambda^{(1)}_{NN}}, \nu_{\lambda^{(1)}_{NN}})$
$\lambda^{(2)}$	a_P	$\lambda^{(2)}_{PP} = P(\mu_{\lambda^{(2)}_{PP}}, \nu_{\lambda^{(2)}_{PP}})$	$\lambda^{(2)}_{PN} = P(\mu_{\lambda^{(2)}_{PN}}, \nu_{\lambda^{(2)}_{PN}})$
	a_B	$\lambda^{(2)}_{BP} = P(\mu_{\lambda^{(2)}_{BP}}, \nu_{\lambda^{(2)}_{BP}})$	$\lambda^{(2)}_{BN} = P(\mu_{\lambda^{(2)}_{BN}}, \nu_{\lambda^{(2)}_{BN}})$
	a_N	$\lambda^{(2)}_{NP} = P(\mu_{\lambda^{(2)}_{NP}}, \nu_{\lambda^{(2)}_{NP}})$	$\lambda^{(2)}_{NN} = P(\mu_{\lambda^{(2)}_{NN}}, \nu_{\lambda^{(2)}_{NN}})$
.	. . .	. . .	. . .
$\lambda^{(m)}$	a_P	$\lambda^{(m)}_{PP} = P(\mu_{\lambda^{(m)}_{PP}}, \nu_{\lambda^{(m)}_{PP}})$	$\lambda^{(m)}_{PN} = P(\mu_{\lambda^{(m)}_{PN}}, \nu_{\lambda^{(m)}_{PN}})$
	a_B	$\lambda^{(m)}_{BP} = P(\mu_{\lambda^{(m)}_{BP}}, \nu_{\lambda^{(m)}_{BP}})$	$\lambda^{(m)}_{BN} = P(\mu_{\lambda^{(m)}_{BN}}, \nu_{\lambda^{(m)}_{BN}})$
	a_N	$\lambda^{(m)}_{NP} = P(\mu_{\lambda^{(m)}_{NP}}, \nu_{\lambda^{(m)}_{NP}})$	$\lambda^{(m)}_{NN} = P(\mu_{\lambda^{(m)}_{NN}}, \nu_{\lambda^{(m)}_{NN}})$

Example 5.1: (Continuation from Example 4.6) TABLE XV depicts a collection of Pythagorean fuzzy loss functions $\{\lambda^{(i)}|i = 1, 2, 3\}$, which are given by three experts $\{E_1, E_2, E_3\}$.

TABLE XV
Pythagorean Fuzzy Loss Functions $\{\lambda^{(i)}|i = 1, 2, 3\}$.

λ	Action	X	$\neg X$
$\lambda^{(1)}$	a_P	$\lambda^{(1)}_{PP} = P(0.1, 0.8)$	$\lambda^{(1)}_{PN} = P(0.9, 0.2)$
	a_B	$\lambda^{(1)}_{BP} = P(0.6, 0.5)$	$\lambda^{(1)}_{BN} = P(0.5, 0.6)$
	a_N	$\lambda^{(1)}_{NP} = P(0.9, 0.3)$	$\lambda^{(1)}_{NN} = P(0.2, 0.8)$
$\lambda^{(2)}$	a_P	$\lambda^{(2)}_{PP} = P(0.2, 0.9)$	$\lambda^{(2)}_{PN} = P(0.8, 0.3)$
	a_B	$\lambda^{(2)}_{BP} = P(0.5, 0.7)$	$\lambda^{(2)}_{BN} = P(0.6, 0.5)$
	a_N	$\lambda^{(2)}_{NP} = P(0.8, 0.2)$	$\lambda^{(2)}_{NN} = P(0.1, 0.9)$
$\lambda^{(3)}$	a_P	$\lambda^{(3)}_{PP} = P(0.3, 0.9)$	$\lambda^{(3)}_{PN} = P(0.8, 0.1)$
	a_B	$\lambda^{(3)}_{BP} = P(0.5, 0.6)$	$\lambda^{(3)}_{BN} = P(0.6, 0.6)$
	a_N	$\lambda^{(3)}_{NP} = P(0.7, 0.1)$	$\lambda^{(3)}_{NN} = P(0.2, 0.9)$

By Theorem 4.3, we have the expected losses $R^{(i)}(a_P|x), R^{(i)}(a_B|x)$ and $R^{(i)}(a_N|x)$ with respect to the Pythagorean fuzzy loss function $\lambda^{(i)}$ as follows:

$$\begin{aligned} R^{(i)}(a_P|x) &= P(\sqrt{1 - (1 - \mu^2_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (1 - \mu^2_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \\ &\quad (\nu_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}); \\ R^{(i)}(a_B|x) &= P(\sqrt{1 - (1 - \mu^2_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (1 - \mu^2_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \\ &\quad (\nu_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}); \\ R^{(i)}(a_N|x) &= P(\sqrt{1 - (1 - \mu^2_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (1 - \mu^2_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \\ &\quad (\nu_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}). \end{aligned}$$

Theorem 5.2: Let $R^{(i)}(a_P|x)$, $R^{(i)}(a_B|x)$ and $R^{(i)}(a_N|x)$ be the expected losses under the actions a_P, a_B and a_N using the Pythagorean fuzzy loss function $\lambda^{(i)}$, respectively, for the object $x \in U$, and $\mathscr{W} = \{w_1, w_2, ..., w_m\}$ be the weight vector

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

of $R^{(i)}(a_\bullet|x)(i = 1, 2, ..., m, \bullet = P, B, N)$. Then

$$\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x)) = P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (1-\mu^2_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x))});$$
$$\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x)) = P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (1-\mu^2_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x))});$$
$$\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x)) = P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (1-\mu^2_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x))} * (\nu_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x))}).$$

Proof: It is straightforward by Theorem 4.3.□

We see that Theorem 5.2 illustrates that the expected losses $\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x))$, $\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x))$ and $\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x))$ are Pythagorean fuzzy numbers. Especially, it implies that how to compute the expected losses $\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x))$, $\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x))$ and $\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x))$ with the closeness index function $\mathscr{P}(\mathscr{R}(x))$.

Definition 5.3: Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, $\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x))$, $\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x))$ and $\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x))$ are the expected losses under the actions a_P, a_B and a_N, respectively, for the agent $x \in U$. Then the Group Expected Loss Table $\mathscr{R}(R(S))$, Group Score Table $\mathscr{S}(R(S))$, Group Closeness Table $\mathscr{P}(R(S))$ and Group Preferred Table $\mathscr{F}(R(S))$ are defined as TABLE XVI, TABLE XVII, TABLE XVIII and TABLE XIX, respectively.

Theorem 5.4: Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, $R(a_P|x)$, $R(a_B|x)$ and $R(a_N|x)$ are the expected losses under the actions a_P, a_B and a_N, respectively, for the agent $x \in U$, and $\bullet = I, S, \mathscr{P}, F$. Then

P^* : If $\bullet(\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x))) \leq \bullet(\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x)))$ and $\bullet(\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x))) \leq \bullet(\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x)))$, then we have $x \in POA(U)$;

B^* : If $\bullet(\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x))) \leq \bullet(\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x)))$ and $\bullet(\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x))) \leq \bullet(\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x)))$, then we have $x \in CTA(U)$;

N^* : If $\bullet(\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x))) \leq \bullet(\mathscr{R}(R^{(1)}(a_P|x), ..., R^{(m)}(a_P|x)))$ and $\bullet(\mathscr{R}(R^{(1)}(a_N|x), ..., R^{(m)}(a_N|x))) \leq \bullet(\mathscr{R}(R^{(1)}(a_B|x), ..., R^{(m)}(a_B|x)))$, then we have $x \in NEA(U)$.

Proof: It is straightforward by Bayesian minimum risk theory.□

Example 5.5: (Continuation from Example 5.1) Taking $w_1 = w_2 = w_3 = \frac{1}{3}$ for Pythagorean fuzzy loss functions $\{\lambda^{(i)}|i = 1, 2, 3\}$, we compute $POA(U), CTA(U)$ and $NEA(U)$ as follows:

(1) First, for $x_j \in U$, by Theorem 5.2, we have

$$I(\mathscr{R}(R^{(1)}(a_P|x_j), ..., R^{(m)}(a_P|x_j))) = P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))});$$
$$I(\mathscr{R}(R^{(1)}(a_B|x_j), ..., R^{(m)}(a_B|x_j))) = P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))});$$
$$I(\mathscr{R}(R^{(1)}(a_N|x_j), ..., R^{(m)}(a_N|x_j))) = P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}).$$

Second, by Definition 5.3, we have the Group Expected Loss Table $\mathscr{R}(R(S))$ as TABLE XX.

TABLE XX
THE GROUP EXPECTED LOSS TABLE $\mathscr{R}(R(S))$.

Action	P	B	N
x_1	P(0.4623,0.6731)	P(0.5412,0.5926)	P(0.7617,0.2503)
x_2	P(0.7203,0.3558)	P(0.5571,0.5769)	P(0.6020,0.4633)
x_3	P(0.7660,0.2929)	P(0.5609,0.5730)	P(0.5186,0.5679)
x_4	P(0.7380,0.3314)	P(0.5586,0.5754)	P(0.5746,0.4986)
x_5	P(0.7344,0.3364)	P(0.5583,0.5757)	P(0.5806,0.4909)
x_6	P(0.6987,0.3852)	P(0.5554,0.5786)	P(0.6295,0.4273)

Third, by Theorem 5.4, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA(U) = \emptyset$. It is difficult to compare the expected losses under the actions a_P, a_B and a_N for the agent x_3 by Definition 2.2. So we can not classify the agent x_3 into $POA(U), CTA(U)$ and $NEA(U)$.

Therefore, $\{x_1\}$ is a positive alliance, and $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance. We find that $\{x_1\}$ and $\{x_2, x_4, x_5, x_6\}$ are difference alliances.

(2) First, for $x_j \in U$, by Theorem 5.2, we have

$$S(\mathscr{R}(R^{(1)}(a_P|x_j), ..., R^{(m)}(a_P|x_j))) = S(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}));$$
$$S(\mathscr{R}(R^{(1)}(a_B|x_j), ..., R^{(m)}(a_B|x_j))) = S(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}));$$
$$S(\mathscr{R}(R^{(1)}(a_N|x_j), ..., R^{(m)}(a_N|x_j))) = S(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}}, \Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))})).$$

Second, by Definition 5.3, we have the Group Score Table $\mathscr{S}(R(S))$ shown in TABLE XXI as follows:

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

TABLE XVI
THE GROUP EXPECTED LOSS TABLE $\mathscr{R}(R(S))$.

Action	P	B	N
x_1	$\mathscr{R}(R^{(1)}(a_P\|x_1), ..., R^{(m)}(a_P\|x_1))$	$\mathscr{R}(R^{(1)}(a_B\|x_1), ..., R^{(m)}(a_B\|x_1))$	$\mathscr{R}(R^{(1)}(a_N\|x_1), ..., R^{(m)}(a_N\|x_1))$
x_2	$\mathscr{R}(R^{(1)}(a_P\|x_2), ..., R^{(m)}(a_P\|x_2))$	$\mathscr{R}(R^{(1)}(a_B\|x_2), ..., R^{(m)}(a_B\|x_2))$	$\mathscr{R}(R^{(1)}(a_N\|x_2), ..., R^{(m)}(a_N\|x_2))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$\mathscr{R}(R^{(1)}(a_P\|x_n), ..., R^{(m)}(a_P\|x_n))$	$\mathscr{R}(R^{(1)}(a_B\|x_n), ..., R^{(m)}(a_B\|x_n))$	$\mathscr{R}(R^{(1)}(a_N\|x_n), ..., R^{(m)}(a_N\|x_n))$

TABLE XVII
THE GROUP SCORE TABLE $\mathscr{S}(R(S))$.

Action	P	B	N
x_1	$S(\mathscr{R}(R^{(1)}(a_P\|x_1), ..., R^{(m)}(a_P\|x_1)))$	$S(\mathscr{R}(R^{(1)}(a_B\|x_1), ..., R^{(m)}(a_B\|x_1)))$	$S(\mathscr{R}(R^{(1)}(a_N\|x_1), ..., R^{(m)}(a_N\|x_1)))$
x_2	$S(\mathscr{R}(R^{(1)}(a_P\|x_2), ..., R^{(m)}(a_P\|x_2)))$	$S(\mathscr{R}(R^{(1)}(a_B\|x_2), ..., R^{(m)}(a_B\|x_2)))$	$S(\mathscr{R}(R^{(1)}(a_N\|x_2), ..., R^{(m)}(a_N\|x_2)))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$S(\mathscr{R}(R^{(1)}(a_P\|x_n), ..., R^{(m)}(a_P\|x_n)))$	$S(\mathscr{R}(R^{(1)}(a_B\|x_n), ..., R^{(m)}(a_B\|x_n)))$	$S(\mathscr{R}(R^{(1)}(a_N\|x_n), ..., R^{(m)}(a_N\|x_n)))$

TABLE XVIII
THE GROUP CLOSENESS TABLE $\mathscr{P}(R(S))$.

Action	P	B	N
x_1	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_P\|x_1), ..., R^{(m)}(a_P\|x_1)))$	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_B\|x_1), ..., R^{(m)}(a_B\|x_1)))$	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_N\|x_1), ..., R^{(m)}(a_N\|x_1)))$
x_2	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_P\|x_2), ..., R^{(m)}(a_P\|x_2)))$	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_B\|x_2), ..., R^{(m)}(a_B\|x_2)))$	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_N\|x_2), ..., R^{(m)}(a_N\|x_2)))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_P\|x_n), ..., R^{(m)}(a_P\|x_n)))$	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_B\|x_n), ..., R^{(m)}(a_B\|x_n)))$	$\mathscr{P}(\mathscr{R}(R^{(1)}(a_N\|x_n), ..., R^{(m)}(a_N\|x_n)))$

TABLE XIX
THE GROUP PREFERRED TABLE $\mathscr{F}(R(S))$.

Action	P	B	N
x_1	$F(\mathscr{R}(R^{(1)}(a_P\|x_1), ..., R^{(m)}(a_P\|x_1)))$	$F(\mathscr{R}(R^{(1)}(a_B\|x_1), ..., R^{(m)}(a_B\|x_1)))$	$F(\mathscr{R}(R^{(1)}(a_N\|x_1), ..., R^{(m)}(a_N\|x_1)))$
x_2	$F(\mathscr{R}(R^{(1)}(a_P\|x_2), ..., R^{(m)}(a_P\|x_2)))$	$F(\mathscr{R}(R^{(1)}(a_B\|x_2), ..., R^{(m)}(a_B\|x_2)))$	$F(\mathscr{R}(R^{(1)}(a_N\|x_2), ..., R^{(m)}(a_N\|x_2)))$
.	.	.	.
.	.	.	.
.	.	.	.
x_n	$F(\mathscr{R}(R^{(1)}(a_P\|x_n), ..., R^{(m)}(a_P\|x_n)))$	$F(\mathscr{R}(R^{(1)}(a_B\|x_n), ..., R^{(m)}(a_B\|x_n)))$	$F(\mathscr{R}(R^{(1)}(a_N\|x_n), ..., R^{(m)}(a_N\|x_n)))$

TABLE XXI
THE GROUP SCORE TABLE $\mathscr{S}(R(S))$.

Action	P	B	N
x_1	-0.2393	-0.0583	0.5176
x_2	0.3922	-0.0224	0.1478
x_3	0.5009	-0.0137	-0.0536
x_4	0.4349	-0.0191	0.0816
x_5	0.4263	-0.0198	0.0961
x_6	0.3398	-0.0262	0.2137

Third, by Theorem 5.4, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA(U) = \{x_3\}$. Therefore, $\{x_1\}$ is a positive alliance, $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance, and $\{x_3\}$ is a negative alliance.

(3) First, for $x_j \in U$, by Theorem 5.2, we have

$$\mathscr{P}(\mathscr{R}(R^{(1)}(a_P|x_j), ..., R^{(m)}(a_P|x_j))) =$$
$$\mathscr{P}(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}},$$
$$\Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}));$$
$$\mathscr{P}(\mathscr{R}(R^{(1)}(a_B|x_j), ..., R^{(m)}(a_B|x_j))) =$$
$$\mathscr{P}(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}},$$
$$\Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}));$$
$$\mathscr{P}(\mathscr{R}(R^{(1)}(a_N|x_j), ..., R^{(m)}(a_N|x_j))) =$$
$$\mathscr{P}(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}},$$
$$\Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))})).$$

Second, by Definition 5.3, we have the Group Closeness Table $\mathscr{P}(R(S))$ shown in TABLE XXII as follows:

TABLE XXII
THE GROUP CLOSENESS TABLE $\mathscr{P}(R(S))$.

Action	P	B	N
x_1	0.4103	0.4785	0.6907
x_2	0.6448	0.4918	0.5519
x_3	0.6887	0.4950	0.4810
x_4	0.6616	0.4930	0.5287
x_5	0.6582	0.4927	0.5338
x_6	0.6246	0.4903	0.5752

Third, by Theorem 5.4, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA(U) = \{x_3\}$. Therefore, $\{x_1\}$ is a positive alliance, $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance, and $\{x_3\}$ is a negative alliance.

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

(4) First, for $x_j \in U$, by Theorem 5.2, we have

$$F(\mathscr{R}(R^{(1)}(a_P|x_j), ..., R^{(m)}(a_P|x_j))) =$$
$$F(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}},$$
$$\Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{PP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{PN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}));$$
$$F(\mathscr{R}(R^{(1)}(a_B|x_j), ..., R^{(m)}(a_B|x_j))) =$$
$$F(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}},$$
$$\Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{BP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{BN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}));$$
$$F(\mathscr{R}(R^{(1)}(a_N|x_j), ..., R^{(m)}(a_N|x_j))) =$$
$$F(P(\Sigma_{i=1}^m w_i * \sqrt{1-(1-\mu^2_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (1-\mu^2_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))}},$$
$$\Sigma_{i=1}^m w_i * (\nu_{\lambda^{(i)}_{NP}})^{\mathscr{P}(\mathscr{R}(x_j))} * (\nu_{\lambda^{(i)}_{NN}})^{1-\mathscr{P}(\mathscr{R}(x_j))})).$$

Second, by Definition 5.3, we have the Group Preferred Table $\mathscr{F}(R(S))$ shown in TABLE XXIII as follows:

TABLE XXIII
THE GROUP PREFERRED TABLE $\mathscr{F}(R(S))$.

Action	P	B	N
x_1	0.4046	0.4768	0.7389
x_2	0.6670	0.4911	0.5626
x_3	0.7193	0.4946	0.4778
x_4	0.6871	0.4924	0.5343
x_5	0.6830	0.4922	0.5404
x_6	0.6430	0.4896	0.5916

Third, by Theorem 5.4, we have $POA(U) = \{x_1\}, CTA(U) = \{x_2, x_4, x_5, x_6\}$ and $NEA(U) = \{x_3\}$. Therefore, $\{x_1\}$ is a positive alliance, $\{x_2, x_4, x_5, x_6\}$ is a neutral alliance, and $\{x_3\}$ is a negative alliance.

(5) First, we list the alliances computed using $\mathscr{R}(R(S)), \mathscr{S}(R(S)), \mathscr{P}(R(S))$ and $\mathscr{F}(R(S))$ in Table XXIV. Concretely, we have the same alliances with $\mathscr{S}(R(S)), \mathscr{P}(R(S))$ and $\mathscr{F}(R(S))$, which are different from the results with $\mathscr{R}(R(S))$; all agents are classified into the positive, neutral, and negative alliances with $\mathscr{S}(R(S)), \mathscr{P}(R(S))$ and $\mathscr{F}(R(S))$, but we can not put x_3 into any alliance with $\mathscr{R}(R(S))$; almost all agents are classified into the neutral alliances with $\mathscr{R}(R(S)), \mathscr{S}(R(S)), \mathscr{P}(R(S))$ and $\mathscr{F}(R(S))$, and less agents are put into the positive and negative alliances. Second, we have the positive alliance $\{x_1\}$ and the neutral alliance $\{x_2, x_4, x_5, x_6\}$ by $\mathscr{R}(R(S))$; we get the positive alliance $\{x_1\}$, the neutral alliance $\{x_2, x_4, x_5, x_6\}$ and the negative alliance $\{x_3\}$ by $\mathscr{S}(R(S)), \mathscr{P}(R(S))$ and $\mathscr{F}(R(S))$. So we find that $\{x_1\}$ and $\{x_3\}$ belong to the positive alliance and the neutral alliance, respectively, so they hold different opinions on most of issues. In other words, they are opponents. We also see that $\{x_1\}$ and $\{x_3\}$ are single alliances, if they want to get supports from other countries, then they must change opinions on some issues. Third, we put the agent x_3 into the neutral alliance in Example 4.6 with a loss function, and we assign the agent x_3 to the negative alliance in Example 5.5 with three loss functions. So we find that three-way group method is more effective than three-way method for conflict analysis of Pythagorean fuzzy information systems. Therefore, we should study how to compute the expected losses of actions with more loss functions or other types of Pythagorean fuzzy loss functions and provide more effective approaches for studying Pythagorean fuzzy information systems for conflicts in the future.

TABLE XXIV
THREE ALLIANCES BASED ON $\mathscr{R}(R(S)), \mathscr{S}(R(S)), \mathscr{P}(R(S))$ AND $\mathscr{F}(R(S))$.

Method	$POA(U)$	$CTA(U)$	$NEA(U)$
$\mathscr{R}(R(S))$	$\{x_1\}$	$\{x_2, x_4, x_5, x_6\}$	$\emptyset$
$\mathscr{S}(R(S))$	$\{x_1\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_3\}$
$\mathscr{P}(R(S))$	$\{x_1\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_3\}$
$\mathscr{F}(R(S))$	$\{x_1\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_3\}$

VI. CONCLUSIONS AND FUTURE WORK

In the era of Big Data, the study of conflicts is of greatest importance both practically and theoretically for Human Society. In this paper, we have presented the concepts of positive, neutral, and negative alliances with two thresholds, and employed examples to illustrate how to construct the positive, neutral and negative alliances in Pythagorean fuzzy information systems for conflicts. Moreover, we have studied three-way conflict analysis of Pythagorean fuzzy information systems based on Bayesian minimum risk theory and employed examples to illustrate how to compute different alliances with a Pythagorean fuzzy loss function given by an expert. Finally, we have investigated three-way group conflict analysis of Pythagorean fuzzy information systems and explored examples to illustrate how to calculate different alliances with a group of Pythagorean fuzzy loss functions given by more experts.

In the future, we will study dynamic Pythagorean fuzzy information systems for conflicts. Furthermore, we will provide effective algorithms for conflict analysis of dynamic Pythagorean fuzzy information systems in the future.

ACKNOWLEDGEMENTS

We would like to thank the reviewers very much for their professional comments and valuable suggestions.

REFERENCES

[1] Z. Pawlak, "An inquiry into anatomy of conflicts," *Information Sciences*, vol. 109, no. 1-4, pp. 65-78, Aug. 1998.

[2] Z. Pawlak, "Some remarks on conflict analysis," *European Journal of Operational Research*, vol. 166, no. 3, pp. 649-654, Nov. 2005.

[3] Z. Pawlak, "On conflicts," *International Journal of Man-Machine Studies*, vol. 21, no. 2, pp. 127-134, 1984.

[4] L. Cholvy, L. Perrussel, J.M. Thevenin, "Using inconsistency measures for estimating reliability," *International Journal of Approximate Reasoning*, vol. 89 pp. 41-57, Oct. 2017.

[5] R. Deja, "Conflict model with negotiation," *Bulletin of the Polish Academy of Sciences*, vol. 44, no. 4, pp. 475-498, 1996.

[6] R. Deja, "Conflict analysis, rough set methods and applications," *Studies in Fuzzyness and Soft Computing*, 2000, pp. 491-520.

[7] R. Deja, "Conflict analysis," *International Journal of Intelligent Systems*, vol. 17, pp. 235-253, 2002.

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

[8] Y.C. Jiang, Y. Tang, Q.M. Chen, Z.M. Cao, "Semantic operations of multiple soft sets under conflict," *Computers and Mathematics with Applications*, vol. 62, no. 4, pp. 1923-1939, Aug. 2011.

[9] S. Jabbour, Y. Ma, B. Raddaoui, L. Sais, "Quantifying conflicts in propositional logic through prime implicates," *International Journal of Approximate Reasoning*, vol. 89, pp. 27-40, Oct. 2017.

[10] Y. Liu, Y. Lin, "Intuitionistic fuzzy rough set model based on conflict distance and applications," *Applied Soft Computing*, vol. 31, pp. 266-273, June 2015.

[11] N.T. Nguyen, M. Malowiecki, "Consistency measures for conflict profiles," *Transactions on Rough Sets I*, vol. 3100, pp. 169-186, 2004.

[12] S. Ramanna, A. Skowron, "Requirements interaction and conflicts: A rough set approach," *in: Proc. IEEE Symposium Series on Foundations of Computational Intelligence*, 2007.

[13] S. Ramanna, J. Peters, A. Skowron, "Approaches to conflict dynamics based on rough sets," *Fundamenta Informaticae*, vol. 75, no. 1-4, pp. 453-468, 2007.

[14] L.G. de O. Silva, A.T. de Almeida-Filho, "A multicriteria approach for analysis of conflicts in evidence theory," *Information Sciences*, vols. 346-347, pp. 275-285, June 2016.

[15] A. Skowron, R. Deja, "On some conflict models and conflict resolutions," *Romanian Journal of Information Science and Technology*, vol. 5, nos. 1-2, pp. 69-82, 2002.

[16] A. Skowron, S. Ramanna, J. Peters, "Conflict analysis and information systems: a rough set approach," *Rough Sets and Current Trends in Computing*, vol. 4062, pp. 233-240, 2006.

[17] B.Z. Sun, W.M. Ma, "Rough approximation of a preference relation by multi-decision dominance for a multi-agent conflict analysis problem," *Information Sciences*, vol. 315, pp. 39-53, Sep. 2015.

[18] B.Z. Sun, W.M. Ma, H.Y. Zhao, "Rough set-based conflict analysis model and method over two universes," *Information Sciences*, vol. 372, pp. 111-125, Dec. 2016.

[19] J.P. Yang, H.Z. Huang, Q. Miao, R. Sun, "A novel information fusion method based on Dempster-Shafer evidence theory for conflict resolution," *Intelligent Data Analysis*, vol. 15, pp. 399-411, 2011.

[20] C. Yu, J. Yang, D. Yang, X. Ma, H. Min, "An improved conflicting evidence combination approach based on a new supporting probability distance," *Expert Systems with Applications*, vol. 42, no. 12, pp. 5139-5149, July 2015.

[21] W. Zhu, F.Y. Wang, "Covering based granular computing for conflict analysis," *Rough Sets Current Trends Computing*, vol. 3975, pp. 566-571, 2006.

[22] Y.Y. Yao, "Three-way decisions with probabilistic rough sets," *Information Sciences*, vol. 180, no. 3, pp. 341-353, Feb. 2010.

[23] Y.F. Chen, X.D. Yue, H. Fujita, S.Y. Fu, "Three-way decision support for diagnosis on focal liver lesions," *Knowledge-Based Systems*, vol. 127, pp. 85-99, July 2017.

[24] T. Feng, H.T. Fan, J.S. Mi, "Uncertainty and reduction of variable precision multigranulation fuzzy rough sets based on three-way decisions," *International Journal of Approximate Reasoning*, vol. 85, pp. 36-58, June 2017.

[25] B.Q. Hu, H. Wong, K.C. Yiu, "On two novel types of three-way decisions in three-way decision spaces," *International Journal of Approximate Reasoning*, vol. 82, pp. 285-306, Mar. 2017.

[26] M.T. Khan, N. Azam, S. Khalid, J.T. Yao, "A three-way approach for learning rules in automatic knowledge-based topic models," *International Journal of Approximate Reasoning*, vol. 82, pp. 210-226, Mar. 2017.

[27] G.M. Lang, D.Q. Miao, M.J. Cai, "Three-way decision approaches to conflict analysis using decision-theoretic rough set theory," *Information Sciences*, vol. 406, pp. 185-207, Sep. 2017.

[28] J.H. Li, C.C. Huang, J.J. Qi, Y.H. Qian, W.Q. Liu, "Three-way cognitive concept learning via multi-granularity," *Information Sciences*, vol. 378, pp. 244-263, Feb. 2017.

[29] X.N. Li, H.J. Yi, Y.H. She, B.Z. Sun, "Generalized three-way decision models based on subset evaluation," *International Journal of Approximate Reasoning*, vol. 83, pp. 142-159, April 2017.

[30] H.X. Li, L.B. Zhang, X.Z. Zhou, B. Huang, "Cost-sensitive sequential three-way decision modeling using a deep neural network," *International Journal of Approximate Reasoning*, vol. 85, pp. 68-78, June 2017.

[31] J. Qian, C.Y. Dang, X.D. Yue, N. Zhang, "Attribute reduction for sequential three-way decisions under dynamic granulation," *International Journal of Approximate Reasoning*, vol. 85, pp. 196-216, June 2017.

[32] B.Z. Sun, W.M. Ma, X. Xiao, "Three-way group decision making based on multigranulation fuzzy decision-theoretic rough set over two universes," *International Journal of Approximate Reasoning*, vol. 81, pp. 87-102, Feb. 2017.

[33] J.F. Xu, D.Q. Miao, Y.J. Zhang, Z.F. Zhang, "A three-way decisions model with probabilistic rough sets for stream computing," *International Journal of Approximate Reasoning*, vol. 88, pp. 1-22, Sep. 2017.

[34] Y.Y. Yao, "Probabilistic rough set approximations," International *Journal of Approximate Reasoning*, vol. 49, no. 2, pp. 255-271, Oct. 2008.

[35] Y.Y. Yao, "Three-way decision and granular computing," *International Journal of Approximate Reasoning*, vol. 103, pp. 107-123, Sep. 2018.

[36] Y.Y. Yao, S. Wang, X.F. Deng, "Constructing shadowed sets and three-way approximations of fuzzy sets," *Information Sciences*, vols. 412-413, pp. 132-153, Oct. 2017.

[37] X. Yang, T.R. Li, H. Fujita, D. Liu, Y.Y. Yao, "A unified model of sequential three-way decisions and multilevel incremental processing," *Knowledge-Based Systems*, vol. 134, pp. 172-188, Oct. 2017.

[38] H.R. Zhang, F. Min, B. Shi, "Regression-based three-way recommendation," *Information Sciences*, vol. 378, pp. 444-461, Feb. 2017.

[39] R.R. Yager, A.M. Abbasov, "Pythagorean membership grades, complex numbers, and decision making," *International Journal of Intelligent Systems*, vol. 28, no. 5, pp. 436-452, May 2013.

[40] K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87-96, Aug. 1986.

[41] G. Beliakov, S. James, "Averaging aggregation functions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs," *in: Proceedings of 2014 IEEE International Conference on Fuzzy Systems(FUZZ-IEEE)*, IEEE, 2014, pp. 298-305.

[42] H. Bustince, E. Barrenechea, M. Pagola, J. Fernandez, Z.S. Xu, B. Bedregal, J. Montero, H. Hagras, F. Herrera, B. DeBaets, "A historical account of types of fuzzy sets and their relationships," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 1, pp. 179-194, Feb. 2016.

[43] G.M. Lang, D.Q. Miao, Z.F. Zhang, N. Yao, "Conflict Analysis for Pythagorean Fuzzy Information Systems," *Lecture Notes in Computer Science*, vol. 10314, pp. 359-367, Jun. 2017.

[44] D.C. Liang, Z.S. Xu, "The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets," *Applied Soft Computing*, vol. 60, pp. 167-179, Jun. 2017.

[45] X.D. Peng, Y. Yang, "Some results for Pythagorean Fuzzy Sets," *International Journal of Intelligent Systems*, vol. 30, no. 11, pp. 1133-1160, Nov. 2015.

[46] X.D. Peng, G. Selvachandran, "Pythagorean fuzzy set: state of the art and future directions," *Artificial Intelligence Review*, https://doi.org/10.1007/s10462-017-9596-9, Nov. 2017.

[47] X.D. Peng, J.G. Dai, "Approaches to Pythagorean Fuzzy Stochastic Multi-criteria Decision Making Based on Prospect Theory and Regret Theory with New Distance Measure and Score Function," *International Journal of Intelligent Systems*, vol. 32, pp. 1187-1214, Nov. 2017.

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TFUZZ.2019.2908123, IEEE Transactions on Fuzzy Systems

[48] X.D. Peng, H. Yuan, Y.Yang, "Pythagorean fuzzy information measures and their applications," *International Journal of Intelligent Systems*, *International Journal of Intelligent Systems*, vol. 32, no. 10, pp. 991-1029, Feb. 2017.

[49] X.D. Peng, "New Operations for Interval-valued Pythagorean Fuzzy Set," *Scientia Iranica*, https://doi.10.24200/SCI.2018.5142.1119, Mar. 2018.

[50] M.Z. Reformat, R.R. Yager, "Suggesting recommendations using Pythagorean fuzzy sets illustrated using netflix movie data," *in: Information Processing and Management of Uncertainty in Knowledge-Based Systems*, Springer, 2014, pp. 546-556.

[51] P.J. Ren, Z.S. Xu, X.J. Gou, "Pythagorean fuzzy TODIM approach to multi-criteria decision making," *Applied Soft Computing*, vol. 42, pp. 246-259, May 2016.

[52] Z.P. Wu, Y.Y. Liu, "Knowledge augmented policy conflict analysis for services collaboration," *Knowledge-Based Systems*, vol. 62, pp. 11-27, May 2014.

[53] R.R. Yager, "Pythagorean membership grades in multicriteria decision making," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958-965, Aug. 2014.

[54] R.R. Yager, "Pythagorean membership grades in multicriteria decision making," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958-965, Aug. 2014.

[55] Y. Yang, H. Ding, Z.S. Chen, Y.L. Li, "A note on Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets," *International Journal of Intelligent Systems*, vol. 31, no. 1, pp. 68-72, Jan. 2016.

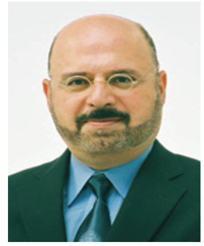
[56] X.L. Zhang, "Multicriteria Pythagorean fuzzy decision analysis: Ahierarchical QUALIFLEX approach with the closeness index-based ranking methods," *Information Sciences*, vol. 330, pp.104-124, Feb. 2016.

[57] X.L. Zhang, Z.S. Xu, "Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets," *International Journal of Intelligent Systems*, vol. 29, no. 12, pp. 1061-1078, Dec. 2014.

[58] C. Zhang, D.Y. Li, R. Ren, "Pythagorean Fuzzy Multigranulation Rough Set over Two Universes and Its Applications in Merger And Acquisition," *International Journal of Intelligent Systems*, vol. 31, no. 9, pp. 921-943, Sep. 2016.

Guangming Lang received the Ph.D. degree in Mathematics from Hunan University, Changsha, China, in 2013. He is currently a Associate Professor in the School of Mathematics and Statistics, Changsha University of Science and Technology. His main research interests include Granular computing, Rough set theory, Fuzzy set theory, Lattice Theory and Three-way decision theory.

Duoqian Miao received the Ph.D. degree in pattern recognition and intelligent system from the Institute of Automation, Chinese Academy of Sciences, Beijing, China, in 1997. He is currently a Professor with the School of Electronics and Information Engineering and the Key Laboratory of Embedded System and Service Computing, Ministry of Education, Tongji University, Shanghai, China. His main research interests include Soft computing, Rough sets, Pattern recognition, Data mining, Machine learning and Intelligent systems.



Hamido Fujita received the Ph.D. degree in Information Science from Tohoku University, Sendai, Japan, in 1988. He is a Chair Professor in the Faculty of Software and Information Science, Iwate Prefectural University. His main research interests include Fuzzy set theory, Cloud computing, Decision support systems, Emergency management, Feature selection, Fuzzy logic, Granular computing, Health care, Image fusion, Social sciences and Medical computing.

1063-6706 (c) 2018 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.