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Trisection-fusion and fusion-trisection methods of three-way conflict analysis with Pythagorean fuzzy information

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ABSTRACT

A basic task of Pawlak conflict analysis is to cluster agents based on their attitudes towards various issues. When ratings of agents are imprecise and uncertain, for example, as represented by a Pythagorean fuzzy situation table, single-measure based methods fall short in ensuring the accuracy of agent clusters. Multi-measure based conflict analysis methods are more effective for generating higher-quality agent clusters. The objective of this paper is to propose two new multi-measure based methods under uncertainty represented by Pythagorean fuzzy sets. We introduce three distinct conflict measures regarding an issue by leveraging the maximum positive and negative agents. The first measure is based on support degrees, the second measure incorporates opposition degrees, and the third measure considers both support and opposition degrees. These measures trisect a set of agents into three disjoint coalitions concerning an individual issue. For multiple issues, we propose two models by combing the two fundamental tasks, namely, trisection and fusion. A trisection-fusion model amalgamates a family of trisections generated from multiple issues based on conflict measures regarding single issues. Matrix representations of trisections are provided, and trisection fusions are transformed into a series of matrix operations. A fusion-trisection model trisects the set of agents according to fused conflict measures on multiple issues. To demonstrate the value of the two models, we apply them in assisting decision-makers in Hunan Province, China, to adjust development plans. The experimental results show that multi-measure based models offer decision-makers more comprehensive guidance.

1. Introduction

Conflicts have naturally arisen since the emergence of human society, and motivated the field of conflict analysis. This field primarily involves the examination of the essence of conflicts and the exploration of potential conflict resolutions. In 1998, Pawlak [1] characterized conflict problems by using a three-valued situation table, and categorized sets of agent pairs into alliance, neutrality, and opposition relations through an auxiliary function. Following this, many researchers have explored three-valued situation tables from various perspectives.

Three-way decisions, proposed by Yao [2–4], represent a conceptual framework that involves contemplating and addressing problems in three aspects. Since its inception in 2010, this theory has merged with machine learning and found applications in diverse fields [5–7]. In particular, three-way decisions has been integrated with conflict analysis. Recently, Lang, Miao, and Fujita [8] investigated Pythagorean fuzzy situation tables for three-way conflict analysis. They divided an agent set into three coalitions concerning an issue set. Notably, in Example 5.5 of their study, it was noted that the majority of agents were grouped into the neutral coalition, while only a few were allocated to the support and opposition coalitions. Two possible reasons for this observation were identified: either the majority of agents genuinely belonged to the neutral coalition based on the aggregation of their attitudes on all issues, or the evaluation functions inadequately captured the systematic attitudes of agents. It is observed that models of conflict analysis frequently utilize evaluation functions such as conflict measures to assess and analyze conflicts. Different evaluation functions consider agents' attitudes on issues from various perspectives, and each one has its advantages and disadvantages. Specifically, they lead to different trisections of an agent set. Drawing inspiration from the Chinese poet Shi Su's words in the West Forest Temple: "It's a range

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viewed in face and peaks viewed from the side, assuming different shapes viewed from far and wide. Of Mountain Lu we cannot make out the true face, for we are lost in the heart of the very place", and another proverb "don't put all your eggs in one basket", there arises a necessity to integrate agents' attitudes towards issues through the utilization of diverse evaluation functions. This motivation drives the formulation of conflict measures that incorporate Pythagorean fuzzy information, as well as the development of conflict analysis models employing multi-measure based methods.

By combining the two basic operations, namely, trisection and fusion, of three-way conflict analysis, we propose a trisection-fusion method and a fusion-trisection method. The contributions of this paper are summarized as follows.

(1) For a Pythagorean fuzzy situation table, we initially identify the maximum positive and negative agents. Subsequently, we formulate three distinct conflict measures concerning a single issue, as well as three measures towards multiple issues. These measures encompass the utilization of the support degree, opposition degree, and a combination of both support and opposition degrees. It is important to note that the initial three conflict measures can be regarded as specific instances derived from the latter three measures. Their design is tailored for trisecting an agent set using diverse fusion mechanisms. The three types of conflict measures provide insights into the relationship between two agents from three unique perspectives.
(2) We achieve a trisection of an agent set through the application of three distinct conflict measures regarding an individual issue. Subsequently, we establish a comprehensive trisection of the agent set across multiple issues by amalgamating a collection of trisections towards individual issues. This involves fusing a family of trisections directed at a single issue. Following this, we proceed to trisect the agent set using three conflict measures with regard to multiple issues. The final trisection of the agent set is then derived by merging a family of trisections towards various issues collectively. The former approach initiates the trisection process by focusing on the trisection action related to an individual issue. In contrast, the latter approach prioritizes the fusion of Pythagorean fuzzy information regarding multiple issues as the primary step.
(3) We introduce matrix representations of the support, neutral, and opposition coalitions concerning both a single issue and multiple issues. We then translate the fusion of a family of trisections into a series of matrix operations. Most importantly, we utilize trisection-fusion and fusion-trisection models of conflict analysis to aid decision-makers in refining policies during the formulation of development plans. In contrast to conflict analysis relying on a single measure, the multi-measure approach furnishes a more comprehensive set of information, and guides decision-makers towards more reasoned actions.

The rest of the paper is structured as follows. Related works are introduced in Section 2. Section 3 recalls some concepts of three-way conflict analysis. In Section 4, a trisection-fusion model of conflict analysis is developed, while Section 5 presents a fusion-trisection model of conflict analysis. Applications of the two conflict analysis models are explored in Section 6, and Section 7 concludes this paper and outlines future work.

2. Related works

In this section, we review the literature on three-way conflict analysis and Pythagorean fuzzy sets.

2.1. Three-way conflict analysis

In 2019, Yao [9] pioneered three-way conflict analysis by integrating three-way decisions with conflict analysis, and reformulated Pawlak's model of conflict analysis with distance functions. After that, many researchers have integrated three-way decisions with conflict analysis. For instance, Feng, Yang, and Guo [10] utilized the CRITIC method to assign issue weights and achieved a trisection of the agent set using Bayesian minimum risk theory. Hu [11] applied quantitative subsethood measures to formulate relations among agents and issues, and interpreted various popular relations on agents and issues within the designed framework. Lang and Yao [12] explored the relationship between two agents using novel alliance and conflict measures, and presented generalized models for conflict analysis when trisecting agent pairs. Lang and Yao [13] converted three-valued situation tables into formal contexts, and offered formal concept analysis insights into three-way conflict analysis. Li et al. [14] depicted conflict problems by utilizing triangular fuzzy situation tables, and demonstrated how to cluster agents whose attitudes on issues are reflected by triangular fuzzy numbers. Li and Yan [15] designed new fuzzy granules and operators for three-way conflict analysis models, and studied how to perform three-way conflict analysis with adaptive thresholds in dynamic situation tables. Du et al. [16] introduced conflict functions utilizing Pythagorean fuzzy information and considering both absolute and relative conflicts, and proposed models of three-way conflict analysis aimed at identifying optimal feasible strategies for conflict resolution. Jiang et al. [17] exploited complex situation tables, and conducted three-way conflict analysis by considering decision makers's varying preferences. Instead of a solitary auxiliary function, Luo et al. [18] defined alliance, neutrality, and conflict relations using both alliance and conflict functions, and crafted strategies for conflict resolution presented in the format of issue-rating pairs. To tackle conflict problems represented by q-rung orthopair fuzzy information, Li, Qiao, and Ding [19] formulated a model for three-way conflict analysis grounded in three-way decisions and decision-theoretic rough sets. Sun et al. [20] developed three-way decision approaches to finding a feasible consensus strategy for conflict situations with the probabilistic rough sets over two universes. Suo and Yang [21] demonstrated the process of designing conflict analysis models specifically for clustering agent pairs when certain attitudes of agents are not available. Wang et al. [22] designed models of three-way conflict analysis aimed at categorizing alliance, neutrality, and conflict relations with interval-valued Pythagorean fuzzy sets and prospect theory. Xu and Jia [23] partitioned agent pairs into three segments based on multiple issues, and redefined three-level conflicts and alliances using similarity and difference functions. Yang, Yao, and Qin [24] introduced opposition-alliance and support-alliance measures to categorize the nine pairs of ratings, and presented a lattice-theoretic framework for three-way conflict analysis. Zhang and Chen [25] devised three hierarchical three-way decision models to enhance the Pawlak's conflict analysis model. Zhi, Qi, Qian et al. [26] employed one-vote veto and approximate three-way concepts to derive the allied, conflict, and neutral attributes of cliques. Zhi, Li, and Li [27] provided a comprehensive framework for multi-level conflict analysis from the perspective of fuzzy formal concept analysis, which demonstrates how to update conflict analysis results as analysis levels increase. The research on three-way conflict analysis is growing as three-way decisions evolves.

2.2. Pythagorean fuzzy sets

In 2013, Yager [28,29] introduced Pythagorean fuzzy sets as an extension of Intuitionistic fuzzy sets. Subsequently, numerous researchers have enriched the field of Pythagorean fuzzy sets from various perspectives. For instance, Garg [30] introduced linguistic Pythagorean fuzzy sets to articulate uncertain information, and developed aggregation operators to fuse linguistic Pythagorean fuzzy numbers. Jia and

Herrera-Viedma [31] transformed the parameters A and B in Z-numbers into the membership and non-membership of Pythagorean fuzzy sets, and calculated the weighted information entropy of Z-numbers for decision making. Kumar and Chen [32] proposed a novel entropy measure for Pythagorean fuzzy sets, and tackled decision-making challenges within Pythagorean fuzzy environments. Borrowing advantages from hesitant fuzzy sets, Liang and Xu [33] introduced hesitant Pythagorean fuzzy sets, and designed the TOPSIS method for handling uncertain problems in hesitant fuzzy environments. Liang and Li [34] performed failure mode and effect analysis by improving the ORESTE method with hesitant Pythagorean fuzzy information. Pan, Gao, Deng, et al. [35] defined constrained Pythagorean fuzzy sets to describe fuzzy information and stochastic information, and designed similarity measures for such sets. Pan, Deng, and Cheong [36] defined quaternion Pythagorean fuzzy sets to describe fuzzy information, and studied its applications in expert evaluation and data-driven environments. Peng, Zhang, and Luo [37] defined new score functions to compare two Pythagorean fuzzy sets, and designed the objective weight by the Criteria Importance Through Inter-criteria Correlation method. Peng, Huang, and Luo [38] derived Pythagorean fuzzy aggregation operators to amalgamate multiple group information, and offered methods for multi-criteria group decision-making within a Pythagorean fuzzy set pair analysis environment. Rani, Chen, and Mishra [39] utilized standard deviations to ascertain attribute weights with Pythagorean fuzzy information. They devised multiple attribute decision-making approaches to discern the preference orders of alternatives. Sarkar, Chakrabarty, and Biswas [40] defined type-2 Pythagorean fuzzy sets to address multicriteria decision-making problems, and applied this methodology to the selection of sustainable transport systems. Motivated by Jensen–Shannon divergence, Xiao and Ding [41] defined a divergence measure to describe the difference between two Pythagorean fuzzy sets, and employed it to handle uncertain information in medical diagnosis. Zhang and Chen [42] delineated the concept of multiplicative consistency for Pythagorean fuzzy preference relations, alongside a group consensus index designed to quantify the degrees of similarity among these relations. Their aim is to facilitate group decision-making in Pythagorean fuzzy environments.

3. An overview of three-way concept analysis with pythagorean fuzzy sets

This section briefly reviews basic concepts of Pythagorean fuzzy sets and three-way conflict analysis.

Definition 1 (*Yager [28], 2013*)**.** Let **Ob** be a set of objects, and two functions $\varphi : \mathbf{Ob} \rightarrow [0,1]$ and $\psi : \mathbf{Ob} \rightarrow [0,1]$. A Pythagorean fuzzy set $\mathcal{P}$ is defined by:

$$\mathcal{P} = \{(ob, \mathcal{P}(\varphi(ob), \psi(ob))) \mid ob \in \mathbf{Ob}\},$$

where $\varphi^2(ob) + \psi^2(ob) \leq 1$. Furthermore, $\varphi(ob)$, $\psi(ob)$, and $\pi(ob)$ represent the membership, non-membership, and hesitant degrees of ob in $\mathcal{P}$, where $\pi(ob) = \sqrt{1 - \varphi^2(ob) - \psi^2(ob)}$. The two-tuple $\gamma = \mathcal{P}(\varphi(ob), \psi(ob))$ is known as a Pythagorean fuzzy number (PFN). For convenience, we use $\gamma = \mathcal{P}(\varphi_\gamma, \psi_\gamma)$ and $\mathbb{P}$ to denote PFN $\gamma = \mathcal{P}(\varphi(ob), \psi(ob))$ and the set of all PFNs, respectively.

Definition 2 (*Lang et al. [8], 2020*)**.** A Pythagorean fuzzy situation table is defined by a 4-tuple $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, where

- $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$ is a set of agents;
- $\mathcal{D} = \{d_1, d_2, \ldots, d_m\}$ is a set of issues;
- $\rho : \mathcal{A} \times \mathcal{D} \rightarrow \mathcal{V}$ is a function from $\mathcal{A} \times \mathcal{D}$ to $\mathcal{V}$;
- $\mathcal{V} = \{V_d \mid d \in \mathcal{D}\} \subseteq 2^{\mathbb{P}}$ is a family of sets of PFNs, where V_d represents the attitude set of all agents on the issue $d \in \mathcal{D}$.

A Pythagorean fuzzy number (PFN) $\mathcal{P}(\varphi_\gamma, \psi_\gamma)$ captures an agent's attitude by considering two fundamental aspects: support and opposition. Notably, the ranges of both the support degree and opposition degree are broader compared to those defined by Intuitionistic fuzzy numbers (IFN). Furthermore, the Pythagorean fuzzy numbers $\mathcal{P}(1.0, 0.0)$ and $\mathcal{P}(0.0, 1.0)$ specifically denote an unequivocal support attitude and an unequivocal opposition attitude, respectively, regarding a given issue.

Table 1
A Pythagorean fuzzy situation table [8].

$\mathcal{A}$ \ $\mathcal{D}$	d_1	d_2	d_3	d_4	d_5
a_1	$\mathcal{P}(1.0, 0.0)$	$\mathcal{P}(0.9, 0.3)$	$\mathcal{P}(0.8, 0.2)$	$\mathcal{P}(0.9, 0.1)$	$\mathcal{P}(0.9, 0.2)$
a_2	$\mathcal{P}(0.9, 0.1)$	$\mathcal{P}(0.6, 0.8)$	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.3, 0.8)$	$\mathcal{P}(0.1, 0.9)$
a_3	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.2, 0.8)$	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.6, 0.8)$
a_4	$\mathcal{P}(0.8, 0.4)$	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.3, 0.8)$	$\mathcal{P}(0.5, 0.8)$	$\mathcal{P}(0.1, 0.9)$
a_5	$\mathcal{P}(0.9, 0.2)$	$\mathcal{P}(0.4, 0.8)$	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.1, 0.9)$	$\mathcal{P}(0.3, 0.9)$
a_6	$\mathcal{P}(0.0, 1.0)$	$\mathcal{P}(0.9, 0.1)$	$\mathcal{P}(0.2, 0.9)$	$\mathcal{P}(0.8, 0.5)$	$\mathcal{P}(0.8, 0.4)$

Example 1. We utilize Table 1 to present a Pythagorean fuzzy situation table. In this table, a_1, a_2, a_3, a_4, a_5, and a_6 represent six agents, while d_1, d_2, d_3, d_4, and d_5 denote five issues. For instance, $\rho(a_2, d_4) = \mathcal{P}(0.3, 0.8)$ indicates that the support and opposition degrees of agent a_2 on issue d_5 are 0.3 and 0.8, respectively.

Definition 3 (*Yager [29], 2014*)**.** Consider a set of PFNs $\mathbf{P} = \{\gamma_k \mid \gamma_k = \mathcal{P}(\varphi_{\gamma_k}, \psi_{\gamma_k}),\ 1 \leq k \leq m\}$, where w_k represents the weight assigned to γ_k. The weights satisfy $\sum_{k=1}^{m} w_k = 1$ and $w_k \geq 0$. The weighted average operator **W**, mapping from $\mathbb{P}^m$ to $\mathbb{P}$, is defined as follows:

$$\mathbf{W}(\gamma_1, \gamma_2, \ldots, \gamma_m) = \mathcal{P}\left(\sum_{k=1}^{m} w_k \varphi_{\gamma_k}, \sum_{k=1}^{m} w_k \psi_{\gamma_k}\right).$$

According to Definition 3, we aggregate a set of attitudes $\{\gamma_k \mid \gamma_k = r(a, d_k), k = 1, 2, \ldots, m\}$ into a Pythagorean fuzzy number $\mathbf{W}(\gamma_1, \gamma_2, \ldots, \gamma_m)$, where w_k stands for the weight associated with the issue $d_k \in \mathcal{D}$. As the set $\{\gamma_k \mid \gamma_k = \rho(a, d_k), k = 1, 2, \ldots, m\}$ represents the attitudes of an agent a, we denote $\mathbf{W}(\gamma_1, \gamma_2, \ldots, \gamma_m)$ as $\mathbf{W}(a)$.

Definition 4 (*Lang et al. [8], 2020*)**.** Let $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$ be a Pythagorean fuzzy situation table, and $\mathbf{E} : (\mathcal{A} \times \mathcal{D})^m \rightarrow V_{\mathbf{E}}$ be an evaluation function. Consider two thresholds τ and η, the support, neutral, and opposition coalitions $\mathbb{SC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D})$, $\mathbb{NC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D})$, and $\mathbb{OC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D})$ towards $\mathcal{D}$ are defined as follows:

$$\mathbb{SC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D}) = \{a \in \mathcal{A} \mid \mathbf{E}(\mathbf{W}(a)) \geq \tau\};$$
$$\mathbb{NC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D}) = \{a \in \mathcal{A} \mid \eta < \mathbf{E}(\mathbf{W}(a)) < \tau\};$$
$$\mathbb{OC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D}) = \{a \in \mathcal{A} \mid \mathbf{E}(\mathbf{W}(a)) \leq \eta\}.$$

The thresholds τ and η can take various numerical types, and the domain $V_{\mathbf{E}}$ encompasses diverse sets depending on the specific evaluation functions. Furthermore, the agent set $\mathcal{A}$ is partitioned into three coalitions: $\mathbb{SC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D})$, $\mathbb{NC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D})$, and $\mathbb{OC}_{(\tau,\eta)}(\mathcal{A}, \mathcal{D})$ with respect to $\mathcal{D}$.

Remark. In Sections 4 and 5, we will formulate three distinct conflict measures from various vantage points. One measure is grounded on the maximum positive agent, another on the maximum negative agent, and the third takes into account both the maximum positive and negative agents. This approach allows us to establish models for three-way conflict analysis that offer diverse perspectives. Subsequently, leveraging different conflict measures and fusion mechanisms, we create trisection-fusion and fusion-trisection models for three-way conflict analysis. It not only attempts to construct conflict measures by incorporating the maximum positive and negative agents but also facilitates the development of models for three-way conflict analysis within the context of a Pythagorean fuzzy situation table. To simplify the expression, we will use similar symbols for the support, neutral,

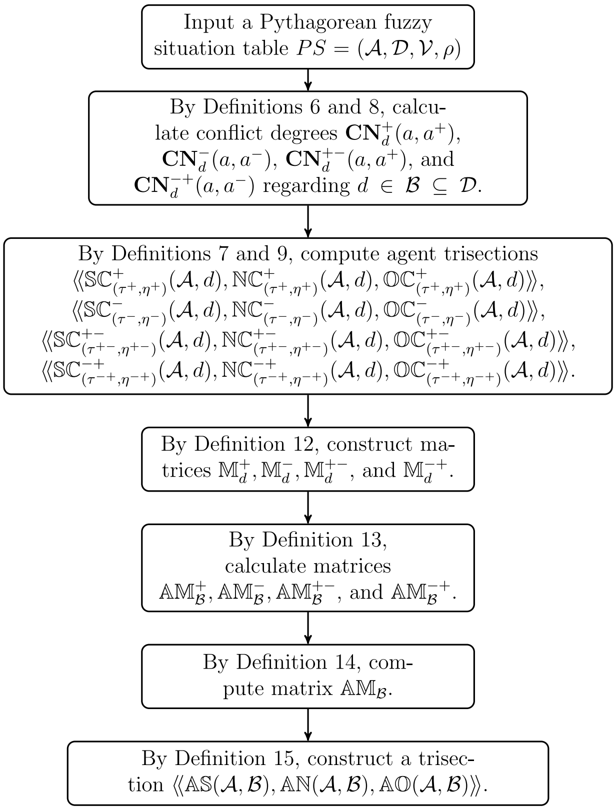

Fig. 1. The framework of trisection-fusion method for conflict analysis.

and opposition coalitions as defined by Definition 4 in the subsequent discussion. However, it is crucial to note that they are two distinct types of support, neutral, and opposition coalitions. One type only involves amalgamating agents' attitudes towards issues, while the other entails aggregating information from various levels relative to the most positively and negatively inclined agents. In essence, the two types of support, neutral, and opposition coalitions are grounded in different mechanisms.

4. Trisection-fusion methods of three-way conflict analysis

This section introduces trisection-fusion methods for analyzing conflict problems based on conflict measures associated with a single issue. Sections 4.1 and 4.2 divide the agent set into three disjoint parts using conflict measures related to a single issue. Section 4.3 establishes a trisection by combining a set of trisections, all associated with a single issue. The framework of the trisection-fusion method for conflict analysis is illustrated in Fig. 1.

4.1. Trisection of agent set based on the maximum positive or maximum negative agent towards an individual issue

Follows, we present two conflict measures for three-way conflict analysis regarding an individual issue, one measure is based on the maximum positive agent, while the other uses the maximum negative agent.

Definition 5. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, an agent characterized by an attitude of absolute support, denoted as $\mathcal{P}(1, 0)$ towards all issues, is referred to as the maximum positive agent a^+. Conversely, an agent with an attitude of absolute opposition, represented by $\mathcal{P}(0, 1)$ towards all issues, is termed as the maximum negative agent a^-.

The maximum positive agent maintains unwavering support for every issue, while the maximum negative agent consistently opposes each issue without hesitation. Indeed, the maximum positive and negative agents share a subtle connection with the best and worst ideal solutions in the TOPSIS method; however, their interpretations differ. It is essential to note that these two agents may not necessarily exist and could be representative of two hypothetical or imaginary entities.

Remark. In three-way conflict analysis, the maximum positive and negative agents are regarded as two reference points. In practice, there are various types of reference points that do not necessarily need to share the same opinion on all issues. In this study, we focus solely on the most maximum positive and negative agents as our reference points. In the future, we will incorporate additional reference points for three-way conflict analysis.

Definition 6. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, where $\rho(a, d) = \mathcal{P}(\varphi_{\gamma_d}, \psi_{\gamma_d})$ represents the attitude of $a \in \mathcal{A}$ towards $d \in \mathcal{D}$:

(1) the conflict measure $\mathbf{CN}^+_d$ between an agent $a \in \mathcal{A}$ and the maximum positive agent a^+ with respect to d is given by:

$$\mathbf{CN}^+_d(a, a^+) = \frac{1}{2}(|\varphi^2_{\gamma_d} - 1| + |\psi^2_{\gamma_d} - 0| + |\pi^2_{\gamma_d} - 0|) = 1 - \varphi^2_{\gamma_d};$$

(2) the conflict measure $\mathbf{CN}^-_d$ between an agent $a \in \mathcal{A}$ and the maximum negative agent a^- with respect to d is given by:

$$\mathbf{CN}^-_d(a, a^-) = \frac{1}{2}(|\varphi^2_{\gamma_d} - 0| + |\psi^2_{\gamma_d} - 1| + |\pi^2_{\gamma_d} - 0|) = 1 - \psi^2_{\gamma_d}.$$

Next, we utilize the two conflict measures to partition the set of agents into the support, neutral, and opposition coalitions.

Definition 7. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, $0 \le \eta^+ \le \tau^+ \le 1$, $0 \le \eta^- \le \tau^- \le 1$, and $d \in \mathcal{D}$,

(1) we define the support, neutral, and opposition coalitions $\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d)$, $\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d)$, and $\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d)$ with respect to the single issue d as follows:

$$\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \mathbf{CN}^+_d(a, a^+) \le 1 - \tau^+\};$$
$$\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid 1 - \tau^+ < \mathbf{CN}^+_d(a, a^+) < 1 - \eta^+\};$$
$$\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \mathbf{CN}^+_d(a, a^+) \ge 1 - \eta^+\}.$$

(2) we define the support, neutral, and opposition coalitions $\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d)$, $\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d)$, and $\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d)$ with respect to the single issue d as follows:

$$\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \mathbf{CN}^-_d(a, a^-) \ge \tau^-\};$$
$$\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \eta^- < \mathbf{CN}^-_d(a, a^-) < \tau^-\};$$
$$\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \mathbf{CN}^-_d(a, a^-) \le \eta^-\}.$$

We partition the set of agents A into three distinct regions: $\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d)$, $\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d)$, and $\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d)$, determined by their values of $\mathbf{CN}^+_d$. Furthermore, we categorize the agent set into three non-overlapping segments: $\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d)$, $\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d)$, and $\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A}, d)$, based on $\mathbf{CN}^-_d$.

Theorem 1. *For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, $0 \le \eta^+ \le \tau^+ \le 1$, $0 \le \eta^- \le \tau^- \le 1$, and $d \in \mathcal{D}$, we have*

(1) $$\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \varphi_{\gamma_d} \ge (\tau^+)^{\frac{1}{2}}\};$$
$$\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid (\tau^+)^{\frac{1}{2}} > \varphi_{\gamma_d} > (\eta^+)^{\frac{1}{2}}\};$$
$$\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A}, d) = \{a \in \mathcal{A} \mid \varphi_{\gamma_d} \le (\eta^+)^{\frac{1}{2}}\};$$

(2) $\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \psi_{\gamma_d} \le (1-\tau^-)^{\frac{1}{2}}\}$;

$\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d) = \{a \in \mathcal{A} \mid (1-\tau^-)^{\frac{1}{2}} < \psi_{\gamma_d} < (1-\eta^-)^{\frac{1}{2}}\}$;

$\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \psi_{\gamma_d} \ge (1-\eta^-)^{\frac{1}{2}}\}$.

Theorem 1(1) classifies an agent set into three distinct regions by examining the support degrees of all agents on a given issue. Similarly, Theorem 1(2) demonstrates that an agent set is partitioned into three separate parts by evaluating the opposition degrees of all agents regarding the same issue.

Example 2. Continuing from Example 1 and choosing $\tau^+ = 0.70$, $\eta^+ = 0.30$, $\tau^- = 0.80$, and $\eta^- = 0.20$,

(1) by Definition 7(1), we determine the support, neutral, and opposition coalitions $\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A},d_1)$, $\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A},d_1)$, and $\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A},d_1)$ for $d_1 \in \mathcal{D}$ as follows:

$\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A},d_1) = \{a_1,a_2,a_5\}$;
$\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A},d_1) = \{a_4\}$;
$\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A},d_1) = \{a_3,a_6\}$.

(2) by Definition 7(2), we identify the support, neutral, and opposition coalitions $\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d_1)$, $\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d_1)$, and $\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d_1)$ for $d_1 \in \mathcal{D}$ as follows:

$\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d_1) = \{a_1,a_2,a_4,a_5\}$;
$\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d_1) = \emptyset$;
$\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A},d_1) = \{a_3,a_6\}$.

4.2. Trisection of agent set based on the maximum positive agent and the maximum negative agent regarding a single issue

By simultaneously leveraging the maximum positive and negative agents, we formulate two conflict measures for three-way conflict analysis regarding an individual issue.

Definition 8. For $PS = (\mathcal{A},\mathcal{D},\mathcal{V},\rho)$,

(1) the conflict measure $\mathbf{CN}^{+-}_d$ between an agent $a \in \mathcal{A}$ and the maximum positive agent a^+ relative to the maximum negative agent a^- for the issue $d \in \mathcal{D}$ is defined by:

$$\mathbf{CN}^{+-}_d(a,a^+) = \frac{\mathbf{CN}^+_d(a,a^+)}{\mathbf{CN}^+_d(a,a^+)+\mathbf{CN}^-_d(a,a^-)} = \frac{1-\varphi^2_{\gamma_d}}{2-\varphi^2_{\gamma_d}-\psi^2_{\gamma_d}};$$

(2) the conflict measure $\mathbf{CN}^{-+}_d$ between an agent $a \in \mathcal{A}$ and the maximum negative agent a^- relative to the maximum positive agent a^+ for the issue $d \in \mathcal{D}$ is given by:

$$\mathbf{CN}^{-+}_d(a,a^-) = \frac{\mathbf{CN}^-_d(a,a^-)}{\mathbf{CN}^+_d(a,a^+)+\mathbf{CN}^-_d(a,a^-)} = \frac{1-\psi^2_{\gamma_d}}{2-\varphi^2_{\gamma_d}-\psi^2_{\gamma_d}}.$$

Remark. For three-way conflict analysis, we define three types of conflict measures regarding an issue: one is based on the maximum positive agent, another is based on the maximum negative agent, and the third is based on both the maximum positive and negative agents. Actually, the former is used when all attitudes are closer to the support attitude $\mathcal{P}(1.0,0.0)$; the second is employed when all opinions are closer to the opposition attitude $\mathcal{P}(0.0,1.0)$; and the third is adopted when opinions span between $\mathcal{P}(1.0,0.0)$ and $\mathcal{P}(0.0,1.0)$. Sometimes, the selection of a conflict measure can be determined by experts or decision-makers, and all conflict measures can be simultaneously employed in three-way conflict analysis.

Following this, we apply the two conflict measures to divide the set of agents into the support, neutral, and opposition coalitions.

Definition 9. For $PS = (\mathcal{A},\mathcal{D},\mathcal{V},\rho)$, $0 \le \eta^{+-} \le \tau^{+-} \le 1$, $0 \le \eta^{-+} \le \tau^{-+} \le 1$, and $d \in \mathcal{D}$,

(1) we define the support, neutral, and opposition coalitions $\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d)$, $\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d)$, and $\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d)$ regarding $d \in \mathcal{D}$ as follows:

$\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \mathbf{CN}^{+-}_d(a,a^+) \le 1-\tau^{+-}\}$;
$\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d) = \{a \in \mathcal{A} \mid 1-\tau^{+-} < \mathbf{CN}^{+-}_d(a,a^+) < 1-\eta^{+-}\}$;
$\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \mathbf{CN}^{+-}_d(a,a^+) \ge 1-\eta^{+-}\}$;

(2) we define the support, neutral, and opposition coalitions $\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d)$, $\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d)$, and $\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d)$ with respect to $d \in \mathcal{D}$ as follows:

$\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \mathbf{CN}^{-+}_d(a,a^-) \ge \tau^{-+}\}$;
$\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \eta^{-+} < \mathbf{CN}^{-+}_d(a,a^-) < \tau^{-+}\}$;
$\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d) = \{a \in \mathcal{A} \mid \mathbf{CN}^{-+}_d(a,a^-) \le \eta^{-+}\}$.

Theorem 2. *For $PS = (\mathcal{A},\mathcal{D},\mathcal{V},\rho)$, $0 \le \eta^{+-} \le \tau^{+-} \le 1$, $0 \le \eta^{-+} \le \tau^{-+} \le 1$, and $d \in \mathcal{D}$, we have*

(1) $\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d)$
$= \{a \in \mathcal{A} \mid \varphi_{\gamma_d} \ge [(\psi^2_{\gamma_d} - \tau^{+-}\psi^2_{\gamma_d} + 2\tau^{+-} - 1)/\tau^{+-}]^{\frac{1}{2}}\}$;
$\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d)$
$= \{a \in \mathcal{A} \mid [(\psi^2_{\gamma_d} - \tau^{+-}\psi^2_{\gamma_d} + 2\tau^{+-} - 1)/\tau^{+-}]^{\frac{1}{2}} > \varphi_{\gamma_d} > [(\psi^2_{\gamma_d} - \eta^{+-}\psi^2_{\gamma_d} + 2\eta^{+-} - 1)/\eta^{+-}]^{\frac{1}{2}}\}$;
$\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d)$
$= \{a \in \mathcal{A} \mid \varphi_{\gamma_d} \le [(\psi^2_{\gamma_d} - \eta^{+-}\psi^2_{\gamma_d} + 2\eta^{+-} - 1)/\eta^{+-}]^{\frac{1}{2}}\}$;

(2) $\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d)$
$= \{a \in \mathcal{A} \mid \psi_{\gamma_d} \le [(\tau^{-+}\varphi^2_{\gamma_d} + 1 - 2\tau^{-+})/(1-\tau^{-+})]^{\frac{1}{2}}\}$;
$\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d)$
$= \{a \in \mathcal{A} \mid [(\tau^{-+}\varphi^2_{\gamma_d} + 1 - 2\tau^{-+})/(1-\tau^{-+})]^{\frac{1}{2}} < \psi_{\gamma_d} < [(\eta^{-+}\varphi^2_{\gamma_d} + 1 - 2\eta^{-+})/(1-\eta^{-+})]^{\frac{1}{2}}\}$;
$\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d)$
$= \{a \in \mathcal{A} \mid \psi_{\gamma_d} \ge [(\eta^{-+}\varphi^2_{\gamma_d} + 1 - 2\eta^{-+})/(1-\eta^{-+})]^{\frac{1}{2}}\}$.

Example 3. Following Example 1, for $\tau^{+-} = 0.70, \eta^{+-} = 0.30, \tau^{-+} = 0.80$, and $\eta^{-+} = 0.20$,

(1) by Definition 9(1), the support, neutral, and opposition coalitions $\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d_1)$, $\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d_1)$, and $\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d_1)$ regarding the issue $d_1 \in \mathcal{D}$ are given by:

$\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d_1) = \{a_1,a_2,a_4,a_5\}$;
$\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d_1) = \emptyset$;
$\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},d_1) = \{a_3,a_6\}$.

(2) by Definition 9(2), the support, neutral, and opposition coalitions $\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d_1)$, $\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d_1)$, and $\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d_1)$ towards the issue $d_1 \in \mathcal{D}$ are given by:

$\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d_1) = \{a_1,a_2,a_5\}$;
$\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d_1) = \{a_4\}$;
$\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},d_1) = \{a_3,a_6\}$.

4.3. A trisection-fusion model of conflict analysis

This section introduces a trisection-fusion model designed for three-way conflict analysis. It achieves this by integrating a series of trisections related to individual issues.

Definition 10. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, the conditional weight $w(d|\mathcal{B})$ of $d \in \mathcal{B}$ with respect to $\mathcal{B} \subseteq \mathcal{D}$ is expressed as follows:

$$w(d|\mathcal{B}) = \frac{w_d}{\sum_{d\in\mathcal{B}} w_d},$$

where w_d is the weight of an issue $d \in \mathcal{D}$, $\sum_{d\in\mathcal{D}} w_d = 1$, and $0 \leq w_d \leq 1$.

The conditional weight of an issue is determined based on the weights assigned to all issues. There are primarily two methods for establishing the weight of an issue: (1) experts provide the weight, or (2) the weight is derived through measures such as alliance and conflict metrics. Since the objective of this study is to propose a framework for three-way conflict analysis, we do not delve deeply into the computation of issue weights. Future research will concentrate on determining how to assign weights to individual issues.

Definition 11. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, the vector representation of a subset of agents $\mathcal{X} \subseteq \mathcal{A}$ is defined as:

$$\mathbb{V}_{\mathcal{X}} = [\mathbb{V}_{\mathcal{X}}(k)]_{1\times n},$$

where

$$\mathbb{V}_{\mathcal{X}}(k) = \begin{cases} 1, & \text{if } a_k \in \mathcal{X}, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 12. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, we define the matrix representation of a trisection $\langle\langle \mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d), \mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d), \mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)\rangle\rangle$ based on $\mathbf{CN}^*_d$ with respect to $d \in \mathcal{D}$ as follows:

$$\mathbb{M}^*_d = [\mathbb{M}^*_d(i,j)]_{3\times n} = \begin{bmatrix} \mathbb{V}_{\mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A},d)} \\ \mathbb{V}_{\mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A},d)} \\ \mathbb{V}_{\mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A},d)} \end{bmatrix},$$

where $\mathbb{V}_{\mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A},d)}$, $\mathbb{V}_{\mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A},d)}$, and $\mathbb{V}_{\mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A},d)}$ stand for the vector representations of $\mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)$, $\mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)$, and $\mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)$, respectively, and $* \in \mathcal{T}$.

Definition 13. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, the matrix $\mathbb{AM}^*_{\mathcal{B}}$ of a trisection with respect to multiple issues $\mathcal{B} \subseteq \mathcal{D}$ based on $\mathbf{CN}^*_d$ is defined as follows:

$$\mathbb{AM}^*_{\mathcal{B}} = [\mathbb{AM}^*_{\mathcal{B}}(i,j)]_{3\times n} = \sum_{b\in\mathcal{B}} [w(b|\mathcal{B}) \times \mathbb{M}^*_b].$$

Definition 14. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, the trisection-fusion matrix $\mathbb{AM}_{\mathcal{B}}$ of a trisection with respect to multiple issues $\mathcal{B} \subseteq \mathcal{D}$ based on $\mathbf{CN}^+_d, \mathbf{CN}^-_d, \mathbf{CN}^{+-}_d$, and $\mathbf{CN}^{-+}_d$ is given by:

$$\mathbb{AM}_{\mathcal{B}} = [\mathbb{AM}_{\mathcal{B}}(i,j)]_{3\times n} = \sum_{*\in\mathcal{T}} [W^* \times \mathbb{AM}^*_{\mathcal{B}}],$$

where $\sum_{*\in\mathcal{T}} W^* = 1$ and $0 \leq W^* \leq 1$.

The weight W^* represents the importance of the matrix $\mathbb{AM}^*_{\mathcal{B}}$, and the matrix $\mathbb{AM}_{\mathcal{B}}$ is constructed by aggregating the matrices $\mathbb{AM}^+_{\mathcal{B}}$, $\mathbb{AM}^-_{\mathcal{B}}$, $\mathbb{AM}^{+-}_{\mathcal{B}}$, and $\mathbb{AM}^{-+}_{\mathcal{B}}$. Moreover, there are two main approaches to determining the weight of a trisection concerning multiple issues: (1) experts assign the weight, or (2) decision makers provide the weight based on their preferences or metrics. As the primary goal of this study is to present a framework for three-way conflict analysis, we do not extensively explore the calculation of trisection weights across multiple issues. Future research will focus on methods for assigning weights to trisections regarding multiple issues.

Remark. To construct the trisection-fusion matrix $\mathbb{AM}_{\mathcal{B}}$, we exclusively employ the conflict measures $\mathbf{CN}^+_d$, $\mathbf{CN}^-_d$, $\mathbf{CN}^{+-}_d$, and $\mathbf{CN}^{-+}_d$. It is worth noting that numerous conflict measures can be defined for the design of the trisection-fusion matrix. Our intention is to present a foundational framework for trisection-fusion methods of three-way conflict analysis.

Definition 15. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, the support, neutral, and opposition coalitions $\mathbb{AS}(\mathcal{A}, \mathcal{D})$, $\mathbb{AN}(\mathcal{A}, \mathcal{D})$, and $\mathbb{AO}(\mathcal{A}, \mathcal{D})$ with respect to $\mathcal{B} \subseteq \mathcal{D}$ are given by:

$$\mathbb{AS}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbb{AM}_{\mathcal{B}}(1,j) > [\mathbb{AM}_{\mathcal{B}}(2,j) \vee \mathbb{AM}_{\mathcal{B}}(3,j)]\};$$
$$\mathbb{AN}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbb{AM}_{\mathcal{B}}(2,j) > [\mathbb{AM}_{\mathcal{B}}(1,j) \vee \mathbb{AM}_{\mathcal{B}}(3,j)]\};$$
$$\mathbb{AO}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbb{AM}_{\mathcal{B}}(3,j) > [\mathbb{AM}_{\mathcal{B}}(1,j) \vee \mathbb{AM}_{\mathcal{B}}(2,j)]\}.$$

If $\mathbb{AM}_{\mathcal{B}}(1,j) = \mathbb{AM}_{\mathcal{B}}(2,j) > \mathbb{AM}_{\mathcal{B}}(3,j)$, $\mathbb{AM}_{\mathcal{B}}(1,j) < \mathbb{AM}_{\mathcal{B}}(2,j) = \mathbb{AM}_{\mathcal{B}}(3,j)$, and $\mathbb{AM}_{\mathcal{B}}(1,j) = \mathbb{AM}_{\mathcal{B}}(2,j) = \mathbb{AM}_{\mathcal{B}}(3,j)$, then we consider that the agent a_j belongs to $\mathbb{AS}(\mathcal{A}, \mathcal{B})$, $\mathbb{AO}(\mathcal{A}, \mathcal{B})$, and $\mathbb{AN}(\mathcal{A}, \mathcal{B})$, respectively. Furthermore, if $\mathbb{AM}_{\mathcal{B}}(1,j) = \mathbb{AM}_{\mathcal{B}}(3,j) > \mathbb{AM}_{\mathcal{B}}(2,j)$, then we consider that the agent a_j belongs to $\mathbb{AN}(\mathcal{A}, \mathcal{B})$. For simplicity, we denote the support, neutral, and opposition coalitions constructed by the matrix $\mathbb{AM}^*_{\mathcal{B}}$ as $\mathbb{AS}^*(\mathcal{A}, \mathcal{B})$, $\mathbb{AN}^*(\mathcal{A}, \mathcal{B})$, and $\mathbb{AO}^*(\mathcal{A}, \mathcal{B})$, respectively.

Algorithm 1 The Trisection-Fusion Algorithm for Constructing Coalitions $\mathbb{AS}(\mathcal{A}, \mathcal{B})$, $\mathbb{AN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{AO}(\mathcal{A}, \mathcal{B})$.

Input: $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$ and $\mathcal{B} \subseteq \mathcal{D}$;
Output: $\mathbb{AS}(\mathcal{A}, \mathcal{B})$, $\mathbb{AN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{AO}(\mathcal{A}, \mathcal{B})$.

1: For any $a \in \mathcal{A}$, compute conflict degrees $\mathbf{CN}^+_d(a, a^+)$, $\mathbf{CN}^-_d(a, a^-)$, $\mathbf{CN}^{+-}_d(a, a^+)$, and $\mathbf{CN}^{-+}_d(a, a^-)$ towards an issue $d \in \mathcal{B}$;
2: Construct the support, neutral, and opposition coalitions $\mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)$, $\mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)$, and $\mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A}, d)$ towards the issue $d \in \mathcal{B}$, where $* \in \mathcal{T}$;
3: Calculate the matrices $\mathbb{AM}^*_{\mathcal{B}}$ and $\mathbb{AM}_{\mathcal{B}}$ regarding multiple issues $\mathcal{B}$, where $* \in \mathcal{T}$;
4: Construct the support, neutral, and opposition coalitions $\mathbb{AS}(\mathcal{A}, \mathcal{B})$, $\mathbb{AN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{AO}(\mathcal{A}, \mathcal{B})$ towards multiple issues $\mathcal{B}$;
5: Output $\mathbb{AS}(\mathcal{A}, \mathcal{B})$, $\mathbb{AN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{AO}(\mathcal{A}, \mathcal{B})$.

In Algorithm 1, Step 1 computes the conflict degree between an agent and the maximum positive agent (respectively, the maximum negative agent) and the relative conflict degree between an agent and the maximum positive agent (respectively, the maximum negative agent) towards an issue. Step 2 constructs the support, neutral, and opposition coalitions towards an individual issue. Step 3 computes four matrices of trisections with respect to multiple issues by aggregating a set of trisections related to individual issues and derives a matrix by combining the four matrices. Step 4 gives the support, neutral, and opposition coalitions towards multiple issues.

Example 4. According to Examples 1, 2, and 3, and taking $w_k = 1/5$, $k = 1, 2, 3, 4, 5$, and $\mathcal{B} = \{d_1, d_3, d_5\}$, we obtain matrices $\mathbb{M}^+_{d_1}$, $\mathbb{M}^+_{d_3}$, $\mathbb{M}^+_{d_5}$, $\mathbb{M}^-_{d_1}$, $\mathbb{M}^-_{d_3}$, $\mathbb{M}^-_{d_5}$, $\mathbb{M}^{+-}_{d_1}$, $\mathbb{M}^{+-}_{d_3}$, $\mathbb{M}^{+-}_{d_5}$, $\mathbb{M}^{-+}_{d_1}$, $\mathbb{M}^{-+}_{d_3}$, and $\mathbb{M}^{-+}_{d_5}$ as follows:

$$\mathbb{M}^+_{d_1} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{M}^+_{d_3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$\mathbb{M}^+_{d_5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix};$$

$$\mathbb{M}^-_{d_1} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{M}^-_{d_3} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$\mathbb{M}^{-}_{d_5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix};$$

$$\mathbb{M}^{+-}_{d_1} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{M}^{+-}_{d_3} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$\mathbb{M}^{+-}_{d_5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix};$$

$$\mathbb{M}^{-+}_{d_1} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbb{M}^{-+}_{d_3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$\mathbb{M}^{-+}_{d_5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

Second, we have the matrices $\mathbb{AM}^{+}_{\mathcal{B}}$, $\mathbb{AM}^{-}_{\mathcal{B}}$, $\mathbb{AM}^{+-}_{\mathcal{B}}$, and $\mathbb{AM}^{-+}_{\mathcal{B}}$ with respect to multiple issues $\mathcal{B}$ as follows:

$$\mathbb{AM}^{+}_{\mathcal{B}} = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 & 1/3 \\ 0 & 2/3 & 2/3 & 2/3 & 2/3 & 2/3 \end{bmatrix},$$

$$\mathbb{AM}^{-}_{\mathcal{B}} = \begin{bmatrix} 1 & 1/3 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 2/3 & 1/3 & 1/3 & 2/3 & 2/3 \end{bmatrix},$$

$$\mathbb{AM}^{+-}_{\mathcal{B}} = \begin{bmatrix} 1 & 1/3 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 2/3 & 2/3 & 2/3 & 2/3 & 2/3 \end{bmatrix},$$

$$\mathbb{AM}^{-+}_{\mathcal{B}} = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 1/3 & 0 & 2/3 & 2/3 & 0 & 1/3 \\ 0 & 2/3 & 1/3 & 1/3 & 2/3 & 2/3 \end{bmatrix}.$$

Third, according to Definition 14, by taking $W^{+} = W^{-} = W^{+-} = W^{-+} = 1/4$, we have the trisection-fusion matrix regarding multiple issues $\mathcal{B}$:

$$\mathbb{AM}_{\mathcal{B}} = \begin{bmatrix} 5/6 & 1/3 & 0 & 1/6 & 1/3 & 1/6 \\ 1/6 & 0 & 1/2 & 1/3 & 0 & 1/6 \\ 0 & 2/3 & 1/2 & 1/2 & 2/3 & 2/3 \end{bmatrix}.$$

Finally, by Definition 15, we have the support, neutral, and opposition coalitions towards multiple issues $\mathcal{B}$:

$\mathbb{AS}(\mathcal{A}, \mathcal{B}) = \{a_1\}$;

$\mathbb{AN}(\mathcal{A}, \mathcal{B}) = \emptyset$;

$\mathbb{AO}(\mathcal{A}, \mathcal{B}) = \{a_2, a_3, a_4, a_5, a_6\}$.

5. Fusion-trisection methods of three-way conflict analysis

In this section, we employ the maximum positive and negative agents to design three types of conflict measures towards multiple issues. Sections 5.1 and 5.2 trisect an agent set into three disjoint parts using conflict measures related to multiple issues. Section 5.3 constructs a trisection by fusing a family of trisections with respect to multiple issues. The framework of the fusion-trisection method for conflict analysis is depicted in Fig. 2.

5.1. *Trisection of agent set based on the maximum positive or negative agent towards multiple issues*

This section first presents two conflict measures concerning multiple issues for three-way conflict analysis. One measure is constructed on the maximum positive agent across multiple issues, while the other is derived by the maximum negative agent regarding multiple issues.

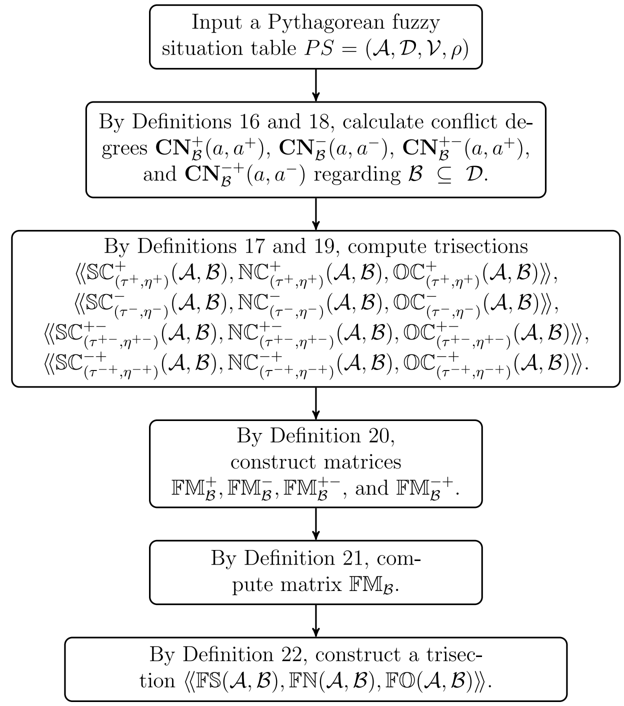

Fig. 2. The framework of the fusion-trisection model of conflict analysis.

Definition 16. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$,

(1) the conflict measure $\mathbf{CN}^{+}_{\mathcal{B}}$ between an agent $a \in \mathcal{A}$ and the maximum positive agent a^{+} with respect to $\mathcal{B}$ is given by:

$$\mathbf{CN}^{+}_{\mathcal{B}}(a, a^{+}) = \sum_{d \in \mathcal{B}} [w(d|\mathcal{B}) \times \mathbf{CN}^{+}_{d}(a, a^{+})];$$

(2) the conflict measure $\mathbf{CN}^{-}_{\mathcal{B}}$ between an agent $a \in \mathcal{A}$ and the maximum negative agent a^{-} with respect to $\mathcal{B}$ is given by:

$$\mathbf{CN}^{-}_{\mathcal{B}}(a, a^{-}) = \sum_{d \in \mathcal{B}} [w(d|\mathcal{B}) \times \mathbf{CN}^{-}_{d}(a, a^{-})].$$

In the subsequent steps, we apply the two conflict measures to categorize the set of agents into the support, neutral, and opposition coalitions.

Definition 17. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, where $0 \leq \eta^{+} \leq \tau^{+} \leq 1$, $0 \leq \eta^{-} \leq \tau^{-} \leq 1$, and $\mathcal{B} \subseteq \mathcal{D}$,

(1) we define the support, neutral, and opposition coalitions $\mathbb{SC}^{+}_{(\tau^{+},\eta^{+})}(\mathcal{A}, \mathcal{B})$, $\mathbb{NC}^{+}_{(\tau^{+},\eta^{+})}(\mathcal{A}, \mathcal{B})$, and $\mathbb{OC}^{+}_{(\tau^{+},\eta^{+})}(\mathcal{A}, \mathcal{B})$ with respect to $\mathcal{B}$ as follows:

$$\mathbb{SC}^{+}_{(\tau^{+},\eta^{+})}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{+}_{\mathcal{B}}(a, a^{+}) \leq 1 - \tau^{+}\};$$
$$\mathbb{NC}^{+}_{(\tau^{+},\eta^{+})}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid 1 - \tau^{+} < \mathbf{CN}^{+}_{\mathcal{B}}(a, a^{+}) < 1 - \eta^{+}\};$$
$$\mathbb{OC}^{+}_{(\tau^{+},\eta^{+})}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{+}_{\mathcal{B}}(a, a^{+}) \geq 1 - \eta^{+}\};$$

(2) we define the support, neutral, and opposition coalitions $\mathbb{SC}^{-}_{(\tau^{-},\eta^{-})}(\mathcal{A}, \mathcal{B})$, $\mathbb{NC}^{-}_{(\tau^{-},\eta^{-})}(\mathcal{A}, \mathcal{B})$, and $\mathbb{OC}^{-}_{(\tau^{-},\eta^{-})}(\mathcal{A}, \mathcal{B})$ with respect to $\mathcal{B}$ as follows:

$$\mathbb{SC}^{-}_{(\tau^{-},\eta^{-})}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{-}_{\mathcal{B}}(a, a^{-}) \geq \tau^{-}\};$$
$$\mathbb{NC}^{-}_{(\tau^{-},\eta^{-})}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \eta^{-} < \mathbf{CN}^{-}_{\mathcal{B}}(a, a^{-}) < \tau^{-}\};$$
$$\mathbb{OC}^{-}_{(\tau^{-},\eta^{-})}(\mathcal{A}, \mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{-}_{\mathcal{B}}(a, a^{-}) \leq \eta^{-}\}.$$

Theorem 3. *For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, $0 \le \eta^+ \le \tau^+ \le 1$, $0 \le \eta^- \le \tau^- \le 1$, and $\mathcal{B} \subseteq \mathcal{D}$, we have*

(1)
$$\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \sum_{b\in\mathcal{B}}[w(b|\mathcal{B}) \times \varphi^2_{\gamma_b}] \ge \tau^+\};$$
$$\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \tau^+ > \sum_{b\in\mathcal{B}}[w(b|\mathcal{B}) \times \varphi^2_{\gamma_b}] > \eta^+\};$$
$$\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \sum_{b\in\mathcal{B}}[w(b|\mathcal{B}) \times \varphi^2_{\gamma_b}] \le \eta^+\};$$

(2)
$$\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \sum_{b\in\mathcal{B}}[w(b|\mathcal{B}) \times \psi^2_{\gamma_b}] \le 1-\tau^-\};$$
$$\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid 1-\tau^- < \sum_{b\in\mathcal{B}}[w(b|\mathcal{B}) \times \psi^2_{\gamma_b}] < 1-\eta^-\};$$
$$\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \sum_{b\in\mathcal{B}}[w(b|\mathcal{B}) \times \psi^2_{\gamma_b}] \ge 1-\eta^-\}.$$

Theorem 4. *For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, $0 \le \eta \le \tau \le 1$, and $\mathcal{B} \subseteq \mathcal{D}$, we have*

$$\mathbb{SC}^+_{(\tau,\eta)}(\mathcal{A},\mathcal{B}) = \mathbb{SC}^-_{(\tau,\eta)}(\mathcal{A},\mathcal{B});$$
$$\mathbb{NC}^+_{(\tau,\eta)}(\mathcal{A},\mathcal{B}) = \mathbb{NC}^-_{(\tau,\eta)}(\mathcal{A},\mathcal{B});$$
$$\mathbb{OC}^+_{(\tau,\eta)}(\mathcal{A},\mathcal{B}) = \mathbb{OC}^-_{(\tau,\eta)}(\mathcal{A},\mathcal{B}).$$

Example 5. According to Example 1, for $w_1 = w_2 = w_3 = w_4 = w_5 = \frac{1}{5}$, $\tau^+ = 0.70$, $\eta^+ = 0.30$, $\tau^- = 0.80$, $\eta^- = 0.20$, and $\mathcal{B} = \{d_1, d_3, d_5\}$,

(1) we obtain the support, neutral, and opposition coalitions $\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B})$, $\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B})$, and $\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B})$ towards multiple issues $\mathcal{B}$ as follows:

$$\mathbb{SC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B}) = \{a_1\};$$
$$\mathbb{NC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B}) = \{a_5\};$$
$$\mathbb{OC}^+_{(\tau^+,\eta^+)}(\mathcal{A},\mathcal{B}) = \{a_2, a_3, a_4, a_6\};$$

(2) we get the support, neutral, and opposition coalitions $\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B})$, $\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B})$, and $\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B})$ towards multiple issues $\mathcal{B}$ as follows:

$$\mathbb{SC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B}) = \{a_1\};$$
$$\mathbb{NC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B}) = \{a_2, a_3, a_4, a_5, a_6\};$$
$$\mathbb{OC}^-_{(\tau^-,\eta^-)}(\mathcal{A},\mathcal{B}) = \emptyset.$$

5.2. Trisection of agent set based on the maximum positive and negative agents towards multiple issues

By utilizing both maximum positive and negative agents concurrently, this section presents two conflict measures towards multiple issues for three-way conflict analysis.

Definition 18. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$,

(1) the conflict measure $\mathbf{CN}^{+-}_{\mathcal{B}}$ between an agent $a \in \mathcal{A}$ and the maximum positive agent a^+ relative to the maximum negative agent a^- with respect to $\mathcal{B} \subseteq \mathcal{D}$ is given by:

$$\mathbf{CN}^{+-}_{\mathcal{B}}(a, a^+) = \frac{\mathbf{CN}^{+}_{\mathcal{B}}(a, a^+)}{\mathbf{CN}^{+}_{\mathcal{B}}(a, a^+) + \mathbf{CN}^{-}_{\mathcal{B}}(a, a^-)};$$

(2) the conflict measure $\mathbf{CN}^{-+}_{\mathcal{B}}$ between an agent $a \in \mathcal{A}$ and the maximum negative agent a^- relative to the maximum positive agent a^+ with respect to $\mathcal{B} \subseteq \mathcal{D}$ is given by:

$$\mathbf{CN}^{-+}_{\mathcal{B}}(a, a^-) = \frac{\mathbf{CN}^{-}_{\mathcal{B}}(a, a^-)}{\mathbf{CN}^{+}_{\mathcal{B}}(a, a^+) + \mathbf{CN}^{-}_{\mathcal{B}}(a, a^-)}.$$

Remark. For three-way conflict analysis, we give three types of conflict measures concerning multiple issues: one is based on the maximum positive agent, another is based on the maximum negative agent, and the third is based on both the maximum positive and negative agents. Specifically, the first measure is employed when all attitudes align closely with the support attitude $\mathcal{P}(1.0, 0.0)$; the second measure is utilized when all opinions lean towards the opposition attitude $\mathcal{P}(0.0, 1.0)$; and the third measure is adopted when opinions fall between $\mathcal{P}(1.0, 0.0)$ and $\mathcal{P}(0.0, 1.0)$. The choice of a conflict measure can be determined by experts or decision-makers, and all conflict measures can be simultaneously applied in three-way conflict analysis.

Subsequently, we utilize the two conflict measures to classify the set of agents into the support, neutral, and opposition coalitions.

Definition 19. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, $0 \le \eta^{+-} \le \tau^{+-} \le 1$, $0 \le \eta^{-+} \le \tau^{-+} \le 1$, and $\mathcal{B} \subseteq \mathcal{D}$,

(1) we give the support, neutral, and opposition coalitions $\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B})$, $\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B})$, and $\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B})$ with respect to $\mathcal{B}$:

$$\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{+-}_{\mathcal{B}}(a, a^+) \le 1-\tau^{+-}\};$$
$$\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid 1-\tau^{+-} < \mathbf{CN}^{+-}_{\mathcal{B}}(a, a^+) < 1-\eta^{+-}\};$$
$$\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{+-}_{\mathcal{B}}(a, a^+) \ge 1-\eta^{+-}\}.$$

(2) we define the support, neutral, and opposition coalitions $\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B})$, $\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B})$, and $\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B})$ with respect to $\mathcal{B}$:

$$\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{-+}_{\mathcal{B}}(a, a^-) \ge \tau^{-+}\};$$
$$\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \eta^{-+} < \mathbf{CN}^{-+}_{\mathcal{B}}(a, a^-) < \tau^{-+}\};$$
$$\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B}) = \{a \in \mathcal{A} \mid \mathbf{CN}^{-+}_{\mathcal{B}}(a, a^-) \le \eta^{-+}\}.$$

Example 6. By Example 1 and taking $w_1 = w_2 = w_3 = w_4 = w_5 = 1/5$, $\tau^{+-} = 0.70$, $\eta^{+-} = 0.30$, $\tau^{-+} = 0.80$, $\eta^{-+} = 0.20$, and $\mathcal{B} = \{d_1, d_3, d_5\}$,

(1) we have the support, neutral, and opposition coalitions $\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B})$, $\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B})$, and $\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B})$ towards multiple issues $\mathcal{B}$:

$$\mathbb{SC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B}) = \{a_1\};$$
$$\mathbb{NC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B}) = \{a_2, a_4, a_5, a_6\};$$
$$\mathbb{OC}^{+-}_{(\tau^{+-},\eta^{+-})}(\mathcal{A},\mathcal{B}) = \{a_3\}.$$

(2) we get the support, neutral, and opposition coalitions $\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B})$, $\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B})$, and $\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B})$ towards multiple issues $\mathcal{B}$:

$$\mathbb{SC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B}) = \{a_1\};$$
$$\mathbb{NC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B}) = \{a_2, a_3, a_4, a_5, a_6\};$$
$$\mathbb{OC}^{-+}_{(\tau^{-+},\eta^{-+})}(\mathcal{A},\mathcal{B}) = \emptyset.$$

5.3. A fusion-trisection model of conflict analysis

This section gives a fusion-trisection model of three-way conflict analysis, and it is achieved through the amalgamation of a set of trisections related to multiple issues.

Definition 20. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, we define the matrix representation of a trisection $\langle\!\langle \mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A},\mathcal{B}), \mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A},\mathcal{B}), \mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A},\mathcal{B}) \rangle\!\rangle$ based on $\mathbf{CN}^*_{\mathcal{B}}$ towards multiple issues $\mathcal{B} \subseteq \mathcal{D}$:

$$\mathbb{FM}^*_{\mathcal{B}} = [\mathbb{FM}^*_{\mathcal{B}}(i,j)]_{3\times n} = \begin{bmatrix} \mathbb{V}_{\mathbb{SC}^*_{(\tau^*,\eta^*)}(\mathcal{A},\mathcal{B})} \\ \mathbb{V}_{\mathbb{NC}^*_{(\tau^*,\eta^*)}(\mathcal{A},\mathcal{B})} \\ \mathbb{V}_{\mathbb{OC}^*_{(\tau^*,\eta^*)}(\mathcal{A},\mathcal{B})} \end{bmatrix}.$$

Definition 21. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, the fusion-trisection matrix $\mathbb{FM}_{\mathcal{B}}$ of a trisection based on $\mathbf{CN}^{+}_{\mathcal{B}}, \mathbf{CN}^{-}_{\mathcal{B}}, \mathbf{CN}^{+-}_{\mathcal{B}}$, and $\mathbf{CN}^{-+}_{\mathcal{B}}$ with respect to multiple issues $\mathcal{B} \subseteq \mathcal{D}$ is given by:

$$\mathbb{FM}_{\mathcal{B}} = [\mathbb{FM}_{\mathcal{B}}(i,j)]_{3\times n} = \sum_{*\in\mathcal{T}} [W^{*} \times \mathbb{FM}^{*}_{\mathcal{B}}],$$

where $\sum_{*\in\mathcal{T}} W^{*} = 1$ and $0 \le W^{*} \le 1$.

The weight W^{*} stands for the importance of the matrix $\mathbb{FM}^{*}_{\mathcal{B}}$, and the fusion-trisection matrix $\mathbb{FM}_{\mathcal{B}}$ is constructed by aggregating the matrices $\mathbb{FM}^{+}_{\mathcal{B}}, \mathbb{FM}^{-}_{\mathcal{B}}, \mathbb{FM}^{+-}_{\mathcal{B}}$, and $\mathbb{FM}^{-+}_{\mathcal{B}}$.

Remark. To construct the fusion-trisection matrix $\mathbb{FM}_{\mathcal{B}}$, we exclusively utilize the conflict measures $\mathbf{CN}^{+}_{\mathcal{B}}$, $\mathbf{CN}^{-}_{\mathcal{B}}$, $\mathbf{CN}^{+-}_{\mathcal{B}}$, and $\mathbf{CN}^{-+}_{\mathcal{B}}$. It is worth noting that multiple conflict measures can be defined for the design of the fusion-trisection matrix. Our primary objective is to establish a foundational framework for fusion-trisection methods in three-way conflict analysis. Therefore, we prioritize the development of fusion-trisection methodologies rather than focusing extensively on constructing conflict measures.

Definition 22. For $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$, we define the support, neutral, and opposition coalitions $\mathbb{FS}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}(\mathcal{A}, \mathcal{B})$ with respect to $\mathcal{B} \subseteq \mathcal{D}$:

$$\mathbb{FS}(\mathcal{A}, \mathcal{B}) = \{a_j \in \mathcal{A} \mid \mathbb{FM}_{\mathcal{B}}(1,j) > [\mathbb{FM}_{\mathcal{B}}(2,j) \vee \mathbb{FM}_{\mathcal{B}}(3,j)]\};$$
$$\mathbb{FN}(\mathcal{A}, \mathcal{B}) = \{a_j \in \mathcal{A} \mid \mathbb{FM}_{\mathcal{B}}(2,j) > [\mathbb{FM}_{\mathcal{B}}(1,j) \vee \mathbb{FM}_{\mathcal{B}}(3,j)]\};$$
$$\mathbb{FO}(\mathcal{A}, \mathcal{B}) = \{a_j \in \mathcal{A} \mid \mathbb{FM}_{\mathcal{B}}(3,j) > [\mathbb{FM}_{\mathcal{B}}(1,j) \vee \mathbb{FM}_{\mathcal{B}}(2,j)]\}.$$

If $\mathbb{FM}_{\mathcal{B}}(1,j) = \mathbb{FM}_{\mathcal{B}}(2,j) > \mathbb{FM}_{\mathcal{B}}(3,j)$, $\mathbb{FM}_{\mathcal{B}}(1,j) < \mathbb{FM}_{\mathcal{B}}(2,j) = \mathbb{FM}_{\mathcal{B}}(3,j)$, and $\mathbb{FM}_{\mathcal{B}}(1,j) = \mathbb{FM}_{\mathcal{B}}(2,j) = \mathbb{FM}_{\mathcal{B}}(3,j)$, then we put the agent a_j into $\mathbb{FS}(\mathcal{A}, \mathcal{B})$, $\mathbb{FO}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FN}(\mathcal{A}, \mathcal{B})$. Furthermore, if $\mathbb{FM}_{\mathcal{B}}(1,j) = \mathbb{FM}_{\mathcal{B}}(3,j) > \mathbb{FM}_{\mathcal{B}}(2,j)$, then we consider that the agent a_j belongs to $\mathbb{FN}(\mathcal{A}, \mathcal{B})$. For simplicity, we denote the support, neutral, and opposition coalitions constructed by the matrix $\mathbb{FM}^{*}_{\mathcal{B}}$ as $\mathbb{FS}^{*}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}^{*}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}^{*}(\mathcal{A}, \mathcal{B})$, respectively.

Remark. The trisection-fusion and fusion-trisection models for three-way conflict analysis are developed from distinct perspectives utilizing Pythagorean fuzzy information. Specifically, the former involves trisecting the agent set concerning a particular issue and then amalgamating a series of such trisections across multiple issues. On the other hand, the latter entails integrating Pythagorean fuzzy information across various issues and subsequently partitioning the agent set based on conflict measures concerning these multiple issues.

Algorithm 2 The Fusion-Trisection Algorithm for Constructing Three Coalitions $\mathbb{FS}(\mathcal{A}, \mathcal{B}), \mathbb{FN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}(\mathcal{A}, \mathcal{B})$.

Input: $PS = (\mathcal{A}, \mathcal{D}, \mathcal{V}, \rho)$ and $\mathcal{B} \subseteq \mathcal{D}$;
Output: $\mathbb{FS}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}(\mathcal{A}, \mathcal{B})$.

1: Compute conflict degrees $\mathbf{CN}^{+}_{\mathcal{B}}(a, a^{+})$, $\mathbf{CN}^{-}_{\mathcal{B}}(a, a^{-})$, $\mathbf{CN}^{+-}_{\mathcal{B}}(a, a^{+})$, and $\mathbf{CN}^{-+}_{\mathcal{B}}(a, a^{-})$ towards multiple issues $\mathcal{B}$;
2: Calculate the support, neutral, and opposition coalitions $\mathbb{SC}^{*}_{(\tau^*,\eta^*)}(\mathcal{A}, \mathcal{B})$, $\mathbb{NC}^{*}_{(\tau^*,\eta^*)}(\mathcal{A}, \mathcal{B})$, and $\mathbb{OC}^{*}_{(\tau^*,\eta^*)}(\mathcal{A}, \mathcal{B})$ towards multiple issues $\mathcal{B}$, where $*\in \mathcal{T}$;
3: Compute the matrices $\mathbb{FM}^{*}_{\mathcal{B}}$ and $\mathbb{FM}_{\mathcal{B}}$ regarding multiple issues $\mathcal{B}$, where $*\in \mathcal{T}$;
4: Construct the support, neutral, and opposition coalitions $\mathbb{FS}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}(\mathcal{A}, \mathcal{B})$ towards multiple issues $\mathcal{B}$;
5: Output $\mathbb{FS}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}(\mathcal{A}, \mathcal{B})$.

In Algorithm 2, Step 1 computes the conflict degree between an agent and the maximum positive agent (respectively, the maximum negative agent), and the relative conflict degree between an agent and the maximum positive agent (respectively, the maximum negative agent) towards multiple issues. Step 2 constructs the support, neutral, and opposition coalitions towards multiple issues. Step 3 computes four matrices of trisections with respect to multiple issues and derives a matrix by combining the four matrices. Step 4 calculates the support, neutral, and opposition coalitions towards multiple issues.

Example 7 (*Continuation from Examples 1, 5, and 6*). By Definition 20, we have matrix representations $\mathbb{FM}^{+}_{\mathcal{B}}$, $\mathbb{FM}^{-}_{\mathcal{B}}$, $\mathbb{FM}^{+-}_{\mathcal{B}}$, and $\mathbb{FM}^{-+}_{\mathcal{B}}$:

$$\mathbb{FM}^{+}_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}; \mathbb{FM}^{-}_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$
$$\mathbb{FM}^{+-}_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}; \mathbb{FM}^{-+}_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Second, according to Definition 21, by taking $W^{*} = 1/4$, we have the fusion-trisection matrix regarding multiple issues $\mathcal{B}$:

$$\mathbb{FM}_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3/4 & 1/2 & 3/4 & 1 & 3/4 \\ 0 & 1/4 & 1/2 & 1/4 & 0 & 1/4 \end{bmatrix}.$$

Finally, by Definition 15, we have the support, neutral, and opposition coalitions towards multiple issues $\mathcal{B}$:

$$\mathbb{FS}(\mathcal{A}, \mathcal{B}) = \{a_1\};$$
$$\mathbb{FN}(\mathcal{A}, \mathcal{B}) = \{a_2, a_4, a_5, a_6\};$$
$$\mathbb{FO}(\mathcal{A}, \mathcal{B}) = \{a_3\}.$$

6. Applications of two models of three-way conflict analysis

In China, decision-makers in Hunan Province generate proposals comprising diverse sets of issues. If more than half of the cities endorse a particular issue set, the proposal will be implemented. In the process of formulating viable proposals, decision-makers gather opinions from all cities in Hunan Province on a range of issues, as illustrated in Table 2, where a_1-Changsha, a_2-Zhuzhou, a_3-Xiangtan, a_4-Hengyang, a_5-Shaoyang, a_6-Yueyang, a_7-Changde, a_8-Zhangjiajie, a_9-Yiyang, a_{10}-Cenzhou, a_{11}-Yongzhou, a_{12}-Huaihua, a_{13}-Loudi, and a_{14}-Xiangxi Tujia and Miao Autonomous Prefecture, d_1-Environmental Conservation, d_2-Agricultural and Rural Modernization, d_3-Information Development, d_4-Protection of Water Resource, d_5-Development of Old Revolutionary base Areas, d_6-Growing Population Aging, d_7-Health Service, d_8-Entertainment, d_9-Industrial Development, d_{10}-Education Development, and d_{11}-Public Transport.

6.1. Applications of trisection-fusion model

Next, we demonstrate the computation of support, neutral, and opposition coalitions and how the trisection-fusion model of conflict analysis aids decision-makers in adjusting policies.

First, by taking $w_k = 1/11$, $\tau^{*} = 0.90$, and $\eta^{*} = 0.30$, where $k = 1, 2, \ldots, 11$ and $*\in \mathcal{T}$, for multiple issues $\mathcal{B} = \{d_1, d_3, d_5, d_7\}$, we have the matrices $\mathbb{AM}^{+}_{\mathcal{B}}$, $\mathbb{AM}^{-}_{\mathcal{B}}$, $\mathbb{AM}^{+-}_{\mathcal{B}}$, and $\mathbb{AM}^{-+}_{\mathcal{B}}$:

$$\mathbb{AM}^{+}_{\mathcal{B}} = \begin{bmatrix} 1/4 & 0 & 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/4 & 0 & 0 & 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 0 \\ 1/4 & 3/4 & 3/4 & 1/2 & 1 & 1 & 1/2 & 3/4 & 1/2 & 1/2 & 1 & 1/2 & 3/4 & 1 \end{bmatrix};$$
$$\mathbb{AM}^{-}_{\mathcal{B}} = \begin{bmatrix} 1 & 1/4 & 1/4 & 1/2 & 1/4 & 1/2 & 1 & 1/4 & 3/4 & 1/2 & 0 & 3/4 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 1/4 & 0 & 1/4 & 0 & 0 & 0 & 1/4 & 1/2 & 0 \\ 0 & 1/4 & 3/4 & 1/2 & 3/4 & 1/4 & 0 & 1/2 & 1/4 & 1/2 & 1 & 0 & 1/2 & 1 \end{bmatrix};$$
$$\mathbb{AM}^{+-}_{\mathcal{B}} = \begin{bmatrix} 1/2 & 0 & 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 & 1/2 & 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 & 1/4 & 3/4 & 3/4 & 0 & 1/4 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & 3/4 & 1/2 & 3/4 & 1/4 & 0 & 3/4 & 1/4 & 1/2 & 1 & 0 & 1/2 & 1 \end{bmatrix};$$
$$\mathbb{AM}^{-+}_{\mathcal{B}} = \begin{bmatrix} 1/2 & 0 & 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 & 1/2 & 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 & 1/4 & 3/4 & 3/4 & 0 & 1/4 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & 3/4 & 1/2 & 3/4 & 1/4 & 0 & 3/4 & 1/4 & 1/2 & 1 & 0 & 1/2 & 1 \end{bmatrix}.$$

Table 2
All attitudes of cities on issues for proposals.

$\mathcal{A}$ \ $\mathcal{D}$	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}
a_1	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.8,0.1)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.6,0.5)$	$\mathcal{P}(0.2,0.8)$	$\mathcal{P}(0.9,0.3)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(1.0,0.0)$
a_2	$\mathcal{P}(0.4,0.6)$	$\mathcal{P}(0.8,0.1)$	$\mathcal{P}(0.2,0.8)$	$\mathcal{P}(0.6,0.6)$	$\mathcal{P}(0.6,0.4)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.9,0.2)$	$\mathcal{P}(0.3,0.7)$
a_3	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.0,0.9)$	$\mathcal{P}(0.0,0.9)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.4,0.5)$
a_4	$\mathcal{P}(0.7,0.5)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(1.0,0.0)$	$\mathcal{P}(0.9,0.2)$	$\mathcal{P}(0.3,0.9)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.8,0.4)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.4,0.9)$	$\mathcal{P}(0.3,0.9)$
a_5	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.1)$
a_6	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.4,0.7)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.9,0.0)$	$\mathcal{P}(0.0,0.9)$
a_7	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(1.0,0.0)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.4,0.2)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.6,0.5)$	$\mathcal{P}(0.9,0.2)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.3)$	$\mathcal{P}(0.6,0.5)$
a_8	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.1,0.8)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.9,0.4)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.9,0.1)$
a_9	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.8,0.4)$	$\mathcal{P}(1.0,0.0)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.1,0.9)$
a_{10}	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.3,0.9)$	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.9,0.2)$	$\mathcal{P}(0.3,0.9)$	$\mathcal{P}(0.9,0.2)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.1,0.8)$	$\mathcal{P}(1.0,0.0)$
a_{11}	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.0,0.9)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.0,0.9)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.3,0.9)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.3,0.9)$	$\mathcal{P}(0.5,0.6)$	$\mathcal{P}(0.3,0.9)$
a_{12}	$\mathcal{P}(0.5,0.6)$	$\mathcal{P}(0.8,0.2)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.5,0.6)$
a_{13}	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.5,0.6)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(0.6,0.7)$	$\mathcal{P}(0.9,0.1)$	$\mathcal{P}(0.2,0.9)$	$\mathcal{P}(0.5,0.5)$	$\mathcal{P}(1.0,0.0)$
a_{14}	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.0,0.9)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.4,0.6)$	$\mathcal{P}(0.1,0.9)$	$\mathcal{P}(0.3,0.9)$	$\mathcal{P}(0.4,0.9)$	$\mathcal{P}(0.4,0.5)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.0,1.0)$	$\mathcal{P}(0.2,0.9)$

Table 3
The support, neutral, and opposition coalitions with conflict measures regarding an issue.

Measure \ Trisection	Support coalition	Neutral coalition	Opposition coalition
$\mathbf{CN}_d^+$	$\emptyset$	$\{a_1, a_{10}, a_{12}\}$	$\{a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{11}, a_{13}, a_{14}\}$
$\mathbf{CN}_d^-$	$\{a_1, a_6, a_7, a_9, a_{12}\}$	$\{a_2, a_4, a_{10}\}$	$\{a_3, a_5, a_8, a_{11}, a_{13}, a_{14}\}$
$\mathbf{CN}_d^{+-}$	$\{a_1, a_9, a_{12}\}$	$\{a_6, a_7, a_{10}\}$	$\{a_2, a_3, a_4, a_5, a_8, a_{11}, a_{13}, a_{14}\}$
$\mathbf{CN}_d^{-+}$	$\{a_1, a_9, a_{12}\}$	$\{a_6, a_7, a_{10}\}$	$\{a_2, a_3, a_4, a_5, a_8, a_{11}, a_{13}, a_{14}\}$
$\{\mathbf{CN}_d^* \mid * \in \mathcal{T}\}$	$\{a_1, a_7, a_{12}\}$	$\{a_9, a_{10}\}$	$\{a_2, a_3, a_4, a_5, a_6, a_8, a_{11}, a_{13}, a_{14}\}$

Accordingly, we have the support, neutral, and opposition coalitions $\mathbb{AS}^*(\mathcal{A},\mathcal{B})$, $\mathbb{AN}^*(\mathcal{A},\mathcal{B})$, and $\mathbb{AO}^*(\mathcal{A},\mathcal{B})$:

$$
\begin{aligned}
\mathbb{AS}^+(\mathcal{A},\mathcal{B}) &= \emptyset,\\
\mathbb{AN}^+(\mathcal{A},\mathcal{B}) &= \{a_1, a_{10}, a_{12}\},\\
\mathbb{AO}^+(\mathcal{A},\mathcal{B}) &= \{a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{11}, a_{13}, a_{14}\};\\
\mathbb{AS}^-(\mathcal{A},\mathcal{B}) &= \{a_1, a_6, a_7, a_9, a_{12}\},\\
\mathbb{AN}^-(\mathcal{A},\mathcal{B}) &= \{a_2, a_4, a_{10}\},\\
\mathbb{AO}^-(\mathcal{A},\mathcal{B}) &= \{a_3, a_5, a_8, a_{11}, a_{13}, a_{14}\};\\
\mathbb{AS}^{+-}(\mathcal{A},\mathcal{B}) &= \{a_1, a_9, a_{12}\},\\
\mathbb{AN}^{+-}(\mathcal{A},\mathcal{B}) &= \{a_6, a_7, a_{10}\},\\
\mathbb{AO}^{+-}(\mathcal{A},\mathcal{B}) &= \{a_2, a_3, a_4, a_5, a_8, a_{11}, a_{13}, a_{14}\};\\
\mathbb{AS}^{-+}(\mathcal{A},\mathcal{B}) &= \{a_1, a_9, a_{12}\},\\
\mathbb{AN}^{-+}(\mathcal{A},\mathcal{B}) &= \{a_6, a_7, a_{10}\},\\
\mathbb{AO}^{-+}(\mathcal{A},\mathcal{B}) &= \{a_2, a_3, a_4, a_5, a_8, a_{11}, a_{13}, a_{14}\}.
\end{aligned}
$$

According to Definition 14, by taking $W^* = 1/4$, we have the fusion-trisection matrix:

$$
\mathbb{AM}_{\mathcal{B}} = \begin{bmatrix}
9/16 & 1/16 & 4/16 & 5/16 & 1/16 & 2/16 & 7/16 & 4/16 & 8/16 & 8/16 & 0 & 9/16 & 0 & 0\\
6/16 & 7/16 & 0 & 3/16 & 2/16 & 7/16 & 7/16 & 1/16 & 0 & 0 & 0 & 5/16 & 7/16 & 0\\
1/16 & 8/16 & 12/16 & 8/16 & 13/16 & 7/16 & 2/16 & 11/16 & 8/16 & 8/16 & 1 & 2/16 & 9/16 & 1
\end{bmatrix}.
$$

By Definition 15, we have the support, neutral, and opposition coalitions $\mathbb{AS}(\mathcal{A},\mathcal{B})$, $\mathbb{AN}(\mathcal{A},\mathcal{B})$, and $\mathbb{AO}(\mathcal{A},\mathcal{B})$:

$$
\begin{aligned}
\mathbb{AS}(\mathcal{A},\mathcal{B}) &= \{a_1, a_7, a_{12}\};\\
\mathbb{AN}(\mathcal{A},\mathcal{B}) &= \{a_9, a_{10}\};\\
\mathbb{AO}(\mathcal{A},\mathcal{B}) &= \{a_2, a_3, a_4, a_5, a_6, a_8, a_{11}, a_{13}, a_{14}\}.
\end{aligned}
$$

Subsequently, we present the support, neutral, and conflict coalitions generated by the five models in Table 3. It is evident that the five models yield distinct support, neutral, and conflict coalitions concerning multiple issues d_1, d_3, d_5, and d_7. For instance, none of the agents are trisected into the support coalition by the conflict measure $\mathbf{CN}_d^+(\cdot, a^+)$. On the other hand, five agents, namely a_1, a_6, a_7, a_9, and a_{12}, are part of the support coalition identified by the conflict measure $\mathbf{CN}_d^-(\cdot, a^-)$. Additionally, only agents a_1, a_9, and a_{12} are included in the support coalition according to the conflict measures $\mathbf{CN}_d^{+-}(\cdot, a^+)$ and $\mathbf{CN}_d^{-+}(\cdot, a^-)$. Based on the support, neutral, and conflict coalitions, decision-makers will formulate development plans. Different combinations of support, neutral, and conflict coalitions will result in varying development plans. Some of these plans may fail to comprehensively consider the complex situation. This observation emphasizes the limitation of relying on results derived from a single conflict measure.

To sum up, based on the outcomes of the trisection-fusion model, we observe that agents a_1, a_7, and a_{12} express support for issues d_1, d_3, d_5, and d_7; agents a_9 and a_{10} maintain neutral attitudes towards issues d_1, d_3, d_5, and d_7; and agents $a_2, a_3, a_4, a_5, a_6, a_8, a_{11}, a_{13}$, and a_{14} exhibit opposition to issues d_1, d_3, d_5, and d_7. Notably, only three agents lend their support to the specified set of issues. Consequently, passing the proposal directly is impossible. Moreover, with only two agents adopting neutral stances, passing the proposal is again infeasible when agents a_9 and a_{10} endorse the set of issues $\mathcal{B}$. Given that nine agents oppose the set of issues $\mathcal{B}$, the passage of the proposal becomes exceedingly challenging. In summary, it is deemed impossible to pass the proposal encompassing issues d_1, d_3, d_5, and d_7. Therefore, decision-makers must adjust the proposal based on the feedback from the fourteen cities.

6.2. Applications of fusion-trisection model

Next, we apply the fusion-trisection model of conflict analysis to calculate the support, neutral, and opposition coalitions. This analysis assists decision-makers in evaluating the feasibility of a proposal.

First, by setting $w_k = 1/11$, $\tau^* = 0.70$, and $\eta^* = 0.30$, where $k = 1, 2, \ldots, 11$ and $* \in \mathcal{T}$, for the subset of issues $\mathcal{B} = \{d_1, d_3, d_5, d_7\}$, we obtain the matrices $\mathbb{FM}_{\mathcal{B}}^+$, $\mathbb{FM}_{\mathcal{B}}^-$, $\mathbb{FM}_{\mathcal{B}}^{+-}$, and $\mathbb{FM}_{\mathcal{B}}^{-+}$:

$$
\mathbb{FM}_{\mathcal{B}}^+ = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0\\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix};
$$

$$
\mathbb{FM}_{\mathcal{B}}^- = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{bmatrix};
$$

Table 4
The support, neutral, and opposition coalitions with conflict measures regarding multiple issues.

Measure \ Trisection	Support coalition	Neutral coalition	Opposition coalition
$\mathbf{CN}^{+}_{\mathcal{B}}$	$\emptyset$	$\{a_1, a_4, a_7, a_9, a_{10}, a_{12}\}$	$\{a_2, a_3, a_5, a_6, a_8, a_{11}, a_{13}, a_{14}\}$
$\mathbf{CN}^{-}_{\mathcal{B}}$	$\{a_1, a_7, a_{12}\}$	$\{a_2, a_3, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{13}\}$	$\{a_{11}, a_{14}\}$
$\mathbf{CN}^{+-}_{\mathcal{B}}$	$\emptyset$	$\{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}, a_{12}\}$	$\{a_5, a_{11}, a_{13}, a_{14}\}$
$\mathbf{CN}^{-+}_{\mathcal{B}}$	$\emptyset$	$\{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}, a_{12}\}$	$\{a_5, a_{11}, a_{13}, a_{14}\}$
$\{\mathbf{CN}^{*}_{\mathcal{B}} \mid * \in \mathcal{T}\}$	$\emptyset$	$\{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}, a_{12}\}$	$\{a_5, a_{11}, a_{13}, a_{14}\}$

$$\mathbb{FM}^{+-}_{\mathcal{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix};$$

$$\mathbb{FM}^{-+}_{\mathcal{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Accordingly, we have the support, neutral, and opposition coalitions $\mathbb{FS}^{*}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}^{*}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}^{*}(\mathcal{A}, \mathcal{B})$:

$$\mathbb{FS}^{+}(\mathcal{A}, \mathcal{B}) = \emptyset,$$
$$\mathbb{FN}^{+}(\mathcal{A}, \mathcal{B}) = \{a_1, a_4, a_7, a_9, a_{10}, a_{12}\},$$
$$\mathbb{FO}^{+}(\mathcal{A}, \mathcal{B}) = \{a_2, a_3, a_5, a_6, a_8, a_{11}, a_{13}, a_{14}\};$$
$$\mathbb{FS}^{-}(\mathcal{A}, \mathcal{B}) = \{a_1, a_7, a_{12}\},$$
$$\mathbb{FN}^{-}(\mathcal{A}, \mathcal{B}) = \{a_2, a_3, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{13}\},$$
$$\mathbb{FO}^{-}(\mathcal{A}, \mathcal{B}) = \{a_{11}, a_{14}\};$$
$$\mathbb{FS}^{+-}(\mathcal{A}, \mathcal{B}) = \emptyset,$$
$$\mathbb{FN}^{+-}(\mathcal{A}, \mathcal{B}) = \{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}, a_{12}\},$$
$$\mathbb{FO}^{+-}(\mathcal{A}, \mathcal{B}) = \{a_5, a_{11}, a_{13}, a_{14}\};$$
$$\mathbb{FS}^{-+}(\mathcal{A}, \mathcal{B}) = \emptyset,$$
$$\mathbb{FN}^{-+}(\mathcal{A}, \mathcal{B}) = \{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}, a_{12}\},$$
$$\mathbb{FO}^{-+}(\mathcal{A}, \mathcal{B}) = \{a_5, a_{11}, a_{13}, a_{14}\}.$$

According to Definition 21, by taking $W^{*} = 1/4$, we have the fusion-trisection matrix:

$$\mathbb{FM}_{\mathcal{B}} = \begin{bmatrix} 1/4 & 0 & 0 & 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 1/4 & 0 & 0 \\ 3/4 & 3/4 & 3/4 & 1 & 1/4 & 3/4 & 3/4 & 3/4 & 1 & 1 & 0 & 3/4 & 1/4 & 0 \\ 0 & 1/4 & 1/4 & 0 & 3/4 & 1/4 & 0 & 1/4 & 0 & 0 & 1 & 0 & 3/4 & 1 \end{bmatrix}.$$

By Definition 22, we have the support, neutral, and opposition coalitions $\mathbb{FS}(\mathcal{A}, \mathcal{B})$, $\mathbb{FN}(\mathcal{A}, \mathcal{B})$, and $\mathbb{FO}(\mathcal{A}, \mathcal{B})$:

$$\mathbb{FS}(\mathcal{A}, \mathcal{B}) = \emptyset;$$
$$\mathbb{FN}(\mathcal{A}, \mathcal{B}) = \{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}, a_{12}\};$$
$$\mathbb{FO}(\mathcal{A}, \mathcal{B}) = \{a_5, a_{11}, a_{13}, a_{14}\}.$$

Subsequently, we illustrate the support, neutral, and conflict coalitions generated by five models in Table 4. While the five models generally yield similar support, neutral, and conflict coalitions towards multiple issues d_1, d_3, d_5, and d_7, there are subtle differences among the results. For instance, agents a_1, a_7, and a_{12} are categorized into the support coalition by the conflict measure $\mathbf{CN}^{-}_{\mathcal{B}}(\cdot, a^{-})$. Conversely, no agents are included in the support coalition by conflict measures $\mathbf{CN}^{+}_{\mathcal{B}}(\cdot, a^{+})$, $\mathbf{CN}^{+-}_{\mathcal{B}}(\cdot, a^{+})$, and $\mathbf{CN}^{-+}_{\mathcal{B}}(\cdot, a^{-})$. The fusion-trisection model amalgamates these results from the four conflict measures, and provides more detailed trisections for decision-making. Based on the support, neutral, and conflict coalitions, decision-makers will formulate more reasonable development plans that comprehensively consider the complex situation.

As a final point, based on the fusion-trisection model's results, we observe that no agents support the set of issues d_1, d_3, d_5, and d_7. Agents $a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9, a_{10}$, and a_{12} maintain neutral attitudes towards issues d_1, d_3, d_5, and d_7, while agents a_5, a_{11}, a_{13}, and a_{14} oppose the set of issues $\mathcal{B}$. With ten agents holding neutral attitudes, it is challenging to garner sufficient support for these issues. Consequently, passing the proposal, which includes issues d_1, d_3, d_5, and d_7, is deemed impossible. The government must, therefore, adjust the proposal in response to the feedback from the fourteen cities.

In summary, both the trisection-fusion and fusion-trisection models of conflict analysis provide consistent advice: decision-makers in Hunan Province must adjust the proposal that includes issues d_1, d_3, d_5, and d_7. In practice, decision-makers have the option to select either the trisection-fusion model or the fusion-trisection model for three-way conflict analysis. Of course, they can also utilize both models to analyze conflict problems.

7. Conclusions and future work

In the context of conflict problems, we recognize that conflict measures form the foundation for models of conflict analysis. However, a single conflict measure may not fully capture the nuanced relationship between two agents. First, we utilized the concepts of maximum positive and negative agents to introduce three types of conflict measures concerning a single issue. We subsequently devised a trisection-fusion model for conflict analysis. Second, we extended our approach to encompass multiple issues, and defined three types of conflict measures based on the maximum positive and negative agents. This led to the development of a fusion-trisection model for conflict analysis. Additionally, we translated the fusion of support, neutral, and conflict coalitions into a series of matrix operations. Finally, we applied two proposed models of conflict analysis to assist decision-makers in adjusting proposals, and demonstrated that the trisection-fusion and fusion-trisection models offer comprehensive insights for decision-making.

In the future, our research will delve into exploring methods for incorporating issue weights when calculating conflict degrees related to multiple issues. Additionally, we will investigate how to consider the weights of trisections in the development of trisection-fusion and fusion-trisection models of three-way conflict analysis. Moreover, we plan to introduce novel conflict measures to analyze the relationship between two agents and build effective models of three-way conflict analysis within the context of Pythagorean fuzzy situation tables. Most importantly, we aim to design models tailored to address complex conflict problems and identify effective strategies for conflict resolution.

CRediT authorship contribution statement

Guangming Lang: Writing – review & editing, Writing – original draft, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Weiping Ding:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization. **Duoqian Miao:** Writing – review & editing, Methodology, Investigation. **Hamido Fujita:** Writing – review & editing, Methodology, Conceptualization. **Yiyu Yao:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization.

Declaration of competing interest

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Data availability

No data was used for the research described in the article.

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