Conflict Analysis for Pythagorean Fuzzy Information Systems

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Abstract. Pythagorean fuzzy sets as generalizations of intuitionistic fuzzy sets are effective for dealing with uncertainty information, but little effort has been paid to conflict analysis of Pythagorean fuzzy information systems. In this paper, we present the concepts of the maximum positive alliance, central alliance, and negative alliance with the two thresholds α and β. Then we show how to compute the thresholds α and β for conflict analysis based on decision-theoretic rough set theory. Finally, we employ several examples to illustrate how to compute the maximum positive alliance, central alliance, and negative alliance from the view of matrix.

Keywords: Pythagorean fuzzy sets · Pythagorean fuzzy information systems · Three-way decision · Decision-theoretic rough sets

1 Introduction

Pythagorean fuzzy sets (PFSs), as generalizations of intuitionistic fuzzy sets (IFSs), are characterized by a membership degree and a non-membership degree satisfying the condition that the square sum of its membership degree and non-membership degree is equal to or less than 1, and they have more powerful ability than IFSs satisfying the condition that the sum of its membership degree and non-membership degree is equal to or less than 1 to model the uncertain information in decision making problems. So far, much effort [1,2,15,19] has been paid to Pythagorean fuzzy sets. For example, Beliakov et al. [1] provided the averaging aggregation functions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs. Bustince et al. [2] investigated a historical account of types of fuzzy sets and discussed their relationships. Reformat et al. [15] proposed a novel collaborative-based recommender system that provides a

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user with the ability to control a process of constructing a list of suggested items. Yager [19] introduced a variety of aggregation operations for Pythagorean fuzzy subsets.

Many scholars [3,9,11–14,16,17] focused on conflict analysis of information systems, and improved the relationship between the two sides of a conflict by finding the essence of the conflict issue. For example, Deja [3] examined nature of conflicts as we are formally defining the conflict situation model. Pawlak [12] initially considered the auxiliary functions and distance functions and offered deeper insight into the structure of conflicts and enables the analysis of relationships between parties and the issues being debated. Silva et al. [13] presented a multicriteria approach for analysis of conflicts in evidence theory. Sun et al. [16] subsequently proposed a conflict analysis decision model and developed a matrix approach for conflict analysis based on rough set theory over two universes. Skowron et al. [17] explained the nature of conflict and defined the conflict situation model in a way to encapsulate the conflict components in a clear manner.

In practice, if opinions of agents on issues are expressed by Pythagorean fuzzy sets, then they are more effective than intuitionistic fuzzy sets for describing imprecise information. But little effort focus on conflict analysis of Pythagorean fuzzy information systems now. Much research [4–8,10,18,20] has illustrated three-way decision theory and matrix theory are effective for knowledge discovery of information systems, we will study conflict analysis of Pythagorean fuzzy information systems based on decision-theoretic rough sets. The contributions of this paper are as follows. Firstly, we provide the concept of Pythagorean fuzzy information system, Pythagorean matrix, Pythagorean closeness index matrix, whole Pythagorean closeness index, and whole Pythagorean closeness index matrix. Secondly, we provide the concepts of maximum positive alliance, central alliance, and negative alliance with the thresholds α and β. Thirdly, we investigate how to compute the thresholds α and β based on decision-theoretic rough sets. We also employ examples to illustrate how to conduct conflict analysis of Pythagorean fuzzy information systems.

The rest of this paper is shown as follows. Section 2 reviews the basic concepts of Pythagorean fuzzy sets and decision-theoretic rough sets. Section 3 provides conflict analysis models for Pythagorean fuzzy information systems. The conclusion is given in Sect. 4.

2 Preliminaries

In this section, we review concepts of Pythagorean fuzzy sets and decision-theoretic rough sets.

Definition 1 [19]. *Let U be an arbitrary non-empty set, a Pythagorean fuzzy set (PFS) P is a mathematical object of the form as follows: $P = \{< x, (\mu(x), \nu(x)) > | x \in U\}$, where $\mu(x), \nu(x) : U \rightarrow [0,1]$ such as $\mu^2(x) + \nu^2(x) \leq 1$, for every $x \in U$, $\mu(x)$ and $\nu(x)$ denote the membership degree and the non-membership degree of the element x to U in P, respectively.*

By Definition 1, we see that an intuitionstic fuzzy set is a Pythagorean fuzzy set, but a Pythagorean fuzzy set is not always an intuitionstic fuzzy set, and Pythagorean fuzzy sets are generalizations of intuitionstic fuzzy sets. Furthermore, the hesitant degree of $x \in U$ is defined as $\pi(x) = \sqrt{1 - \mu^2(x) - \nu^2(x)}$. For convenience, we denote the Pythagorean fuzzy number (PFN) as $\gamma = P(\mu_\gamma, \nu_\gamma)$ satisfying $\mu_\gamma, \nu_\gamma \in [0, 1]$ and $\mu_\gamma^2 + \nu_\gamma^2 \leq 1$, and the hesitant degree $\pi_\gamma(x) = \sqrt{1 - \mu_\gamma^2(x) - \nu_\gamma^2(x)}$.

Definition 2 [21]. *Let $\gamma_1 = (\mu_{\gamma_1}, \nu_{\gamma_1})$ and $\gamma_2 = (\mu_{\gamma_2}, \nu_{\gamma_2})$ be PFNs. Then the Euclidean distance between γ_1 and γ_2 is defined as: $d(\gamma_1, \gamma_2) = \frac{1}{2}(|\mu_{\gamma_1}^2 - \mu_{\gamma_2}^2| + |\nu_{\gamma_1}^2 - \nu_{\gamma_2}^2| + |\pi_{\gamma_1}^2 - \pi_{\gamma_2}^2|)$.*

By Definition 2, we get the Euclidean distance between two Pythagorean fuzzy numbers, which describes the similarity degree between Pythagorean fuzzy numbers. Then we provide the concept of the closeness index of a Pythagorean fuzzy number.

Definition 3 [21]. *Let $\gamma = (\mu_\gamma, \nu_\gamma)$ be a PFN, $\mathscr{O}^+ = (1, 0)$ be the positive ideal PFN, and $\mathscr{O}^- = (0, 1)$ be the negative ideal PFN. Then the closeness index of γ is defined as: $\mathscr{P}(\gamma) = \frac{d(\gamma, \mathscr{O}^-)}{d(\gamma, \mathscr{O}^+) + d(\gamma, \mathscr{O}^-)} = \frac{1 - \nu_\gamma^2}{2 - \mu_\gamma^2 - \nu_\gamma^2}$.*

By Definition 3, we have the closeness index of a Pythagorean fuzzy number, and obtain the relationship between the Pythagorean fuzzy number and the positive ideal PFN, the negative ideal PFN.

Definition 4 [20]. *Let $S = (U, A)$ be an information system, and $X \subseteq U$. Then the probabilistic lower and upper approximations of X are defined as follows: $\underline{apr}_{(\alpha,\beta)}(X) = \{x \in U \mid P(X|[x]) \geq \alpha\}$; $\overline{apr}_{(\alpha,\beta)}(X) = \{x \in U \mid P(X|[x]_A) \geq \beta\}$, where $P(X|[x]_A) = \frac{|[x]_A \cap X|}{|[x]_A|}$, $[x]_A$ is the equivalence class of x with respect to A, and $0 \leq \beta \leq \alpha \leq 1$.*

The probabilistic lower and upper approximation operators are better than Pawlak's model for handling the uncertain and imprecise information. By Definition 4, Prof. Yao presented the concepts of the probabilistic positive, boundary, and negative regions as follows.

Definition 5 [20]. *Let $S = (U, A)$ be an information system, $X \subseteq U$, and $0 \leq \beta \leq \alpha \leq 1$. Then the probabilistic positive, boundary, and negative regions of X are defined as: $POS_{(\alpha,\beta)}(X) = \{x \in U \mid P(X|[x]) \geq \alpha\}$, $BND_{(\alpha,\beta)}(X) = \{x \in U \mid \beta < P(X|[x]) < \alpha\}$, $NEG_{(\alpha,\beta)}(X) = \{x \in U \mid P(X|[x]) \leq \beta\}$.*

3 Conflict Analysis for Pythagorean Fuzzy Information Systems

In this section, we provide conflict analysis models for Pythagorean fuzzy information systems.

Definition 6. *A Pythagorean fuzzy information system is a 4-tuple* $S = (U, A, V, f)$*, where* $U = \{x_1, x_2, \ldots, x_n\}$ *is a finite set of objects,* $A = \{c_1, c_2, \ldots, c_m\}$ *is a finite set of attributes,* $V = \{V_a \mid a \in A\}$*, where* V_a *is the set of attribute values on a, all attribute values are PFNs, and f is a function from* $U \times A$ *into* V*.*

Pythagorean fuzzy information systems are generalizations of intuitionistic fuzzy information systems, which are more effective for depicting uncertain information in practical situations.

Table 1. The Pythagorean fuzzy information system for the middle east conflict.

U	c_1	c_2	c_3	c_4	c_5
x_1	(1.0, 0.0)	(0.9, 0.3)	(0.8, 0.2)	(0.9, 0.1)	(0.9, 0.2)
x_2	(0.9, 0.1)	(0.5, 0.5)	(0.1, 0.9)	(0.3, 0.8)	(0.1, 0.9)
x_3	(0.1, 0.9)	(0.1, 0.9)	(0.2, 0.8)	(0.1, 0.9)	(0.5, 0.5)
x_4	(0.5, 0.5)	(0.1, 0.9)	(0.3, 0.7)	(0.5, 0.5)	(0.1, 0.9)
x_5	(0.9, 0.2)	(0.4, 0.6)	(0.1, 0.9)	(0.1, 0.9)	(0.3, 0.9)
x_6	(0.0, 1.0)	(0.9, 0.1)	(0.2, 0.9)	(0.5, 0.5)	(0.8, 0.4)

Example 1. Table 1 depicts the Pythagorean fuzzy information system for the Middle East conflict, which is given by experts. Concretely, x_1, x_2, x_3, x_4, x_5, and x_6 denotes Israel, Egypt, Palestinians, Jordan, Syria, and Saudi Arabia, respectively. Moreover, c_1 means Autonomous Palestinian state on the West Bank and Gaza; c_2 denotes Israeli military outpost along the Jordan River; c_3 stands for Israeli retains East Jerusalem; c_4 is Israeli military outposts on the Golan Heights; c_5 notes Arab countries grant citizenship to Palestinians who choose to remain with their borders.

Definition 7. *Let* $S = (U, A, V, f)$ *be a Pythagorean fuzzy information system. Then the Pythagorean matrix* $M(S)$ *is defined as follows:*

$$M(S) = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & . & (\mu_{1m}, \nu_{1m}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & . & (\mu_{2m}, \nu_{2m}) \\ . & . & . & . \\ (\mu_{n1}, \nu_{n1}) & (\mu_{n2}, \nu_{n2}) & . & (\mu_{nm}, \nu_{nm}) \end{bmatrix}.$$

By Definition 7, we have the matrix representation of a Pythagorean fuzzy information system, which is helpful for dealing with uncertain information with computers.

Example 2 (Continuation from Example 1). By Definition 7, we have the Pythagorean matrix $M(S)$ as follows:

$$M(S) = \begin{bmatrix} (1.0, 0.0) & (0.9, 0.3) & (0.8, 0.2) & (0.9, 0.1) & (0.9, 0.2) \\ (0.9, 0.1) & (0.5, 0.5) & (0.1, 0.9) & (0.3, 0.8) & (0.1, 0.9) \\ (0.1, 0.9) & (0.1, 0.9) & (0.2, 0.8) & (0.1, 0.9) & (0.5, 0.5) \\ (0.5, 0.5) & (0.1, 0.9) & (0.3, 0.7) & (0.5, 0.5) & (0.1, 0.9) \\ (0.9, 0.2) & (0.4, 0.6) & (0.1, 0.9) & (0.1, 0.9) & (0.3, 0.9) \\ (0.0, 1.0) & (0.9, 0.1) & (0.2, 0.9) & (0.5, 0.5) & (0.8, 0.4) \end{bmatrix}.$$

Definition 8. *Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system. Then the Pythagorean closeness index matrix $MP(S)$ is defined as follows:*

$$MP(S) = \begin{bmatrix} \mathscr{P}(\gamma_{11}) & \mathscr{P}(\gamma_{12}) & . & \mathscr{P}(\gamma_{1m}) \\ \mathscr{P}(\gamma_{21}) & \mathscr{P}(\gamma_{22}) & . & \mathscr{P}(\gamma_{2m}) \\ . & . & . & . \\ \mathscr{P}(\gamma_{n1}) & \mathscr{P}(\gamma_{n2}) & . & \mathscr{P}(\gamma_{nm}) \end{bmatrix},$$

where $\gamma_{ij} = (\mu_{ij}, \nu_{ij})$ denote the attribute value of x_i with respect to c_j.

By Definition 8, we get the matrix representation of Pythagorean closeness indexes for the Pythagorean fuzzy information system, which is helpful for conducting conflict analysis.

Example 3 (Continuation from Example 1). By Definition 8, we have the Pythagorean closeness index matrix $MP(S)$ as follows:

$$MP(S) = \begin{bmatrix} 1.000 & 0.827 & 0.727 & 0.839 & 0.835 \\ 0.839 & 0.500 & 0.161 & 0.283 & 0.161 \\ 0.161 & 0.161 & 0.273 & 0.161 & 0.500 \\ 0.500 & 0.161 & 0.359 & 0.500 & 0.161 \\ 0.835 & 0.432 & 0.161 & 0.161 & 0.173 \\ 0.000 & 0.839 & 0.165 & 0.500 & 0.700 \end{bmatrix}.$$

Definition 9. *Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, γ_i denotes the attribute value of $x \in U$ on $c_i \in A$, $0 \le w_j \le 1$ and $\sum_{j=1}^{m} w_j = 1$. Then the whole Pythagorean closeness index of x on A is defined as: $D(x) = \Sigma_{i=1}^{m} w_i \mathscr{P}(\gamma_i) = \Sigma_{i=1}^{m} \frac{w_i(1-\nu_{\gamma_i}^2)}{2-\mu_{\gamma_i}^2-\nu_{\gamma_i}^2}$.*

By Definition 9, we get the whole Pythagorean closeness index of each object with respect to all attributes and present the concept of the whole Pythagorean closeness index matrix $CP_A(S)$ as follows.

Definition 10. *Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, $0 \le w_j \le 1$, and $\sum_{j=1}^{m} w_j = 1$. Then the whole Pythagorean closeness index matrix $CP_A(S)$ is defined as follows:*

$$CP_A(S) = \begin{bmatrix} D(x_1) \\ D(x_2) \\ . \\ D(x_n) \end{bmatrix} = \begin{bmatrix} w_1\mathscr{P}(\gamma_{11}) + w_2\mathscr{P}(\gamma_{12}) + . + w_m\mathscr{P}(\gamma_{1m}) \\ w_1\mathscr{P}(\gamma_{21}) + w_2\mathscr{P}(\gamma_{22}) + . + w_m\mathscr{P}(\gamma_{2m}) \\ . \\ w_1\mathscr{P}(\gamma_{n1}) + w_2\mathscr{P}(\gamma_{n2}) + . + w_m\mathscr{P}(\gamma_{nm}) \end{bmatrix}.$$

By Definition 10, we have the matrix representation of the whole Pythagorean closeness indexes of all objects with respect to all attributes.

Example 4 (Continuation from Example 3). By Definition 10, we get the whole Pythagorean closeness index matrix $CP_A(S)$ as follows:

$$CP_A(S) = \left[0.84560.38880.25120.33620.35240.4408\right]^T.$$

By Definition 9, we provide the concepts of the maximum positive alliance, central alliance, and negative alliance as follows.

Definition 11. *Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, and $0 \leq \beta \leq \alpha \leq 1$. Then the maximum positive alliance, central alliance, and negative alliance are defined as: $POA_{(\alpha,\beta)}(U) = \{x \in U \mid D(x) \geq \alpha\}$; $CTA_{(\alpha,\beta)}(U) = \{x \in U \mid \beta < D(x) < \alpha\}$; $NEA_{(\alpha,\beta)}(U) = \{x \in U \mid D(x) \leq \beta\}$.*

By Definition 11, we see that the universe is divided into three parts: the maximum positive alliance, central alliance, and negative alliance using two thresholds α and β.

Example 5 (Continuation from Example 5). By Definition 11, we have $POA_{(\alpha,\beta)}(U) = \{x_1\}, CTA_{(\alpha,\beta)}(U) = \{x_2, x_3, x_4, x_5, x_6\}$, and $NEA_{(\alpha,\beta)}(U) = \emptyset$.

Thirdly, we calculate the parameters α and β for conflict analysis based on decision-theoretic rough set theory.

Table 2. Loss function for Pythagorean fuzzy information systems.

Action	$x \in POA_{(\alpha,\beta)}(U)$	$x \in NEA_{(\alpha,\beta)}(U)$
a_P	λ_{PP}	λ_{PN}
a_C	λ_{CP}	λ_{CN}
a_N	λ_{NP}	λ_{NN}

Theorem 1. *Let $S = (U, A, V, f)$ be a Pythagorean fuzzy information system, and the losses $\lambda_{PP}, \lambda_{CP}, \lambda_{NP}, \lambda_{NN}, \lambda_{CN}$, and λ_{PN}, where $0 \leq \lambda_{PP} \leq \lambda_{CP} \leq \lambda_{NP}$ and $0 \leq \lambda_{NN} \leq \lambda_{CN} \leq \lambda_{PN}$. Then*

(1) *If $D_A(x) > \alpha$, then $x \in POA_{(\alpha,\beta)}(U)$;*
(2) *If $\alpha \geq D_A(x) \geq \beta$, then $y \in CTA_{(\alpha,\beta)}(U)$;*
(3) *If $D_A(x) < \beta$, then $y \in NEA_{(\alpha,\beta)}(U)$, where*

$$\alpha = \frac{\lambda_{PN} - \lambda_{NN}}{\lambda_{PN} - \lambda_{CN} + \lambda_{CP} - \lambda_{PP}}, \beta = \frac{\lambda_{CN} - \lambda_{NN}}{\lambda_{CN} - \lambda_{NN} + \lambda_{NP} - \lambda_{CP}}.$$

Proof. By Table 2, we have the expected losses $R(a_P|x)$, $R(a_C|x)$, and $R(a_N|x)$ associated with taking the individual actions for the object x as follows:

$$R(a_P|x) = \lambda_{PP} * D_A(x) + \lambda_{PN} * (1 - D_A(x));$$
$$R(a_C|x) = \lambda_{CP} * D_A(x) + \lambda_{CN} * (1 - D_A(x));$$
$$R(a_N|x) = \lambda_{NP} * D_A(x) + \lambda_{NN} * (1 - D_A(x)).$$

The Bayesian decision procedure suggests the following minimum-cost decision rules:

(P): If $R(a_P|x) \leq R(a_C|x)$ and $R(a_P|x) \leq R(a_N|x)$, then $x \in POA_{(\alpha,\beta)}(U)$;
(N): If $R(a_C|x) \leq R(a_P|x)$ and $R(a_C|x) \leq R^x(a_N|x)$, then $x \in CTA_{(\alpha,\beta)}(U)$;
(A): If $R(a_N|x) \leq R(a_P|x)$ and $R(a_N|x) \leq R(a_C|x)$, then $x \in NEA_{(\alpha,\beta)}(U)$.
Suppose $\lambda_{PP} \leq \lambda_{CP} \leq \lambda_{NP}$, we simplify the rules $(P), (C)$, and (N) as follows:
(C): If $D_A(x) > \alpha$, then $x \in POA_{(\alpha,\beta)}(U)$;
(N): If $\beta \leq D_A(x) \leq \alpha$, then $x \in CTA_{(\alpha,\beta)}(U)$;
(A): If $D_A(x) < \beta$, then $x \in NEA_{(\alpha,\beta)}(U)$.

Table 3. Loss function for Pythagorean fuzzy information systems.

Action	$x \in POA_{(\alpha,\beta)}(U)$	$x \in NEA_{(\alpha,\beta)}(U)$
a_P	$\lambda_{PP} = 0$	$\lambda_{PN} = 10$
a_C	$\lambda_{CP} = 4$	$\lambda_{CN} = 4$
a_N	$\lambda_{NP} = 10$	$\lambda_{NN} = 0$

By Theorem 1, we get the two thresholds α and β for computing the maximum positive alliance, central alliance, and negative alliance using loss functions, which supplies a theoretical foundation for decision making with reasonable thresholds α and β.

Example 6 (Continued from Example 5). By Theorem 1, we have the thresholds α and β using Table 3 as follows:

$$\alpha = \frac{\lambda_{PN} - \lambda_{CN}}{\lambda_{PN} - \lambda_{CN} + \lambda_{CP} - \lambda_{PP}} = \frac{5-2}{5-2+2-0} = \frac{3}{5},$$
$$\beta = \frac{\lambda_{CN} - \lambda_{NN}}{\lambda_{CN} - \lambda_{NN} + \lambda_{NP} - \lambda_{CP}} = \frac{2-0}{2-0+6-2} = \frac{1}{3}.$$

By Definition 11, we get $POA_{(\alpha,\beta)}(U) = \{x_1\}, CTA_{(\alpha,\beta)}(U) = \{x_2, x_4, x_5, x_6\}$, and $NEA_{(\alpha,\beta)}(U) = \{x_3\}$.

In Example 6, we compute the thresholds $\alpha = \frac{3}{5}$ and $\beta = \frac{1}{3}$ for computing the maximum positive alliance, central alliance, and negative alliance using loss functions in Table 3, and give a theoretical foundation for decision making with reasonable thresholds.

4 Conclusions

In this paper, we have introduced the concepts of the maximum positive alliance, central alliance, and negative alliance. Furthermore, we have shown how to compute the thresholds for conflict analysis of Pythagorean fuzzy information systems. Finally, we have employed several examples to illustrate how to conduct conflict analysis of Pythagorean fuzzy information systems from the view of matrix.

In practice, Pythagorean fuzzy information systems are effective for describing uncertain information, and there will be more Pythagorean fuzzy information systems, and we will further study conflict analysis of Pythagorean fuzzy information systems in the future.

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