Relation of Relative Reduct Based on Nested Decision Granularity

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Abstract— **A granulation structure for stratified rough sets' approximation is examined. This paper mainly explores the relationships between their cores and their reductions based on consistent and inconsistent information tables. At the end, some conclusions are drawn as following: The relative core of coarser decision granularity must be included by that of finer decision granularity; A relative reduction of coarser decision granularity can be enlarged to be the one of the finer decision granularity, contrarily, a relative reduction of finer decision granularity is sure to contain that of coarser decision granularity, all of which take on special significance in practice for knowledge reduction and dynamic solution problems.**

Index Terms— **Relative reduction, Inclusion degree, Granular partition, Nested decision granularity.**

I. Introduction

KNOWLEDGE Representation System[1] based on the decision table generally includes a mass of data and information records. These records make up of a relational database and reflect connections between condition attributes and decision attributes to a certain extent, therefore, they are carriers of domain knowledge. The main objective of Knowledge Discovery In Database[2] (abbreviation, KDD) is how to acquire potential, novel, correct and available knowledge that involves vague, incomplete, uncertain, partly truth and vast information, then we can solve many problems in correlative fields via these acquired knowledge. In the course of problem-solving and information-processing, a granularity and abstraction of information is necessary in most cases. Observing the real world by using various grain sizes, we can abstract and consider only those that serve our present interest, Information Granulation[3] has a great influence on the design and application of intelligent system. Our ability to conceptualize the world at different granularities and to switch among these granularities is fundamental to our intelligence and flexibility. It enables us to map the complexities of the world around us into simpler theories that are computationally tractable to reason in.

The principles of the granularity theory have been applied in many studies. Giunchigalia and Walsh put forward a theory of abstraction which is thought of as "the process which allows people to consider what is relevant and to forget a lot of irrelevant details which would get in the way of what they are trying to do". In fact, Abstraction methods are essential ideas of information granulation. Zadeh and Pawlak are exploiters in the research domain, namely Granular Computing, subsequently Skowron .A ., Yao. Y.Y., T.Y.Lin., Zhong.N., Zhang.L and Zhang B et al, have developed the theory, which recently received much attention from computational intelligence communities[4,5,6].

Because rough set is one of main tools for granular computing and knowledge reduction plays an important role in information and knowledge predigest, we analyze some notions of rough sets and explore relationships among relative cores / relative reducts of nested sequence of granulations about decision attribute and prove it based on inclusion degree concept in the end.

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II. Basic Concepts and Theorems

Definition 1[7] Partitioning and Covering: Let U denote a finite and non-empty set called the universe. $R \subseteq U \times U$ denotes an equivalence relation on U, where "$\times$" denotes the Cartesian product of sets, (namely R is reflective, symmetric as well as transitive.) By the equivalence relation R on the universe U, one can divide the set U into a family of subsets $G_i(G_i = [x]_R, i = 1, 2, \cdots, n)$ that satisfies: (1)$G_i \neq \emptyset$; (2)$G_i \cap G_j = \emptyset, i \neq j$ (namely there are no overlaps among them); (3)$\bigcap_{i=1}^{n} G_i = U$, then call $\pi(U/R) = \{[x]_R : \forall x \in U\} = \{G_1, G_2, \cdots, G_n\} = \{G_i\}_{i=1}^{n}$ the partition on the universe induced by R, which can be denoted by $\pi(R)$ when U and R are explicit, where $[x]_R = \{y|xRy, \forall x, y \in U\}$.

Because intersection of finite equivalence relations on a universe is still an equivalence relation, one can define the partition by intersection of finite equivalence relations on a universe. Given a universe U and a family of equivalence relations $\Re$ on U, if $B \subseteq \Re \wedge B \neq \emptyset$, then $ind(B) = \bigcap_{\forall a \in B} \alpha$ is still an equivalence relation on the U, it is called a indiscernibility relation on U and is denoted by $ind(B), B$, therefore $\pi(B)$ is called the granular partition of the universe induced by B,where $[x]_B = \{y| \bigwedge_{\forall R \in B} xRy, \forall x, y \in U\}$. In addition, $\zeta(\pi) = \{\pi|\pi = B, \forall B \subseteq \Re \vee B = \emptyset \vee B = " = "\}$ denotes all the partitions of the universe U induced by the family of finite equivalence relations.

Definition 2 The size of granular partitioning: Let $\pi_1(R_1) = \{G_{11}, G_{12}, \cdots, G_{1m}\}$ and $\pi_2(R_2) = \{G_{21}, G_{22}, \cdots, G_{2n}\}$ be two granular partitions on a universe. If $\forall G_{1i} \in \pi_1$, always $\exists G_{2j} \in \pi_2$ satisfies $G_{1i} \subseteq G_{2j}$, then we

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call π_1 finer granular partitioning than π_2, denoted by $\pi_1 \leq \pi_2$. There also has the case on which the sizes of two granular partitions can't be compared with.

Definition 3 The size of equivalence relations on the universe U : Let $\Re$ denote the set of all the equivalence relations on the given universe U, if $\forall R_1, R_2 \in \Re, \forall x, y \in U$ satisfies (1) $xR_1y \Rightarrow xR_2y$ then call R_1 finer than R_2, simply denoted $R_1 \leq R_2$;(2) $xR_1y \Leftrightarrow xR_2y$ then call the size of R_1 is equal to that of R_2; (3) if (1),(2) are not valid, then call the size of R_1 unable to compare with the one of R_2.

Definition 4 Inclusion degree[8] **:** Let $(U, \leq)$ be a partial order set, if for $\forall x, y \in U$,there is a real number $\partial(y/x)$ with respect to x and y and it satisfies (1) $0 \leq \partial(y/x) \leq 1$, (2) $x \leq y \Rightarrow \partial(y/x) = 1$, (3) $x \leq y \leq z \Rightarrow \partial(x/z) \leq \partial(x/y)$, then $\partial(y/x)$ is called a inclusion degree in the partial order set. One can integrate many granular computing and rough sets models by inclusion degree, which leads to a more general approximation structure for computing and reasoning with information granules.

Example 1: Let U denote a finite and non-empty set called the universe. $P(U)$ is the power set of the universe U, where $(P(U), \subseteq)$ constitutes a partial order set. For $\forall X, Y \in P(U)$, one can define:

$$\partial(Y/X) = |X \cap Y|/|X| \tag{1}$$

Then it's a kind of inclusion degrees on the $P(U)$.

Proof: ① for $\forall X, Y \in P(U)$, $\because 0 \leq |X \cap Y| \leq |X|, \therefore 0 \leq \partial(Y/X) = |X \cap Y|/|X| \leq 1$;
② $x \subseteq y, \partial(Y/X) = |X \cap Y|/|X| = |X|/|X| = 1$;
③ $X \subseteq Y \subseteq Z$, so $|X| \leq |Y| \leq |Z|$, $\partial(X/Z) = |X \cap Z|/|Z| = |X|/|Z| \leq |X|/|Y| = |X \cap Y|/|Y| = \partial(X/Y)$.

So the formula (1) is the inclusion degree on the $P(U)$ according to the definition 4.

Definition 5[9] **The significance degree of condition attribute** $a(\forall a \in C)$ **with respect to the decision attribute** D**:** Let $(U, C \cup D, V, f)$ be a decision table, $\pi(D)$ denotes a granular partitioning on the universe induced by the decision attribute D. For $\forall a \in C$, $sig(a) = 1 - \partial(\pi(D)/\pi(C - \{a\})$ is called the significance degree of condition attribute ($\forall a \in C$) with respect to the decision attribute D. Where ∂ is an inclusion degree on the $\zeta_U(\pi)$. The significance degree play an important role in analyzing the dependent degree among attributes and constructing heuristic arithmetic of knowledge reduction.

Definition 6[8] **The positive field of relation** Q **with respect to relation** P: Let U be a universe. Q and P are two equivalence relations on the U , then $POS_PQ = \bigcup X \in U/Q$. $\underline{P}(X)$ is called the positive field of relation Q with respect to relation P. It shows the sets of all objects on the U that are partitioned exactly into the equivalence classes of relation Q by the classification information of U/P, where $\underline{P}(X) = \{Y_i | (Y_i \in U/P) \wedge (Y_i \subseteq X)\}$

Definition 7[8] **The core attribute and the relative core to decision attribute** D **of a decision table:** Let a quaternion (U, A_t, V, f) be a decision table. For $\forall a \in C$, if the single attribute a satisfies one of the conditions as following: (1) $POS_C(D) \neq POS_{C-\{a\}}(D)$. (2) In the discernibility matrix $M = (r_{ij})_{n \times n}$ based on objects, for $\forall x_i, x_j \in U, i, j = 1, 2, \cdots, n.(n = |U|), \exists r_{ij} = \{a\}$ is a single attribute set (3) In a given consistent decision table, $ind(C - \{a\}) \leq ind(D)$ is not held, namely either $\pi(C - \{a\}) \leq \pi(D)$ or $R_{C-\{a\}}) \leq R_D$ is not held.

Then we call the single attribute a is a core attribute. The set of all core attributes of the decision table is called the relative core to D, denoted $core_C(D)$. A core attribute is necessary for keeping the classification ability about the decision table, because if any one of core attribute is deleted, the classification ability about the information table will be weaken.

Definition 8 The relative reduction to decision attribute D[7]**:** Let $(U, C \cup D, V, f)$ be a decision table, if $\exists P \subseteq C$, satisfies
(1) P is independent, namely for $\forall a \in P, POS_{P-\{a\}}(D) \neq POS_P(D)$
(2)$POS_P(D) = POS_C(D)$ then the relation P is called a the relative reduction to decision attribute D about the decision table, denoted by $P \in red_C(D)$.

Definition 9 The consistent decision table: Let $(U, C \cup D, V, f)$ be a decision table. If $\pi(C) \leq \pi(D)$, namely $POS_C(D) = U$, then the decision table is called a consistent decision table. Otherwise, the decision table is called a inconsistent decision table. Knowledge being acquired based on a consistent decision table is determinate, which show the information table does not contain any confliction information /instances, but the case in a inconsistent decision table is reverse.

III. Basic Theorems

Theorem 1[8]**:** Let $\zeta_U(\pi)$ denote the set of all granular partitions on a given universe U. The $(\zeta_U(\pi), \leq)$ constitutes a partial order set, and ∂ is a inclusion degree on the $(P(U), \subseteq)$. For $\forall \pi_1, \pi_2 \in \zeta_U(\pi)$ define $\partial(\pi_2/\pi_1) = \bigwedge_{i=1}^{k} \bigvee_{j=1}^{l} \partial(Y_j/X_i)$, then it is called a inclusion degree on the $(\zeta(\pi), \leq)$. Where $\pi_1 = \{X_i\}_{i=1}^k, \pi_2 = \{Y_j\}_{j=1}^l$ are two kind of granular partitions on the universe U.

Theorem 2[8]: Let $(U, C \cup D, V, f)$ be a given consistent decision table, then for $\forall a \in C$, a is a core attribute, iff (if and only if)

$$sig(a) = 1 - \partial(\pi(D))/\pi(C - \{a\})) > 0 \tag{2}$$

IV. Research on the Relationships among Relative Cores and That of among Relative Reducts of Nested Decision Granularities

A. Based on a family of consistent and nested decision table

When we solves many practice problems, we usually need subdivide or coarsen decision granularity in a information table so as to solve problems dynamically according to accuracy of approximation, hence we will study the relationships of nested decision granularity's relative cores and that of their relative reducts.

Definition 10: A family of consistent and nested decision granularities about a information table: Let $(U, C \cup D_i, V, f)$ be a family of decision tables, if they satisfy: (1) $\pi(C) \leq \pi(D_i)$, (2) $\pi(D_1) \geq \pi(D_2) \geq \cdots \geq \pi(D_N), i =$

$1, 2, \cdots, N$, then it is called a family of consistent and nested decision granularities about the information system (U, C, V, f).

Theorem 3: Let $(U, C \cup D_i, V, f)$ be a family of consistent and nested decision granularities. $i = 1, 2, \cdots, N$, ∂ is a inclusion degree on the $\zeta_U(\pi)$. Then the formulas as following are held:

$$core_C(D_1) \subseteq core_C(D_2) \subseteq \cdots \subseteq core_C(D_N) \qquad (3)$$

$$\forall B^{'} \in red_C(D_{i+1}) \Rightarrow \exists B \subseteq B^{'} \wedge B \in red_C(D_i) \qquad (4)$$

Proof: Let $\pi(D_i) \geq \pi(D_{i+1})$, next to prove: for $\forall a \in core_C(D_i) \Rightarrow a \in core_C(D_{i+1})$.
$\because$ $\pi(C) \leq \pi(D_{i+1}) \leq (D_i)$,for $\forall a \in core_C(D_i), i = 1, 2, \cdots, N$.$\Leftrightarrow$ $sig(a) = 1 - \partial(\pi(D_i)/\pi(C_{\{}a\}) > 0 \Leftrightarrow \partial(\pi(D_i)/\pi(C_{\{}a\}) < 1 \Leftrightarrow \pi(C_{\{}a\}) \leq \pi(D_i)$ is not held. $\Rightarrow$ $\pi(C_{\{}a\}) \leq \pi(D_{i+1})$ is not held.$\Rightarrow$ $a \in core_C(D_{i+1})$. Otherwise,$\pi(C - \{a\}) \leq \pi(D_{i+1})$, and $\because \pi(D_{i+1}) \leq \pi(D_i)$, then $\pi(C - \{a\}) \leq \pi(D_{i+1}) \leq \pi(D_i) \Rightarrow a \notin core_C(D_1)$, which lead to contradiction. So $core_C(D_i) \subseteq core_C(D_{i+1})$.

Owing to values of variable i from 1 to $n-1$ in above the formula ,obviously proposition (1) is right.

Supposed $\pi(D_i) \geq \pi(D_{i+1})$, next to prove:
① for $\forall B' \in red_c(D_{i+1})$, always exists $B \subseteq B'$, gets $B \in red_C(D_i)$,
② for $\forall B \in red_C(D_i) \wedge \pi(B) \leq \pi(D_{i+1}) \Rightarrow (\exists B' \supseteq B) \wedge (B' \in red_C(D_{i+1}))$.

Let $B' \in red_C(D_{i+1}) \Leftrightarrow \pi(B') \leq \pi(D_{i+1})$ and $\forall a \in B', \pi(B' = \{a\}) \leq \pi(D_{i+1})$ not be held. $\because$ $\pi(D_i) \geq \pi(D_{i+1}) \Rightarrow pi(B') \leq \pi(D_{i+1}) \leq \pi(D_i)$, if for $\forall a \in B', \pi(B' - \{a\}) \leq \pi(D_i)$ is not held, then $B' \in red_C(D_i)$. One can get $B = B'$, namely B satisfies $B \subseteq B'$ and $B \in red_C(D_i)$. if $\exists a \in B'$, get $\pi(B' - \{a\}) \leq \pi(D_i)$ can be held, then take $B'' = B' - \{a\}$, in succession to reason for $\forall \beta \in B''$ whether $\pi(B' - \{\beta\}) \leq \pi(D_i)$ comes into existence. If it is not held, then $B'' = B' - \{a\} \in red_C(D_i)$, take $B = B'' \Rightarrow B = B'' \subset B'$ and $B \in red_C(D_i)$.If it is valid, then the above reasoning processes are continued, due to the finite attributes set, we can find always $B \subseteq B'$, get B satisfies $(\pi(B) \leq \pi(D_i)$ and $\forall b(\pi(B - \{b\}) \leq \pi(D_i)$ is not held, so $B \in red_C(D_i)$. Owing to values of variable i from 1 to n, the proposition ① is right. In the same, we can prove the proposition ② is right too.

Theorem 3 shows the relations of relative cores and reductions to D of nested decision granularities, which can be used to acquire knowledge dynamically in information tables and solve practical problems easily based on granular computing.

B. Based on a family of inconsistent and nested decision table induced from relational database

For discussing relationships of relative cores and that of among relative reducts of nested decision granularities induced from the relational database (namely no identical object in decision table), a new inclusion degree (we call δ inclusion degree in the following text) is defined and applied here to measure the inconsistent and nested decision table.

Definition 11: A family of inconsistent and nested decision granularities induced from a relational database: Let $(U, C \cup D_i, V, f)$ be a family of decision tables, if they satisfy:
(1) $\pi(D_1) \geq \pi(D_2) \geq \cdots \geq \pi(D_N), i = 1, 2, \cdots, N$
(2) $POS_C(D_i) \neq U, i = 1, 2, \cdots, N$
(3) $\forall x_i, x_j \in U, i \neq j$,$x_i$ and x_j are not identical.

Then it is called a family of inconsistent and nested decision granularities induced from relational database.

Definition 12: Let U denote a finite and non-empty set called the universe, $P(U)$ is the power set of the universe U. For $\forall X, Y \subseteq P(U)$, δ inclusion degree is defined as:

$$\delta(Y/X) = \begin{cases} 1, X \subseteq Y \\ 0, (\text{others}) \end{cases} \qquad (5)$$

Furthermore, $\delta(Y/X)$ is inclusion degree on the $P(U)$.

Definition 13: Let $\zeta_U(\pi)$ denote the set of all granular partitions on a given universe U. The $(\zeta(\pi), \leq)$ constitutes a partial order set, and δ function is defined on the $(P(U), \subseteq)$. For $\forall \pi_1, \pi_2 \in \zeta_U(\pi)$ define $\delta(\pi_2/\pi_1) = \sum_{i=1}^{k}\sum_{j=1}^{l}\delta(Y_j/X_i)$, Where $\pi_1 = \{X\}_{i=1}^{k}, \pi_2 = \{Y\}_{j=1}^{l}$ are two kind of granular partitions on the universe U.

Lemma 1: Let $(U, C \cup D_i, V, f)$ be a family of inconsistent and nested decision granularities induced from relational database. $i = 1, 2, \cdots, N$. So $\delta(\pi(D_i)/\pi(C)) = |POS_C(D)|$. Where $\pi_1 = \{X\}_{i=1}^{k}, \pi_2 = \{Y\}_{j=1}^{l}$ are two kind of granular partitions on the universe U.

Proof: ① if X_i satisfies $|X_i| = 1$ and $x \in X_i$, always and only exist one $Y_j \in \pi(D_i), X_i \subseteq Y_j \Rightarrow \sum_{j=1}^{l}\delta(Y_j/X_i) = 1$. From the definition 6, we can know object must satisfy $x \in POS_C(D_i)$ (namely $X_i \subseteq POS_C(D)$).
② if X_i satisfies $|X_i| > 1$, that mean the elements of X_i have confliction information or instances. So there is no such Y_j satisfies $(X_i \subseteq Y_j)$ namely $\sum_{j=1}^{l}\delta(Y_j/X_i) = 0$, and $X_i \nsubseteq POS_C(D)$.

From (i)(ii),we can conclude $\delta(\pi(D_i)/\pi(C)) = |POS_C(D)|$ are equal with the number of X_i which satisfies $|X_i| = 1$. Actually in an inconsistent decision table induced from relational database, $|X_i| = 1$ means $x \in X_i$ is consistent object and $\delta(\pi(D_i)/\pi(C))$ denotes the number of consistent object in decision table.

Lemma 2: Let $(U, C \cup D_i, V, f)$ be a family of inconsistent and nested decision granularities induced from relational database. $i = 1, 2, \cdots, N$. The following formula comes into existence. $\delta(\pi(D_i)/\pi(C)) = \delta(\pi(D_{i+1})/\pi(C))$ Where $\pi_1 = \{X\}_{i=1}^{k}, \pi_2 = \{Y\}_{j=1}^{l}$ are two kind of granular partitions on the universe U, $\pi(D_i) \geq \pi(D_{i+1})$.

Proof: $\because \delta(\pi(D_i)/\pi(C)) = |POS_C(D)|$ and fining of the decision attribute won't change the inconsistent degree of the decision table induced from relational database. Therefore the proposition comes into existence.

Theorem 4: Let $(U, C \cup D, V, f)$ be a given inconsistent decision table induced from relational database, then for $\forall a \in C$, a is a core attribute, iff (if and only if)

$$\delta(\pi(D))/\pi(C)) \neq \delta(\pi(D))/\pi(C - \{a\}) \qquad (6)$$

Proof: $\because$ $\delta(\pi(D))/\pi(C)) \neq \delta(\pi(D))/\pi(C-\{a\}) \Leftrightarrow pos_{C-\{a\}}(D) \neq pos_C(D)$, So a is a core attribute of the decision table.

Theorem 5: Let $(U, C \cup D_i, V, f)$ be a family of inconsistent and nested decision granularities induced from relational database. $i = 1, 2, \cdots, N$. The relative reduction about the decision table $B \in red_C(D)$ is equivalent with following equations which are held

$$\delta(\pi(D)/\pi(B)) = \delta(\pi(D)/\pi(C)) \quad (7)$$

$$\forall a \in B, \delta(\pi(D)/\pi(B-\{a\})) \neq \delta(\pi(D)/\pi(C)) \quad (8)$$

Proof:(1) $\because B \in red_C(D) \Rightarrow POS_B(D) = POS_C(D) \wedge \forall a \in BPOS_{B-\{a\}}(D) \neq POS_C(D) \Rightarrow \delta(\pi(D)/\pi(B)) = \delta(\pi(D)/\pi(C)) \wedge \forall a \in B, \delta(\pi(D)/\pi(B-\{a\})) \neq \delta(\pi(D)/\pi(C))$
(2) $\because \delta(\pi(D)/\pi(B)) = \delta(\pi(D)/\pi(C))$. From Lemma 1 we can know $|POS_B(D)| = |POS_C(D)|$ and $\forall X \in \pi(B) \wedge X \subseteq POS_B(D) \Rightarrow |X| = 1 \because \pi(B) < \pi(C) \therefore X$ must satisfies $X \in \pi(C) \wedge X \subseteq POS_C(D)$, so $\delta(\pi(D_i)/\pi(B)) = \delta(\pi(D_i)/\pi(C)) \Rightarrow pos_B(D) = pos_C(D)$. In the same, $foralla \in B, \delta(\pi(D)/\pi(B-\{a\})) \neq \delta(\pi(D)/\pi(C)) \Rightarrow \forall a \in B, pos_{B-\{a\}}(D) \neq pos_C(D)$. Therefore, $\delta(\pi(D)/\pi(B)) = \delta(\pi(D)/\pi(C))$ and $\forall a \in B, \delta(\pi(D)/\pi(B-\{a\})) \neq \delta(\pi(D)/\pi(C)) \Rightarrow B \in red_C(D)$ come into existence.

Theorem 6: Let $(U, C \cup D_i, V, f)$ be a family of inconsistent and nested decision granularities induced from relational database. $i = 1, 2, \cdots, N$. Then following formulas is held:

$$core_C(D_1) \subseteq core_C(D_2) \subseteq \cdots \subseteq core_C(D_N) \quad (9)$$

$$\forall B' \in red_C(D_{i+1}) \Rightarrow \exists B \subseteq B' \wedge B \in red_C(D_i) \quad (10)$$

Proof: Let $\pi(D_i) \geq \pi(D_{i+1})$, next to prove $\forall a \in core_C(D_i) \Rightarrow a \in core_C(D_{i+1})$.

For $\forall a \in core_C(D_i)$, $i = 1, 2, \cdots, N. \Leftrightarrow \delta(\pi(D_i)/\pi(C-\{a\})) \neq \delta(\pi(D_i)/\pi(C)) \Leftrightarrow \pi(D_i) \geq \pi(D_{i+1}) \Leftrightarrow \delta(\pi(D_i)/\pi(C)) = \delta(\pi(D_{i+1})/\pi(C))$ and $\delta(\pi(D_i)/\pi(C-\{a\})) = \delta(\pi(D_{i+1})/\pi(C-\{a\})) \therefore \delta(\pi(D_{i+1})/\pi(C-\{a\})) \neq \delta(\pi(D_{i+1})/\pi(C)) \Leftrightarrow a \in core_C(D_{i+1})$ So next conclusion can be draw

$$core_C(D_i) \subseteq core_C(D_{i+1}) \quad (11)$$

Owing to values of variable i from 1 to $n-1$ in above the formula(14), which show the proposition (1) is right.

Supposed $\pi(D_i) \geq \pi(D_{i+1})$, next to prove:
① for $\forall B' \in red_c(D_{i+1})$,always exists $B \subseteq B'$,gets $B \in red_C(D_i)$,
② for $\forall B \in red_C(D_i) \Rightarrow (\exists B' \supseteq B) \wedge (B' \in red_C(D_{i+1}))$.

Let $B' \in red_C(D_{i+1}) \Leftrightarrow \delta(\pi(D_{i+1})/\pi(B')) = \delta(\pi(D_{i+1})/\pi(C))$ and $\forall a \in B', \delta(\pi(D_{i+1})/\pi(B'-\{a\})) \neq \delta(\pi(D_{i+1})/\pi(C)) \because \pi(D_i) \geq \pi(D_{i+1}) \Rightarrow \delta(\pi(D_i)/\pi(C)) = \delta(\pi(D_{i+1})/\pi(C))$ and $\delta(\pi(D_i)/\pi(B)) = \delta(\pi(D_{i+1})/\pi(B))$, $\therefore \delta(\pi(D_i)/\pi(B')) = \delta(\pi(D_i)/\pi(C))$ if for$\forall a \in B', \delta(\pi(D_i)/\pi(B'-\{a\})) \neq \delta(\pi(D_i)/\pi(C))$ then $B' \in red_C(D_i)$. One can get $B = B'$, namely B satisfies $B \subseteq B'$ and $B \in red_C(D_i)$. if $\exists a \in B'$, get $\delta(\pi(D_i)/\pi(B'-\{a\})) = \delta(\pi(D_i)/\pi(C))$, then take $B'' = B' - \{a\}$, in succession to reason for $\forall \beta \in B''$ whether $\delta(\pi(D_i)/\pi(B''-\{\beta\})) = \delta(\pi(D_i)/\pi(C))$ comes into existence. If it is not held, then $B'' = B' - \{\beta\} \in red_C(D_i)$, take $B = B'' \Rightarrow B = B'' \subset B'$ and $B \in red_C(D_i)$. If it is valid, then the above reasoning process will be went on. Because the attributes set is finite, so we can surely find $B \subseteq B'$ while B satisfies $\delta(\pi(D_i)/\pi(B)) = \delta(\pi(D_i)/\pi(C))$and

$\delta(\pi(D_i)/\pi(B-\{\beta\})) \neq \delta(\pi(D_i)/\pi(C))$. Therefore $B \in red_C(D_i)$. Owing to values of variable i from 1 to n, the proposition ① is right. In the same, we can prove the proposition ② is right too.

V. Conclusion

In this paper, we explored the relationships among relative cores and relative reducts of nested granularities based on consistent and inconsistent information tables, while concluded futher that the cores of nested sequence of granulations about decision attribute keep reverse order nested relations. A reduction of coarser decision granulation can enlarge a reduction of its finer decision granulation; reversely a reduction of finer decision granulation is sure to contain a reduction of its coarser decision granulation. The findings have the certain practical significance to knowledge reduction and dynamically solution problems. At the same time these findings are beneficial in further studies models of revision and recursion of knowledge acquisition based on a decision table.

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