Structured Prior Knowledge and Granular Structures

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Abstract. In this paper, a hierarchical organization of prior knowledge based on multidimensional data model is firstly proposed, it is the basis of structured thinking. Secondly, a representation of granular structures based on multidimensional data model is also proposed, it can represents information from multiview and multilevel. Finally, the relation between structured prior knowledge and granular structures is analyzed.

1 Introduction

Granular computing is a general computation theory for effectively using granules such as classes, clusters, subsets, groups and intervals to build an efficient computational model for complex applications with huge amounts of data, information and knowledge [1]. It is also a way of thinking [2] that relies on the human ability to perceive the real world under various levels of granularity (i.e., abstraction).

Hobbs [3] stated that 'We perceive and represent the world under various grain sizes, and abstract only those things that serve our present interests. The ability to conceptualize the world at different granularities and to switch among these granularities is fundamental to our intelligence and flexibility. This enables us to map the complexities of real world into computationally tractable simpler theories'. Gordon et al. [4] pointed out that human perception benefits from the ability to focus attention at various levels of detail and to shift focus from one level to another. The grain size at which people choose to focus affects not only what they can discern but what becomes indistinguishable, thus permitting the mind to ignore confusing detail. Bradley C. Love [5] noticed that humans frequently utilize and acquire category knowledge at multiple levels of abstraction. Yao [2] proposed the basic ideas of granular computing, i.e., problem solving with different granularities. Granular computing, as a way of thinking, can captures and reflects our ability to perceive the world at different granularity and to change granularities in problem solving.

Although these scholars have noticed that human can solve problem at different levels of granularity, the reason of which has seldom been analyzed until now. If we know how the human intellect works, we could simulate it by machine.

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In our opinion, this is concerned with human cognition, which leads to significant differences between human and machine in problem solving. Firstly, human can use relevant prior knowledge subconsciously in problem solving, but machine can't. For example, we notice that everyone can solve a given problem easily from his/her familiar fields at different levels of granularities. But (s)he even cannot solve a problem from his/her unfamiliar field at single level, not to mention at multiple levels. Secondly, human can use their relevant prior knowledge to generate a good 'structure' or problem representation in problem solving. As we all know that the famous story of young Gauss who gave the answer to the sum of all the numbers from 1 to 100 very quickly, not by very fast mental arithmetic but by noticing a pattern in the number sequence. Namely, that the numbers form pairs (1+100=101, 2+99=101, $\cdots$, 50+51=101). This example indicates that a good structuring, or representation of the problem helps considerably. In essence, the so called 'good' structure of a problem is a hierarchical structure induced from it. Thirdly, J.Hawkins and S.Blakeslee [6] pointed out that there have fundamentally different mechanisms between human brain and machine. The mechanism of human brain is that it retrieves the answers stored in memory a long time ago, but not "compute" the answers to a problem as machine. This indicates that human usually search a relevant or similar answer to the solved problem from his memory in problem solving, or we can say that it depends on relevant prior knowledge to solve problem for human. Maybe these can be used to interpret why human can focus on different levels during the process of problem solving, but machine cannot.

In this paper, the importance of hierarchical structured prior knowledge in granular computing is stressed firstly, where prior knowledge is extended to a broader sense, which includes domain knowledge. A nested and hierarchical organization of prior knowledge based on multidimensional data model is proposed, it is the basis of structured thinking [7]. Secondly, multidimensional data model was introduced into granular computing, it can represent information from multiview and multilevel, and can be used as a representation model of granular structures. Finally, the relation between structured prior knowledge and granular structures is analyzed.

2 Prior Knowledge

In this section, we will give a brief introduction to prior knowledge, and propose an organization of prior knowledge based on multidimensional data model. We also point out that structured prior knowledge is the basis of structured thinking. The main role of structured prior knowledge is to provides humans with a much greater control over the solved problem.

2.1 An Introduction to Prior Knowledge

Prior knowledge is all the knowledge you've acquired in your lifetime. It includes knowledge gained from formal and informal instruction. Prior knowledge has the same meaning as background knowledge, previous knowledge, personal knowledge, etc. In this paper, we will extend the concept "prior knowledge" to a broader sense, which also includes domain knowledge.

Prior knowledge can help us all of the time. When we do something for the first time, we will feel hard because we don't have much prior knowledge about it. After we do something several times and accumulate a lot of relevant prior knowledge, we will feel easier. This indicates that prior knowledge is very helpful to human problem solving.

In practice, prior knowledge is valuable to be incorporated into a practical problem solving. Yu et al. [8] noticed the necessity of extra information to problem solving, and provided a mathematical expression

$$Generalization = Data + knowledge \tag{1}$$

This formula indicates that if you want to solve a problem at higher levels or multiple levels of abstraction, you will have to add extra knowledge to the solved problem. In this formula, data can be obtained from the solved problem. But what's the knowledge here? In our opinion, the knowledge here is what we call prior knowledge in this paper.

Prior knowledge can be obtained by learning. When we learn new knowledge, we will assimilate it and make sense of it by connecting it to what we have already known, or we can say that we will incorporate it into our existing knowledge structure subconsciously. As stated in [9]: "Our representation of the world is not necessarily identical to the actual world. We modify information that is received through our senses, sharpening, selecting, discarding, abstracting, etc. So our internal representation of the world is really our own construction. In other words, human brain is not a sponge that passively absorbs information leaking out from the environment. Instead, they continually search and synthesize." Which structure does prior knowledge be organized in human brain? And which organization of prior knowledge is suitable for granular computing particularly? These will be discussed in the next subsection.

2.2 Structure of Prior Knowledge

Some researchers have noticed the importance of knowledge structure. Mandler [10] pointed out that meaning does not exist until some structure, or organization, is achieved. In [11], the authors pointed out that knowledge structure is a structured collection of concepts and their interrelationships, it includes two dimensions: multilevel structure and multiview structure. This is in accordance with granular structures. In [6], the authors pointed out that "the cortex's hierarchical structure stores a model of the hierarchical structure of the real world. The real world's nested structure is mirrored by the nested structure of your cortex". In an exactly analogous way, our memories of things and the way our brain represents them are stored in the hierarchical of the cortex. In [12], the authors mentioned that humans and other species represent knowledge of routine events or stereotypical action sequences hierarchically. These indicate that prior knowledge can be organized as a hierarchical structure. But which hierarchical structure can reflect prior knowledge more intuitively?

Quillian [13,14,15] pointed out that prior knowledge is stored in the form of semantic networks in human brain. A semantic network or knowledge structure is created with three primitives: concepts, relations, and instances. Yao [16] pointed out human thought and knowledge is normally organized as hierarchical structures, where concepts are ordered by their different levels of specificity or granularity. A plausible reason for such organizations is that they reflect truthfully the hierarchical and nested structures abundant in natural and artificial systems. Human perception and understanding of the real world depends, to a large extent, on such nested and hierarchical structures.

Of course, we don't know the genuine organization of prior knowledge in human brain until now. But from the above we know that prior knowledge should be organized as a nested and hierarchical structures, which is helpful to human problem solving.

2.3 Organization of Prior Knowledge

Maybe there have a lot of knowledge stored in human brain, but only a little part of which is relevant to the solved problem in problem solving. So we should only extract the relevant part of prior knowledge and reorganize them as a nested and hierarchical structure in problem solving. Hierarchical structures not only make a complex problem more easily understandable, but also lead to efficient solutions.

In this subsection, we will provide a nested and hierarchical organization of prior knowledge relevant to the solved problem, which is suitable for problem solving particularly. We will reorganize relevant prior knowledge from one point of view with multiple levels of granularity as a concept hierarchy, and reorganize relevant prior knowledge from multilevel and multiview as a multidimensional data model.

The formal use of concept hierarchies as the most important background knowledge in data mining was introduced by Han, Cai and Cercone [18]. And a multidimensional data model can be regarded as a combination of multiple concept hierarchies. So it is rational to represents prior knowledge as concept hierarchies or multidimensional data model.

Concept hierarchy. Concepts are the basic unit of human thoughts and play a central role in our understanding of the world. Human usually has a rich clustering of concepts for knowledge from his familiar field, in which each concept is related to many other concepts, and the relationships between concepts are clearly understood. Concepts are arranged hierarchically using umbrella concepts to more tightly relate them. The concept hierarchy is such an example.

In what follows, an organization of prior knowledge from one point of view with multiple levels of granularity is proposed based on concept hierarchy. In order to satisfy the need of nested and hierarchical structure, we will organize relevant prior knowledge as a concept hierarchy in the form of tree in this paper.

A concept hierarchy is a graph whose nodes represent concepts and whose arcs represent partial order relation between these concepts. In a concept hierarchy, the meaning of a concept is built on a small number of simpler concepts, which in turn is defined at a lower level using other concepts.

Concept hierarchies are used to express knowledge in concise and high-level terms. As P.Witold [17] pointed out that granular computing is an information processing pyramid, concept hierarchy tree just has the form of pyramid, where more nodes at lower level and they are all specific concepts, less nodes at higher level and they are abstract concepts.

In [1], the authors pointed out that the granulation process transforms the semantics of the granulated entities. At the lowest level in a concept hierarchy, basic concepts are feature values available from a data set. At a higher level, a more complex concept is synthesized from lower level concepts (layered learning for concept synthesis).

In [19], the authors stated "Concepts are not isolated into the human cognitive system. They are immersed into a hierarchical structure that facilitates, among others, classification tasks to the human cognitive system. This conceptual hierarchy expresses a binary relationship of inclusion defined by the following criterions:

Inclusion criterion : Each hierarchy node determines a domain included into domain of its father node. Each hierarchy node determines a domain that includes every domain of its son nodes.

Generalisation-Specialisation criterion: Every node in the hierarchy, has differentiating properties that make it different from its father node, if it exists, and from the others son nodes of its father, if they exist. "

Concept hierarchies may be defined by discretizing or grouping data, that is, discretizing numerical data into interval, and grouping categorical data into a generalized abstract concept. A total or partial order can be defined among groups of data.

For example, we can evaluate the ability of a person from multiview, such as 'education', 'vocation', 'income', etc. And, for each view, e.g., 'education', we can regard it from multilevel by relying on our relevant prior knowledge. For example, prior knowledge about 'education' can be organized as the following concept hierarchy.

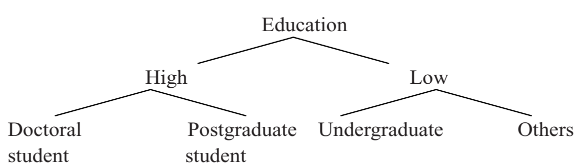

Fig. 1. Concept hierarchy of 'Education'

The concept hierarchy illustrated as figure 1 possesses a nested and hierarchical structure, where every node represent a concept, and arcs represent partial order relation between these concepts. Nodes at lower level are represent specific concepts, and those at higher level are represent abstract concepts.

Different people may own different prior knowledge, and different people may also have different preferences. Thus, for a given problem, different people will construct different concept hierarchy. To illustrate this, we still take 'education' for example. If you are a manager of some college and university, or a manager of a science institute, your prior knowledge relevant to 'education' may be structured as figure 1. But in common people's opinion, 'undergraduate' should also belong to high education. Thus, prior knowledge is relevant to the context of a problem.

A concept hierarchy can only organizes relevant prior knowledge from one particular angle or point of view with multiple levels of granularity, but it can not organizes those from multiview.

In the next section, we will borrow the concept of multidimensional data model from data warehousing, and provide an organization of prior knowledge. This organization can represent relevant prior knowledge from multilevel and multiview.

Multidimensional data model. In 1969, Collins and Quillian [15] made a typical experiment to prove prior knowledge that stored in long-term memory are in network architecture. This experiment suggests that people organize knowledge structurally and stored the features of the concept in different levels of the hierarchical architecture.

How can we represent this network architecture intuitively? In this subsection, we will organize multiple concept hierarchies as an organic whole, which is represented by a multidimensional data model. This organization can not only make relevant prior knowledge more easily understandable, but also represents them from multiview and multilevel intuitively.

Multidimensional data model [20,21] is a variation of the relational model that uses multidimensional structures to organize data and express the relationships among data, in which the data is presented as a data cube, which is a lattice of cuboid. A multidimensional data model includes a number of dimensions that each includes multiple levels of abstraction defined by concept hierarchy. Thus, a multidimensional data model can be treated as a combination of multiple concept hierarchies, and it can represent data from multiview and multilevel. This organization provides users with the flexibility to view data from different perspective. Based on the hierarchical structure of multidimensional data model, it is possible to "scan" a data table from different levels of abstraction and different dimensions.

In a multidimensional data model, each dimension can be represented by a concept hierarchy, which can represents a problem from a particular angle. Concept hierarchies of multiple dimensions be organized as a multidimensional data model, which can represents data from multiview and multilevel. In fact, multidimensional data model itself is a well-organized network structure, it can reflect relevant prior knowledge intuitively and completely.

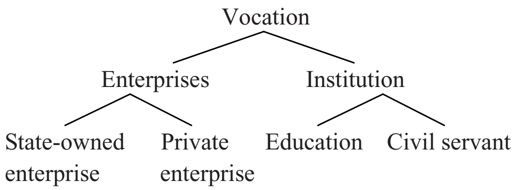

Fig. 2. Concept hierarchy of 'Vocation'

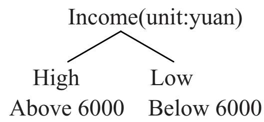

Fig. 3. Concept hierarchy of 'Income'

Next we will illustrate the ability of multidimensional data model represents prior knowledge from multiview and multilevel through an example. As the above mentioned, we can evaluate the ability of a person from multiview, such as 'education', 'vocation', 'income', etc. And for each view, we can regard it from multilevel by relying on our relevant prior knowledge.

We assume that prior knowledge about 'vocation' and 'income' be structured as concept hierarchies illustrated as figure 2 and figure 3, respectively. Then we will organize prior knowledge about 'education', 'vocation', and 'income' as a multidimensional data model, which can used to evaluate a person from multiview and multilevel.

For simplicity, we will denote those concepts in figure 1, figure 2 and figure 3 by some symbols. Such as denote 'education' by 'A', 'vocation' by 'B' and 'income' by 'C'. The correspondence between concepts and symbols is as follows.

'Education' $\leftrightarrow$ 'A'
'High education' $\leftrightarrow$ 'A1' 'Low education' $\leftrightarrow$ 'A2'
'Doctoral student' $\leftrightarrow$ 'A11' 'Postgraduate student' $\leftrightarrow$ 'A12'
'Undergraduate' $\leftrightarrow$ 'A21' 'Others' $\leftrightarrow$ 'A22'

where 'A11', 'A12', 'A21', 'A22' are specific concepts, and 'A1' is abstracted from 'A11' and 'A12', 'A2' is abstracted from 'A21' and 'A22'.

'Vocation' $\leftrightarrow$ 'B'
'Enterprises' $\leftrightarrow$ 'B1' 'Institution' $\leftrightarrow$ 'B2'
'State-owned enterprise' $\leftrightarrow$ 'B11' 'Private enterprise' $\leftrightarrow$ 'B12'
'Education' $\leftrightarrow$ 'B21' 'Civil servant' $\leftrightarrow$ 'B22'

Similarly, 'B11', 'B12', 'B21', 'B22' are specific concepts, and 'B1' is abstracted from 'B11' and 'B12', 'B2' is abstracted from 'B21' and 'B22'.

'Income' $\leftrightarrow$ 'C'
'High income' $\leftrightarrow$ 'C1' 'Low income' $\leftrightarrow$ 'C2'

The multidimensional data model is organized from the above three concept hierarchies as figure 4, which can be used to evaluate a person from multiview (education, vocation and income) and multilevel. For example, the shadow cell in the data cube illustrated as figure 4 represents persons with high education, work as a civil servant and have a high income. Each concept hierarchy corresponds a dimension in figure 4.

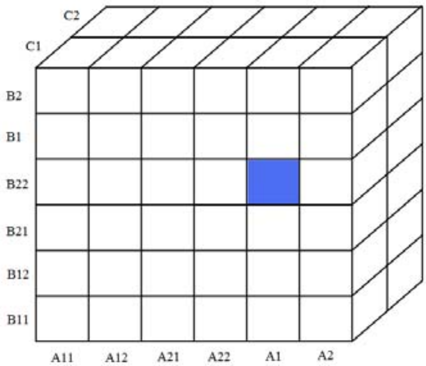

Fig. 4. Multidimensional data model

3 Granular Structure

In this section, we will give a brief introduction to granular structure, and present a representation of granular structure from multiview and multilevel. This representation can be used to represent information from multi-granularity in granular computing.

3.1 What Is Granular Structure

A central notion of granular computing is multilevel granular structures, which consists of inter-connected and inter-acting granules, families of granules interpreted as levels of differing granularity, and partially ordered multiple levels known as hierarchical structures. They are the results of a structured understanding, interpretation, representation, and description of a real-world problem or system. Granular structures provide structured descriptions of a system or a problem under consideration.

Granular computing emphasizes on structures. The study of granular computing depends crucially on granular structures that represent reality through multilevel and multiview. It seems that hierarchical granular structure is a good candidate for developing a theory of granular computing.

3.2 Representation of Granular Structure

In granular computing, we usually represent a problem from one particular angle or point-of-view with multiple levels of granularity by a hierarchy. However, the conceptualization of a problem through multiple hierarchies (i.e., multiview) and multilevel in each hierarchy is general and flexible. A complete understanding of a problem requires a series of granular structures that should reflect multiple views with multiple levels [16,22]. Thus, granular structures need to be modeled as multiple hierarchies and multiple levels in each hierarchy. That is, granular structures should reflect multiview and multilevel in each view. But there has no effective model to represent granular structure in existing literature.

We notice that the structure of multidimensional data model is consistent with the needs of granular structure. A multidimensional data model includes multiple dimensions, and each dimension includes multiple levels of abstraction. So each dimension in a multidimensional data model corresponds to a view with multiple levels of granularity in granular structures. Thus, we will represent a problem from multiview and multilevel by a multidimensional data model, which use multidimensional structure to organize data and express the relationships among data. This representation will facilitate the process of problem solving. Moreover, concept hierarchy and multidimensional data model can not only to elicit the content of knowledge, but also its structure.

Granular structures provide descriptions of a system or a problem under consideration. Yao [23] pointed out that granular structures may be accurately described as a multilevel view given by a single hierarchy and a multiview understanding given by many hierarchies. But he didn't mention how to organize these multiple hierarchies as an organic whole.

In what follows, we will present multidimensional data model representation of granular structures through an example.

Example 1. Representing granular structures of the following problem by a multidimensional data model, where the problem is provided by table 1.

This problem is provided by a table, where every column represents a particular view of the problem. So every column can be represented by a concept hierarchy, as shown in figure 1, figure 2, and figure 3. So this problem can be represented as a multidimensional data model by organizing these concept hierarchies as an organic whole, as shown in figure 5. The objects in cells of data cube as in figure 5 are satisfy properties determined by corresponding coordinate. Or we can say that the objects in cells of data cube is the extension of a concept, and the intension of the concept is designated by coordinate of the corresponding cell.

We can obtain the multidimensional data model representation of table 1 as figure 6 by combining table 1 and granular structure illustrated as figure 5. Or in other words, we can obtain multidimensional data model as figure 6 by loading data in table 1 to granular structure as figure 5. For example, the object $4, 5, 10$ in data cube possesses the properties 'A2', 'B1' and 'C2' simultaneously, that is, these objects possess properties as 'Low education', work in 'Enterprises' and 'Low income'.

For a given table, we can generalize its every attribute to a concept hierarchy tree, and organize these concept hierarchy trees as a multidimensional data model. In essence, the multidimensional structure of this multidimensional data model is the granular structure hid in the given table. Thus a given table can be generalized to multiple tables with different degrees of abstraction by combining this table and granular structures hid in it. These tables with different degree of abstraction is the basis of structured problem solving and structured information processing.

Table 1. Training dataset

U	Education	Vocation	Income(unit:yuan)
1	Doctoral student	Private enterprise	High
2	Postgraduate student	State-owned enterprise	High
3	Others	Education	Low
4	Undergraduate	Private enterprise	Low
5	Undergraduate	State-owned enterprise	Low
6	Postgraduate student	State-owned enterprise	Low
7	Undergraduate	State-owned enterprise	High
8	Undergraduate	Civil servant	Low
9	Doctoral student	Education	Low
10	Others	State-owned enterprise	Low

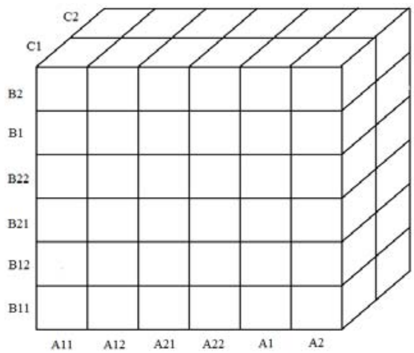

Fig. 5. Granular structure of table 1

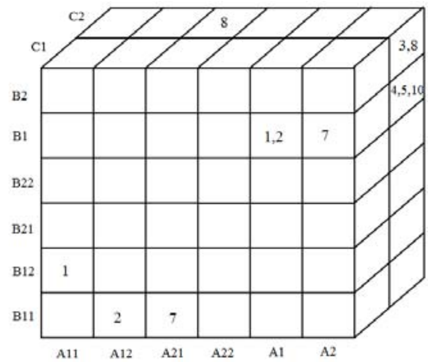

Fig. 6. Multidimensional data model of table 1

4 Relation between Structured Prior Knowledge and Granular Structure

Every one may be possess much knowledge, which usually be organized as a complicated network architecture in human brain. But prior knowledge is always related to a problem. When we solve a problem, we only need to consider prior knowledge relevant to it. Prior knowledge relevant to the solved problem will be extracted from the complicated network structure, and be reorganized as a nested and hierarchical structure, we call it structured prior knowledge, which will helpful to human problem solving.

Granular structure is a central notion of granular computing, it can represents a problem from multilevel and multiview. Granular structure can make implicit knowledge explicit, make invisible knowledge visible, make domain-specific knowledge domain-independent and make subconscious effects conscious [7]. In essence, granular structure is a structure hid in the solved problem.

As stated above, we can see that prior knowledge and granular structure are related together tightly by the solved problem. There have many points of similarity between prior knowledge and granular structure. For instance, they have a similar structure, that is, they all can be modeled as a nested and hierarchical structure, and each hierarchical structure also includes multiple levels of abstraction. They also have a similar ability of representation, that is, they all can represent a problem from one view with multiple levels of abstraction or from multiview and multilevel. In fact, Granular structures is an intuitive reflection of structured prior knowledge relevant to the solved problem, and the construction of granular structures is need the guidance of relevant prior knowledge.

5 Conclusion

In this paper, a nested and hierarchical organization of prior knowledge based on multidimensional data model is proposed, it is the basis of structured thinking. A representation of granular structures based on multidimensional data model is also proposed, it can represents a problem from multiview and multilevel. Finally, the relation between structured prior knowledge and granular structure is analyzed.

From the discussion, we conclude that the reason of human intelligence can solve problem at different levels of granularities is that human can not only use hierarchically organized relevant prior knowledge subconsciously, but also use structured prior knowledge to reorganize the solved problem as a good representation (granular structure) in problem solving.

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