Analyzing Skill Sets With or-Relation Tables in Knowledge Spaces

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Abstract

The disjunctive model of skill map in knowledge spaces can be interpreted based on an or-binary relation table between skills and questions. There may exist skills that are the union of other skills. Omitting these skills will not change the knowledge structure. Finding a minimal skill set may be formulated similar to the problem of attribute reduction in rough set theory, where an and-binary relation table is used. In this paper, an or-relation skill-question table is considered for a disjunctive model of knowledge spaces. A minimal skill set is defined and an algorithm for finding the minimal skill set is proposed. An example is used to illustrate the basic idea.

1. Introduction

Cognitive science [16][18] is an interdisciplinary study related to psychology and artificial intelligence. Its scope covers a wide range of topics, including knowledge acquisition and representation. In fact, a main task of cognition is knowledge acquisition [1].

Cognitive informatics, initiated by Wang and his colleagues [22-28], is born from the marriage of cognitive and information sciences. It investigates into the internal information processing mechanisms and processes of the natural intelligence (i.e., human brains and minds) and artificial intelligence (i.e., machines). An important topic in cognitive informatics is the study of knowledge.

The theory of knowledge spaces [2-5] represents a new paradigm in mathematical psychology for knowledge assessment. Students' knowledge states are represented and assessed systematically by using a finite set of questions. A collection of subsets of questions is called a knowledge space in which each subset is called a knowledge state or a cognitive state. The family of knowledge states may be determined by the dependency of questions or the mastery of different sets of questions by a group of students. To be consistent with the traditional explanatory features of psychometric theory, the notion of skills may be used to interpret students' knowledge states. Specifically, a question can be described by several skills.

In many situations, students' knowledge states can be inferred from data sets that summarize students' ability to answer certain questions. Binary relation tables are used as a common tool to describe the data sets. For such a table, it is assumed that rows denote objects and columns denote attributes of the objects. When the dimensionality of the data set is huge, it may be difficult to find rules and discover useful knowledge from it. One needs to reduce the table, if possible, without loss of the valuable information. A successful method to deal with such a problem is attribution reduction in the rough set theory.

Rough set theory [12], proposed by Pawlak in 1982, is useful for discovering knowledge hidden in a data set. One of the significant contributions of rough set theory is attribute reduction. Typically, there may exist some attributes that do not provide additional information and these redundant attributes can be removed. The notion of a reduct is a minimal subset of the attributes that provides the same information as the

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entire set of attributes [12]. Through the attribute reduction, we can work with a reduced information table in order to find rules efficiently. Attribute reduction may be viewed as a granular computing approach. It reduces the redundant attributes in the table while keeping the granularity of knowledge.

The relationship among the attributes in an information table or a decision table is traditionally interpreted as an and-relation. That is, an object must satisfy the values of *all* attributes at the same time. Rough set researchers focus on this type of and-relation tables [6-11][17][19-21][29-30][32-40]. In this paper, we consider an alternative interpretation of binary relation tables. The relationship among the attributes is interpreted as an or-relation. That is, any one of the attributes is sufficient to describe an object. This kind of binary relation tables is useful in knowledge spaces.

There are two kinds of skill maps in knowledge spaces. One is the disjunctive model, in which a question can be answered if any one of a set of skills is mastered. For instance, if one wants to calculate the value of the expression, $2+2+2+2+2=2\times 5$, one only needs to master of the skills in the set $\{$*addition*, *multiplication*$\}$. The other model is the conjunctive model, in which a question can be answered by mastery of all skills in a set of skills. For example, one can only correctly calculate the value of $2\times 5-1$ if one to masters both skills in the set $\{$*multiplication*, *subtraction*$\}$. Skills in the disjunctive model are modeled by an or-relation, and skills in the conjunctive model are modeled by an and-relation. This paper focuses on discussing the or-relation tables.

We can set up a skill-question table where rows are questions and columns are skills. That is, questions are considered as objects and skills are attributes. If solving a question needs to master some skills, we denote the corresponding skill value as 1 and 0 otherwise. From such a skill-question table, the dependencies of questions can be obtained. In the disjunctive model, for each skill, we can get a question set, i.e., a knowledge state. For an arbitrary combination of skills, some knowledge states are obtained. All knowledge states, together with the empty set and the entire question set, form a knowledge structure. It has been proved that such a knowledge structure is closed under set union, which is named as a knowledge space [3].

In the disjunctive model, it may happen that one skill is the union of other skills in the table. That is, the skill is redundant and removing it will not cause any changes in the derived knowledge structure. A large number of such superfluous skills will lower the efficiency of building, testing and searching for a knowledge space and waste the storage space. It is therefore useful to find a minimal skill set in a skill-question table. We adopt some notions from the rough set theory in designing such an algorithm for finding a minimal skill set that has the same power for constructing a knowledge space.

Knowledge spaces [2-5] and rough sets [12-15] are two related theories. One can adopt ideas from one in attempt to enrich the other, as well as combining ideas from both theories. Rough set approximations have been introduced into knowledge spaces in our previous work [31]. In this paper, we adopt the ideas of attribute reduction from rough set theory to knowledge space. This not only leads to more insights into attribute reduction, but also brings us closer to a common framework for studying the two related theories.

Knowledge representation and processing is important to cognitive computing and cognitive informatics. The study of knowledge spaces is therefore relevant to cognitive informatics. The rest of paper is organized as follows. Section 2 presents some relevant background about knowledge spaces. Section 3 gives an algorithm for finding a minimal skill set. Section 4 summarizes the main results of this study.

2. Overview of Knowledge Spaces

This section presents an overview of knowledge spaces by introducing its basic concepts and notions [2-5].

2.1. Knowledge spaces

The theory of knowledge spaces was proposed and studied by Doignon and Falmagne [2-5] in 1985. It starts out with rather simple psychological assumptions on assessing students' knowledge based on their ability to answer questions. Knowledge spaces may be viewed as a theory of information presentation and information use. The main objective of knowledge spaces is to effectively and economically solve the problem of knowledge assessment. Many researchers have contributed to the theory and several computerized procedures have been implemented [2-5].

In knowledge spaces, one uses a finite set of universe (i.e., a set of questions denoted by Q) and a collection of subsets of the universe (i.e., a knowledge structure denoted by $\mathcal{K}$), where $\mathcal{K}$ contains at least the empty set $\varnothing$ and the whole set Q. The members of

$\mathcal{K}$ are called the knowledge states or cognitive states that are subsets of questions given by experts or correctly answered by students.

There are two types of knowledge structures. One is the knowledge structure associated to a surmise relation, closed under set union and intersection. The other is the knowledge structure associated to a surmise system, closed only under set union. From the view of granular computing, a knowledge state can be referred to as a granule. The knowledge structure may be viewed as a granular structure.

A surmise relation on the set Q of questions is a transitive and reflexive relation S on Q. By aSb, we can surmise that the mastery of a if a student can answer correctly question b. This relation imposes certain conditions on the corresponding knowledge structure. For example, surmising the mastery of question a if a student can answer correctly question b means that if a knowledge state contains b, it must also contain a. The knowledge structure associated to a surmise relation can be formally defined [2]:

Definition 1. For a surmise relation S on the (finite) set Q of questions, the associated knowledge structure $\mathcal{K}$ is defined by:

$$\mathcal{K} = \{\mathbf{K} | (\forall q, q' \in Q, qSq', q' \in \mathbf{K}) \Rightarrow q \in \mathbf{K}\}, \tag{1}$$

where $\mathbf{K}$ is a knowledge state.

With surmise relations, a question can only have one prerequisite. Sometimes, this may be too restrictive. In practice, we may assume that a knowledge structure is closed only under union, called a knowledge space. A knowledge space is a weakened knowledge structure associated to a surmise relation.

We can define a knowledge space with a surmise system. A surmise system on a (finite) set Q is a mapping σ that associates to any element q in Q. A nonempty collection $\sigma(q)$ of the subsets of Q satisfies the following three conditions:

1) $C \in \sigma(q) \Rightarrow q \in C$;

2) $(C \in \sigma(q), q' \in C) \Rightarrow (\exists C' \in \sigma(q'), C' \subseteq C)$;

3) $C \in \sigma(q) \Rightarrow (\forall C' \in \sigma(q), C' \not\subset C)$.

The subsets C, C' in $\sigma(q)$ are the clauses for question q. The corresponding knowledge structure has the following definition.

Definition 2. For a surmise system (Q, σ), the associated knowledge structure is defined by:

$$\mathcal{K} = \{\mathbf{K} | (\forall q \in Q, q \in \mathbf{K}) \Rightarrow (C \in \sigma(q), C \in \mathbf{K})\}. \tag{2}$$

They constitute the knowledge structure associated to (Q, σ). In fact, there is a one-to-one correspondence between surmise systems on Q and knowledge spaces on Q.

There are several approaches to obtain a surmise relation or a surmise system. From the existing research, we can identify at least the following methods:

1. Querying experts;
2. A posteriori analysis of mass data;
3. Analysis of didactics and curricula;
4. Systematical problem construction;
5. Analysis of skills, demands, competence (latent structures).

A preferable method may be the analysis of skills and competence. Many researchers concentrate on this kind of sub-theory. We assume that the surmise relation or the surmise system is obtained by the skills.

2.2. Skill-question tables in knowledge spaces

Given a set of questions Q, the more skills one has, the more questions one is capable of solving. This simple idea is the basis for studying the connection between skills and questions.

Given a set S of skills, for each subset Y of S, there exists a set of questions that can be solved by the skills in Y. With such a connection, it is straightforward to compute all possible knowledge states.

To establish the relation between skills and questions more intuitively, we can use a binary relation table. The rows represent the questions and the columns represent the skills. Mastery of a question needs one to have some skills; we set 1 as the values in the corresponding rows and columns. Otherwise, the values are assumed to be 0. Table 1 is an example of a binary relation table.

With respect to the different meanings of skills, there are two models. A question may be solved if any one skill is mastered. That is, the relationship between skills is an or-relation. Mastery of any one skill is capable of solving the question. It is called the disjunctive model. Alternatively, solving a question needs the mastery of all the skills in a subset of skills. It is called the conjunctive model. In the conjunctive model, the relationship between skills is an and-relation that is the same as an information table in the rough set theory.

This paper uses the idea of attribute reduction in rough set theory. We mainly discusses how to find a minimal skill set to save the storage space and increase of efficiency of knowledge assessment, while keeping the corresponding knowledge structure in an or-relation table.

Table 1. A Skill-Question Table

	s_1	s_2	s_3	s_4	s_5
a	1	1	0	1	0
b	1	1	1	0	0
c	0	1	1	0	0
d	1	1	0	0	1
e	0	0	0	0	1

3. The Minimal Skill Set in the Disjunctive Model in Knowledge Spaces

The notion of a minimal skill set is introduced and an algorithm for finding such a set is given.

3.1. Minimal skill set

Let Q be a nonempty set of questions and S be a nonempty set of skills. Suppose τ is a skill mapping from Q to $2^s \setminus \{\varnothing\}$. For any q in Q, the subset $\tau(q)$ of S is referred to as the set of skills assigned to q (by the skill map τ).

Suppose Y is a subset of S. We say that $\mathbf{K} \subseteq Q$ is the knowledge state delineated by Y via the disjunctive model if

$$\mathbf{K} = \{q \in Q | \tau(q) \cap Y \neq \varnothing\}. \quad (3)$$

From the Table 1, the knowledge state delineated by $Y = \{s_1, s_5\}$ is $\{a,b,d,e\}$. The subset $\{b,d\}$ is not a knowledge state, since it cannot be delineated by any subset Y of S. The knowledge space delineated for Table 1 is given by:

$$\mathcal{K} = \{\varnothing, \{a\}, \{b,c\}, \{d,e\}, \{a,b,d\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}, \{a,b,c,d\}, \{a,b,d,e\}, \{a,b,c,d,e\}\}. \quad (4)$$

In the disjunctive model, a skill may be composed of other skills. In a skill-question table, it is shown that the question subset delineated by a skill can be the union of other question subsets delineated by other skills. Using Table 1, s_2 consists of s_1 and s_3, since the knowledge state delineated by s_2 equals to the union of the knowledge states delineated respectively by s_1 and s_3. That is, s_2 can be represented by s_1 and s_3. We only need to leave one skill subset $\{s_1, s_3, s_4, s_5\}$. We say that s_2 is redundant in computing a knowledge space, because the knowledge state delineated by s_2 is the same as the knowledge state delineated by $\{s_1, s_3\}$. It can be easily shown that the knowledge space delineated by $\{s_1, s_3, s_4, s_5\}$ is the same as the knowledge space delineated by the original skill-question table, but not the same as that delineated by $\{s_2, s_4, s_5\}$.

Redundant skills limit the effective and economic assessment of knowledge and waste memory. For the efficient processes, we need to find a minimal skill. First, we state the properties that must be satisfied by a ***minimal skill set*** in a skill-question table.

Definition 3. A minimal skill set of a skill-question table should satisfy two conditions:

a) The knowledge space delineated by the decreased skill-question table should be the same with that delineated by the initial table;

b) The knowledge state delineated by a single skill cannot be the union of the knowledge states delineated by other skills.

It follows that a ***minimal skill set*** of a skill-question table are the most compact skill-question table which can delineate the same knowledge space as the original table. We can also determine a skill whether it can be replaced by the other skills

Definition 4. A skill that can be removed must satisfy two conditions:

a) The knowledge space delineated by the skill-question table will not change after omitting the column of the skill;

b) The knowledge state delineated by the skill is the union of the knowledge states delineated by other skills.

3.2. Algorithm for finding a minimal skill set

From a skill-question table, we can obtain the question set for each skill. Suppose we have a set of skills $\mathbf{S} = \{s_1, s_2, ..., s_m\}$. Then all the questions that skill $s_i \left(1 \le i \le m\right)$ can solve constitute the question set $d(s_i)$. We denote D_{s_i} as the collection of the question sets for some skill contained in $d(s_i)$. Namely

$$D_{s_i} = \{d(s_k) | d(s_k) \subset d(s_i), 1 \le k \le m, k \ne i\}. \quad (5)$$

With the definition above, we can see that a skill can be removed if it satisfies $d(s_i) = \bigcup D_{s_i}$. A skill which can not be removed should satisfy $D_{s_i} = \{\varnothing\}$. We can define a subset of skills consisting of all the skills satisfying $D_{s_i} = \{\varnothing\}$ as the ***core*** skill of the skill set.

Definition 5. The core skill of a skill set is defined by:

$$CORE(\mathbf{S}) = \{s_i | D_{s_i} = \{\varnothing\}, 1 \le i \le m\}. \quad (6)$$

Base on these notations, we give an algorithm for finding a minimal skill set.

Algorithm : Finding a minimal skill set

STEP 1: For each skill, the corresponding question set is obtained. Namely, for the skill set $\mathbf{S} = \{s_1, s_2, ..., s_m\}$, find the question sets $\{d(s_1), d(s_2), ..., d(s_m)\}$;

STEP 2: Compute every skill's D_{s_i}, namely, $\{D_{s_1}, D_{s_2}, ..., D_{s_m}\}$;

STEP 3: For each skill $s_i \in \mathbf{S}$, remove it from $\mathbf{S}$ if $d(s_i) = \bigcup D_{s_i}$;

STEP 4: Output the rest skill subset $\mathbf{R} = \mathbf{S}$.

By using the algorithm, we can remove the redundant skills from a skill-question table of the disjunctive model. Compared to the attribute reduction in the rough set theory, finding a minimal skill set can also be regarded as reducing the redundant information while keeping the knowledge structure. From the view of granular computing, it maintains the granularity of the knowledge space.

3.3. An example

Based on Table 1, the skill set $\mathbf{S} = \{s_1, s_2, ..., s_5\}$, and the family of the associated question sets is $\{d(s_i) \mid i = 1, 2, ..., 5\} = \{\{a,b,d\}, \{a,b,c,d\}, \{b,c\}, \{a\}, \{d,e\}\}$. The family of D_{s_i} for each skill is $\{D_{s_i} \mid i = 1, 2, ..., 5\} = \{\{d(s_4)\}, \{d(s_1), d(s_3), d(s_4)\}, \{\varnothing\}\}$. It is easy to show that $d(s_2) = \bigcup D_{s_2}$. Therefore, s_2 can be removed. The decreased skill-question table is shown in Table 2:

Table 2. A Decreased Skill-Question Table

	*s*1	*s*3	*s*4	*s*5
a	1	0	1	0
b	1	1	0	0
c	0	1	0	0
d	1	0	0	1
e	0	0	0	1

The knowledge space delineated by the decreased table 2 is given as follows:

$$\mathcal{K} = \{\varnothing, \{a\}, \{b,c\}, \{d,e\}, \{a,b,d\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}, \{a,b,c,d\}, \{a,b,d,e\}, \{a,b,c,d,e\}\}$$

which is the same with the knowledge space delineated by Table 1.

4. Conclusion

We present a new interpretation of a skill-question table under the disjunctive model in knowledge spaces. Similar to the rough set theory, skills can be viewed as the attributes and questions as objects. However, the relationship between attributes is no longer an and-relation but or-relation. That is, a question can be solved by mastery of only one skill. An algorithm for finding a minimal skill set in an or-relation table is proposed.

Or-relation tables are a new type of tables. A skill-question table in the disjunctive model can be considered as a special case in or-relation tables. This paper demonstrates some potential value on studying or-relation tables.

Approximations and reduction are two of the central topics in the rough set theory. We introduce them into knowledge spaces. The results from this study show a strong connection between the two related theories. As future work, we plan to investigate systematically the connections between rough set theory and knowledge spaces based on a common framework of granular computing.

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