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RESEARCH ARTICLE

Conflict Analysis Triggered by Three-Way Decision and Pythagorean Fuzzy Rough Set

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Abstract

Conflict is ubiquitous in human society and has a profound impact on various fields such as the economy, politics, law, and military. Many scholars have focused on exploring the internal mechanisms and potential solutions to conflicts. Notably, describing agents' attitudes is an effective way to construct a conflict model. However, in decision-making, agents' attitudes on issues are often vague and ambiguous. Pythagorean fuzzy set can deal with fuzzy information more accurately than intuitionistic fuzzy set. On the basis of this understanding, we investigate the conflicts from the perspective of Pythagorean fuzzy set. Firstly, we use Pythagorean fuzzy numbers to express the attitudes of agents on issues, and subsequently establish a Pythagorean fuzzy conflict information system. Secondly, we classify agents into three categories by a pair of thresholds to establish a trisected agent set model with risk preference. Thirdly, we construct a three-way conflict analysis model based on multi-granulation Pythagorean fuzzy decision-theoretic rough set and discuss both global and local conflicts by combining conflict analysis with multi-granulation decision-theoretic rough set. Finally, we discuss the relationships and properties of the proposed conflict analysis models.

Keywords Pythagorean fuzzy set · Intuitionistic fuzzy set · Pythagorean fuzzy numbers · Multi-granulation Pythagorean fuzzy decision-theoretic rough set

1 Introduction

Conflicts are prevalent in human society, ranging from disputes over personal interests to their impact on various aspects such as the economy, politics, and military of a country. Many scholars have dedicated their attention to understanding the mechanisms of conflicts, describing and expressing conflicts, as well as exploring ways to avoid them [1–3]. Pawlak introduced a conflict analysis model that utilized values of 1, 0, and -1 to represent an agent's attitudes of support, neutrality, and opposition to issues, respectively. This model has become an important theoretical framework for conflict analysis [4–6]. Deja [7] extended Pawlak's model and identified three fundamental questions for conflict analysis: (1) What are the underlying causes of conflict? (2) How can a workable consensus strategy be identified? (3) Is there a solution that satisfies all agents involved? Skowron et al. [8] presented a requirements determination model based on rough sets, which utilizes a conflict relation to effectively represent agreements or disagreements among agents. Sun and Ma [9] introduced an approach for addressing the problem of multi-agent conflict analysis by leveraging the proposed multi-decision rough set approach. Lang et al. [10] developed a probability model of conflict analysis by employing a pair of thresholds. On the basis of their previous work [10], Lang et al. [11] employed Pythagorean fuzzy numbers to express the attitudes of agents and defined probability conflict set, probability neutrality set and probability alliance set based on Bayesian minimum risk theory. By using a formal concept analysis approach, Lang and Yao [12] investigated the relationships between agent coalitions and issue bundles. Yao [13] reformulated the Pawlak's model by dividing the agent set, the agent's relation set, the issue set,

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and the issue's relation set into trisections. Sun et al. [14] established a conflict decision-making information system based on probabilistic rough set over two universes, and proposed a decision-making model for complex conflicts. Basir et al. [15] proposed a conflict resolution model that utilizes game-theoretic rough sets. This model constructs a game involving all relevant agents, which produces more realistic and accurate results. Ali et al. [16] employed four multi-granulation rough sets with dominance relation and applied them to solve multi-agent conflict analysis decision problem. Zhi et al. [17] utilized an approximate three-way concept lattice to define alliance set, conflict set, and neutral set, respectively, and explored maximal coalitions and minimum conflict sets. Li et al. [18] constructed a triangular fuzzy information system, adopted relative areas to describe the concrete attitudes of agents. They also developed a tripartition agents model through a pair of thresholds. Li et al. [19] formulated a three-way conflict analysis and resolution model in q-rung orthopair fuzzy information system, and provided answers to Deja's three fundamental questions about Pawlak's conflict model. Wang et al. [20] drew inspiration from prospect theory and determined the allied set, neutral set, and conflict set by considering the agent's reference point. In order to clarify conflict semantics, Luo et al. [21] trisected the agent set and issue set by separating the opposite aspects within an auxiliary function, and created a pair of alliance and conflict functions. Combining qualitative and quantitative evaluations, Lang et al. [22] developed a comprehensive model that unifies existing models through rough set and formal concept analysis.

Three-way decision [23], proposed by Yao, describes decision-makers' decision behaviors towards uncertain things.

Due to its ability to effectively explain decision-making under uncertain conditions, three-way decision has attracted a lot of attention of many scholars [24–26]. Zhan et al. [27] incorporated three-way decision into multi-attribute decision-making by employing an outranking relation. They proposed three strategies to design a novel three-way decision model specifically tailored for multi-attribute decision-making. Yang and Yao [28] investigated two potential solutions to the problem of constructing a shadowed set from an intuitionistic fuzzy set by combining three-way decision and shadowed set. Zhang et al. [29] conducted an in-depth exploration of loss functions and developed a Pythagorean fuzzy three-way decision model by incorporating a hesitation description into the Pythagorean fuzzy environment. Zhang and Ma [30] presented three-way decision with decision-theoretic rough set based on Pythagorean fuzzy covering.

In 1965, Zadeh [31] introduced fuzzy set to describe fuzzy phenomenon, and explained the meaning of fuzziness through the definitions and operations of membership degree and membership function. Based on fuzzy set, Zadeh [32] proposed the concept of fuzzy information granularity in 1979. Yao [33] specifically established a granulation computing model in neighborhood system. Qian et al. [34] developed a multi-granulation rough set model and subsequently constructed a multi-granulation decision-theoretic rough set model by using Bayesian risk decision theory [35]. Based on the Zadeh's work, Atanassov [36] developed the concept of intuitionistic fuzzy set and investigated its properties. Later, Yager and Abbasov [37] proposed Pythagorean fuzzy set with the assumption that the sum of the squares of membership and non-membership degrees does not exceed 1. Pythagorean fuzzy set can deal with fuzzy information and fuzzy concepts more accurately than intuitionistic fuzzy set. Based on Pythagorean Fuzzy Bonferroni mean with weighted interaction operator, Yang et al. [38] adopted a decision-making method for addressing the aggregation problem of online multi-attribute interactive ratings. In order to realize the sustainable development of shared e-bikes, Tang and Yang [39] utilized Pythagorean fuzzy decision-making method to help recycling suppliers to make reasonable selection. By employing the concepts of variance and covariance, Ejegwa et al. [40] proposed a three-way approach for computing the correlation coefficient between Pythagorean fuzzy sets. According to the semantics of three-way decision, Zhao et al. [41] established a three-way decision model on Pythagorean fuzzy set based on the dominant relationship of Pythagorean fuzzy set.

In real-life scenarios, agents often encounter some complex decision-making environments that make their attitudes less straightforward. This means that agents' attitudes towards issues are vague and uncertain. Since Pythagorean fuzzy set is more effective than intuitionistic fuzzy set in describing such vague and uncertain information, and there are many Pythagorean fuzzy information systems for conflicts in which attitudes of agents on issues are depicted by Pythagorean fuzzy numbers [11, 42]. These research findings serve as an important reference for us to study the three-way conflict analysis model. Furthermore, multi-granulation conflict is common in life. For example, let's consider a conflict between a company and its unionized employees. At the micro-level, individual employees may be dissatisfied with their wages and working conditions. This might lead to strikes or protests. At the meso-level, the union leaders are negotiating with the company's management to reach a collective bargaining agreement. These negotiations may involve disagreements on issues such as salary increases, benefits, or working hours. At the macro-level, the conflict may be influenced by broader societal factors such as economic conditions, government policies, or cultural norms. Despite the development of various conflict analysis models, the current models have not been investigated from a multi-granulation perspective for practical conflict problems. Particularly, in specific conflict scenarios like military conflicts, agents exercise caution in decision-making. They tend to employ multiple evaluation functions to assess decision outcomes and prevent errors. Since the multi-granulation rough set is approximation of a family of equivalence relations, it can accurately capture the cautious psychology of agents in decision-making. Therefore, researching the integration of multi-granulation rough set and conflict analysis presents an interesting topic worth exploring. Consequently, an important aspect of this paper is the construction of a multi-granulation conflict analysis model to describe conflicts within multi-dimensional data.

The rest of this paper is organized in the following: Sect. 2 provides an overview of Pythagorean fuzzy set, conflict analysis and multi-granulation decision-theoretic rough set. In Sect. 3, we present a Pythagorean fuzzy three-way conflict analysis model which trisects agents set, and establish a trisected agents set model with different risk preference. Section 4 focuses on establishing the tripartite classification of agents based on multi-granulation Pythagorean fuzzy rough approximations and discusses detailly the relations and properties of the proposed conflict analysis models. In Sect. 5, we explore methods for measuring the precision of the proposed rough set model. Finally, Sect. 6 concludes the paper and outlines potential future work.

2 Preliminaries

We will briefly review some necessary concepts about Pythagorean fuzzy set, conflict analysis and multi-granulation rough approximations in this section.

2.1 Pythagorean Fuzzy Set

Pythagorean fuzzy set is utilized to reveal the inherent fuzziness of things. Accordingly, this subsection reviews its definition and some basic properties.

Definition 1 [43]. Let U be the universe of discourse. A Pythagorean fuzzy set (PFS) on U is defined in the following:

$$P = \left\{x, u_P(x), v_P(x) | x \in U\right\} \tag{1}$$

where the functions $u_P(x), v_P(x) : U \to [0, 1]$ are the membership degree and non-membership degree of x to P, respectively. For any $x \in U$, we have $0 \le u_P^2(x) + v_P^2(x) \le 1$, and the hesitant membership of x to P is given as follows:

$$\pi_P(x) = \sqrt{1 - u_P^2(x) - v_P^2(x)} \tag{2}$$

$r(x) = (u_P(x), v_P(x))$ is called Pythagorean fuzzy number (PFN), written as $r = (u_P, v_P)$.

Definition 2 [43]. Assume $r = (u_P, v_P)$, $r_1 = (u_{P_1}, v_{P_1})$ and $r_2 = (u_{P_2}, v_{P_2})$ are PFNS. A quasi-ordering of Pythagorean fuzzy set is defined as follows:

$$r_1 \ge r_2 \text{ iff } u_{P_1} \ge u_{P_2} \text{ and } v_{P_1} \le v_{P_2} \tag{3}$$

In a nutshell, Pythagorean fuzzy set is more flexible in describing and characterizing the fuzziness of things.

2.2 Conflict Analysis

Conflict analysis, an important branch of management science, plays a significant role in our decision-making. Notably, trisecting the agent set is currently an important research area within conflict analysis, as it guides us in making sound decisions. This is also the primary focus of this article. Thus, it is necessary to review some basic definitions of conflict analysis.

Definition 3 [6]. Let $IS = (U, A, V, f)$ be an information system, where U is a non-empty finite set of agents, A is a non-empty finite set of issues, $V = \cup\left\{V_{c_j} | c_j \in A\right\}$, V_{c_j} is the set of values of issues c_j, f is the relationship between U and A, $f : U \times A \to \{-1, 0, 1\}$. For any $x \in U$, $a \in A$, the function f is defined as follows:

$$f(x, a) = \begin{cases} +1, & \textit{agent x support issue a} \\ 0, & \textit{agent x neutralize issue a} \\ -1, & \textit{agent x oppose issue a} \end{cases} \tag{4}$$

To depict clearly the relations between agents and issues, we illustrate it through Example 1 from Table 1.

Example 1 An information system of a conflict problem is shown in Table 1.

where the agents x_1, x_2, x_3, x_4, x_5 and x_6 represent Israel, Egypt, Palestine, Jordan, Syria, Saudi Arabia, respectively.

Table 1 Information system for the Middle East conflict [11]

	a_1	a_2	a_3	a_4	a_5
x_1	− 1	1	1	1	1
x_2	1	0	− 1	− 1	− 1
x_3	1	− 1	− 1	− 1	0
x_4	0	− 1	− 1	0	− 1
x_5	1	− 1	− 1	− 1	− 1
x_6	0	1	− 1	0	1

The issues a_1, a_2, a_3, a_4 and a_5 refer to Autonomous Palestinian state on the West Bank and Gaza, Israeli military outpost along Jordan River, Israeli retains East Jerusalem, Israeli military outposts on the Golan Heights, Arab countries grant citizenship to Palestinians who choose to remain within their borders, respectively [11]. Table 1 reflects the attitudes of agents on issues, e.g. $f(x_3, u_2) = -1$ shows that the agent x_3 oppose the issue u_2, namely, Palestine objects to the presence of Israeli military outposts along the Jordan River.

Definition 4 [6]. Let $IS=(U, A, V, f)$ be an information system, where U is a non-empty finite set of agents, A is a non-empty finite set of issues. For $\forall x, y \in U$ and $\forall c \in A$, the auxiliary function is defined in the following formal:

$$\phi_c(x,y) = \begin{cases} 1 f(x,c)\cdot f(y,c) = 1 \vee x = y \\ 0 f(x,c)\cdot f(y,c) = 0 \wedge x \neq y \\ -1 f(x,c)\cdot f(y,c) = -1 \end{cases} \tag{5}$$

where $f(x,c)$ and $f(y,c)$ denote the attitudes of x and y on c, respectively. When $\phi_c(x,y) = 1$, agent x and agent y have the same attitude to issue c or agent x and agent y are the same agent; when $\phi_c(x,y) = 0$, at least one of agent x and agent y is neutral about issue c; when $\phi_c(x,y) = -1$, agent x and agent y have opposing attitudes towards issue c.

In conflict analysis, the distance function proposed by Pawlak [6] calculates the distance between any two agents as follows.

Definition 5 [6]. Assume $IS=(U, A, V, f)$ is an information system. For $\forall c \in A$, the distance function ρ_A for $x, y \in U$ is defined as follows:

$$\rho_A(x,y) = \frac{\sum\limits_{c\in A} \phi_c^*(x,y)}{|A|} \tag{6}$$

where $\phi_c^*(x,y) = \frac{1-\phi_c(x,y)}{2} = \begin{cases} 0 \quad f(x,c)\cdot f(y,c) = 1 \vee x = y \\ 0.5 \quad f(x,c)\cdot f(y,c) = 0 \wedge x \neq y \\ 1 \quad f(x,c)\cdot f(y,c) = -1 \end{cases}$.

2.3 Multi-granulation Rough Approximations

It is well known that one of the important advantages of rough set is semantic interpretability. Decision-theoretic rough set is used to explain uncertain things from the perspective of probability, and Qian et al. [35] developed the multi-granulation decision-theoretic rough set model based on Bayesian risk decision theory. Multi-granulation decision-theoretic rough set can effectively handle the uncertainty of multi-dimensional uncertain data. Thus, this subsection introduces the key concepts of decision-theoretic rough set and multi-granulation decision-theoretic rough set.

Definition 6 [44]. Assume that (U,R) is an approximate space. The partition formed by the equivalence relation R on the universe U is given as (U,R). For any $X \subseteq U$, the lower approximation and the upper approximation of X are respectively defined by:

$$\begin{aligned} \underline{R}(X) &= \left\{x \in U | [x]_R \subseteq X\right\} \\ \overline{R}(X) &= \left\{x \in U | [x]_R \cap X \neq \varnothing\right\} \end{aligned} \tag{7}$$

where $[x]_R$ is the equivalence class of x under the equivalence relation R.

When $\underline{R}(X) = \overline{R}(X)$, X is called the exact set under equivalence relation. When $\underline{R}(X) \neq \overline{R}(X)$, X is called rough set under equivalence relation. The lower and upper approximations of X divide U into the following three disjoint regions.

$$\begin{aligned} &\text{Positive region of } X: POS(X) = \underline{R}(X) \\ &\text{Negative region of } X: NEG(X) = U - \overline{R}(X) \\ &\text{Boundary region of } X: BND(X) = \overline{R}(X) - \underline{R}(X) \end{aligned} \tag{8}$$

Definition 6 offers a reasonable semantic explanation for describing uncertain concepts.

Definition 7 [44]. Let $IS = (U, A, V, f)$ be an information system, where U is a non-empty finite set of objects, A is a non-empty finite set of attributes, and f is the relationship between U and A. According to Bayes risk decision theory, the state space $\Theta=\{X, \sim X\}$ is constructed to describe the state of objects belonging to the set X and not belonging to the set X, respectively. The set of actions is given by $A = \left\{a_P, a_B, a_N\right\}$, where a_P,a_B and a_N express the actions in classifying an object into $POS(X)$,$BND(X)$ and $NEG(X)$, respectively. The loss function values of the corresponding actions in different states are shown in Table 2.

where λ_{*P} denotes the decision loss of taking action a_* for classifying an object in X into the region specified by *, and λ_{*N} denotes the decision loss of taking action a_* for classifying an object that not belong to X into the region specified by *.

Table 2 Loss function values of different actions in different states

	X	$\sim X$
a_P	λ_{PP}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}

For a given object x, the expected loss of the decision action taken is written as $R(a_*|[x]_R)$, and expressed as follows:

$$\begin{aligned} R(a_P|[x]_R) &= \lambda_{PP}\Pr(X|[x]_R) + \lambda_{PN}\Pr(\sim X|[x]_R) \\ R(a_B|[x]_R) &= \lambda_{BP}\Pr(X|[x]_R) + \lambda_{BN}\Pr(\sim X|[x]_R) \\ R(a_N|[x]_R) &= \lambda_{NP}\Pr(X|[x]_R) + \lambda_{NN}\Pr(\sim X|[x]_R) \end{aligned} \tag{9}$$

where $\Pr\left(X|[x]_R\right)$ represents the conditional probability that the equivalence class $[x]_R$ of object x belongs to set X.

The minimum loss rules produced by Bayes risk decision-making process is as Rule 1.

Rule 1:

(P) If $R(a_P|[x]_R) \le R(a_B|[x]_R)$ and.

$R(a_P|[x]_R) \le R(a_N|[x]_R)$, then $x \in POS(X)$;

(B) If $R(a_B|[x]_R) \le R(a_N|[x]_R)$ and.

$R(a_B|[x]_R) \le R(a_P|[x]_R)$, then $x \in BND(X)$;

(N) If $R(a_N|[x]_R) \le R(a_P|[x]_R)$ and.

$R(a_N|[x]_R) \le R(a_B|[x]_R)$, then $x \in NEG(X)$.

The Rule 1 can also be expressed as Rule 2.

Rule 2:

(PP) If $\Pr(X|[x]_R) \ge \alpha$ and $\Pr(X|[x]_R) \ge \gamma$, then $x \in POS(X)$;

(PB) If $\Pr(X|[x]_R) < \alpha$ and $\Pr(X|[x]_R) > \beta$, then $x \in BND(X)$;

(PN) If $\Pr(X|[x]_R) \le \beta$ and $\Pr(X|[x]_R) \le \gamma$, then $x \in NEG(X)$.

where

$$\begin{aligned} \alpha &= \frac{\lambda_{PN} - \lambda_{BN}}{(\lambda_{PN} - \lambda_{BN}) + (\lambda_{BP} - \lambda_{PP})} \\ \beta &= \frac{\lambda_{BN} - \lambda_{NN}}{(\lambda_{BN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{BP})} \\ \gamma &= \frac{\lambda_{PN} - \lambda_{NN}}{(\lambda_{PN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{PP})} \end{aligned} \tag{10}$$

Obviously, recall that Rule 2, one can classify an object into the positive region, boundary region or negative region. Furthermore, the definitions of the optimistic multi-granulation rough approximations and the pessimistic multi-granulation rough approximations can be given as follows:

Definition 8 [35]. Assume $IS = (U, A, V, f)$ is an information system, $R_1, R_2, \cdots, R_m$ are m granular structures. For any $X \subseteq U$, the lower and upper approximations of the optimistic multi-granulation rough approximations of X about R_i can be defined by:

$$\begin{aligned} \underline{\sum_{i=1}^{m} R_i{}^O}(X) &= \left\{ x | \bigvee_{i=1}^{m} \Pr(X|[x]_{R_i}) \ge \alpha, x \in U \right\} \\ \overline{\sum_{i=1}^{m} R_i{}^O}(X) &= \sim \underline{\sum_{i=1}^{m} R_i{}^O}(\sim X) \end{aligned} \tag{11}$$

where $[x]_{R_i}$ is the equivalence class of x under R_i, $\sim X$ is the complement set of X.

According to the definitions of the lower approximation and the upper approximation of the optimistic multi-granulation rough approximations, the boundary region of the optimistic multi-granulation rough approximations can be defined by

$$BN^O_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i{}^O}(X) - \underline{\sum_{i=1}^{m} R_i{}^O}(X) \tag{12}$$

Analogous to decision-theoretic rough set, one may obtain decision rules of the optimistic multi-granulation rough approximations in the following.

Definition 9 [35]. Assume $IS = (U, AT, V, f)$ is an information system, $R_1, R_2, \cdots, R_m$ are m granular structures. For any $X \subseteq U$, when $\alpha > \beta$, decision rules of the optimistic multi-granulation rough approximations are in the following:

($OP1$) If $\exists i \in \{1, 2, \cdots, m\}$ such that $\Pr(X|[x]_{R_i}) \ge \alpha$, then $x \in POS(X)$;

($ON1$) If $\forall i \in \{1, 2, \cdots, m\}$ such that $\Pr(X|[x]_{R_i}) \le \beta$, then. $x \in NEG(X)$;

($OB1$) Otherwise, $x \in BND(X)$.

Definition 10 [35]. Assume $IS = (U, A, V, f)$ is an information system. $R_1, R_2, \cdots, R_m$ are m granular structures. For any $X \subseteq U$, the lower and upper approximations of the pessimistic multi-granulation rough approximations of X about R_i can be defined by:

$$\begin{aligned} \underline{\sum_{i=1}^{m} R_i{}^P}(X) &= \left\{ x | \bigwedge_{i=1}^{m} \Pr(X|[x]_{R_i}) \ge \alpha, x \in U \right\} \\ \overline{\sum_{i=1}^{m} R_i{}^P}(X) &= \sim \underline{\sum_{i=1}^{m} R_i{}^P}(\sim X) \end{aligned} \tag{13}$$

Formally, based on the definitions of the lower approximation and the upper approximation of the pessimistic multi-granulation rough approximations, the boundary region of the pessimistic multi-granulation rough approximations can be defined by:

Table 3 Conflict information system

U	a_1	a_2	d
x_1	0	0	N
x_2	1	0	B
x_3	1	1	P
x_4	0	0	N
x_5	1	1	P
x_6	0	0	N
x_7	0	1	B

Table 4 Conditional probabilities of objects under R_1 and R_2

U	$\Pr(X\|[x_j]_{R_1})$	$\Pr(X\|[x_j]_{R_2})$
x_1	0.33	0.33
x_2	1	1
x_3	1	1
x_4	0.33	0.33
x_5	1	1
x_6	0.33	0.33
x_7	1	1

Table 5 Decision results of the optimistic multi-granulation rough approximations

P	B	N
$\{x_2, x_3, x_5, x_7\}$	$\{x_1, x_4, x_6\}$	$\{\emptyset\}$

$$BN^{P}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i}^{P}(X) - \underline{\sum_{i=1}^{m} R_i}^{P}(X) \tag{14}$$

Particularly, similar to the optimistic multi-granulation rough approximations, the decision rules of the pessimistic multi-granulation rough approximations can be also acquired as follows.

Definition 11 [35]. Let $IS = (U, AT, V, f)$ be an information system, $R_1, R_2, \cdots, R_m$ be m granular structures. For any $X \subseteq U$, when $\alpha > \beta$, decision rules of the pessimistic multi-granulation rough approximations are as follows:

($PP1$) If $\forall i \in \{1, 2, \cdots, m\}$ such that $\Pr(X|[x]_{R_i}) \geq \alpha$, then $x \in POS(X)$;

($PN1$) If $\exists i \in \{1, 2, \cdots, m\}$ such that $\Pr(X|[x]_{R_i}) \leq \beta$, then $x \in NEG(X)$;

($PB1$) Otherwise, $x \in BND(X)$.

Recall that Definition 9 and Definition 11, one may gain the decision rules of the optimistic multi-granulation rough approximations and the pessimistic multi-granulation rough approximations through Example 2 from Table 3.

Table 6 Decision results of the pessimistic multi-granulation rough approximations

P	B	N
$\{x_2, x_3, x_5, x_7\}$	$\{x_1, x_4, x_6\}$	$\{\emptyset\}$

Table 7 Pythagorean fuzzy conflict information system

	a_1	a_2	a_3	a_4
x_1	(0.4, 0.7)	(0.4, 0.6)	(0.2, 0.8)	(0.6, 0.2)
x_2	(0.8, 0.1)	(0.7, 0.4)	(0.4, 0.3)	(0.7, 0.1)
x_3	(0.5, 0.3)	(0.8, 0.2)	(0.7, 0.2)	(0.5, 0.4)

Example 2 Table 3 is a conflict information system, where $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$,$R_i\,(i = 1, 2, \cdots, m)$ are m granular structures, $AT = \{a_1, a_2\} \cup \{d\}$, the decision attribute $d = \{P, B, N\}$, and P, N and B indicate accepting, rejecting, pending decisions, respectively.

Assume $m = 2$, $R_1 = \{a_1\}$, $R_2 = \{a_1, a_2\}$, and $X = \{x_1, x_2, x_3, x_5, x_7\}$. Then, the conditional probabilities of each object under R_1 and R_2 can be required respectively, and results are expressed in Table 4.

Let $\alpha = 0.6$, $\beta = 0.3$. By Definitions of 9 and 11, the decision results are shown in Table 5 and Table 6, respectively.

3 Three-Way Conflict Analysis in Pythagorean Fuzzy Information System

It is worth noting that there are numerous factors that influence decisions of agents, such as weather, mood, physical condition. Therefore, the attitudes of agents on issues are often vague and ambiguous. In view of the advantage of PFS in characterizing uncertain information, PFNs are used to express the agents' attitudes towards the issues in this paper. Consequently, a novel three-way conflict analysis model is constructed.

3.1 Pythagorean Fuzzy Conflict Information System

For the sake of concrete discussion in detail later, we establish a *Pythagorean Fuzzy Conflict Information System* in this subsection. Henceforth, we abbreviate it as *PFCIS*.

Definition 12 An information system $IS = (U, A, V, f)$ is called a *PFCIS*, where U is a non-empty finite set of agents, A is a non-empty finite set of issues, $V = \cup\left\{ V_{c_j} | c_j \in A \right\}$, V_{c_j} is the set of values of issues c_j, the attitudes of agents to issues are denoted by PFNs and *f* is the relationship between U and A, $f : U \times A \rightarrow V$.

Correspondingly, a *PFCIS* can be established ground on Definition 12. To depict clearly the *PFCIS*, one can state it by Example 3.

Example 3 A *PFCIS* is shown in Table 7.

It can be seen that the attitudes of agents towards the issues from Table 7, for example,(0.8, 0.2) shows that the support degree to the issue a_2 is 0.8, and the oppose degree to the issue a_2 is 0.2 for the agent x_3.

Definition 13 Assume $IS = (U, A, V, f)$ is a *PFCIS*, a PFN is the attitude of any agent $x_i \in U$ on the any issue $a \in A$. M and N are the attitude of agents $x, y \in U$ on the issue set A, and then the Pythagorean fuzzy correlation coefficient is defined as:

$$k(M,N) = \frac{2C(M,N)}{C(M,M) + C(N,N)} \tag{15}$$

where

$$C(M,N) = \sum_{i=1}^{n} [u_M^2(x_i)u_N^2(x_i) + v_M^2(x_i)v_N^2(x_i) + \pi_M^2(x_i)\pi_N^2(x_i)],$$

$$C(M,M) = \sum_{i=1}^{n} [u_M^4(x_i) + v_M^4(x_i) + \pi_M^4(x_i)] \text{ and}$$

$$C(N,N) = \sum_{i=1}^{n} [u_N^4(x_i) + v_N^4(x_i) + \pi_N^4(x_i)].$$

Intuitively, based on Definition 13, one may easily obtain the properties of $k(M,N)$ in the following theorem.

Theorem 1 *Let M and N be PFSs*, $k : PFS \times PFS \rightarrow [0,1]$, *then these properties hold in the following*.

$$\begin{aligned} &(1)\, k(M,N) \in [0,1] \\ &(2)\, k(M,N) = k(N,M) \\ &(3)\, \text{if } k(M,N) = 1, \text{then } M = N \end{aligned} \tag{16}$$

Proof (1) According to mean inequalities $ab \leq \frac{a^2+b^2}{2}$, there exists $u_M^2(x_i)u_N^2(x_i) + v_M^2(x_i)v_N^2(x_i) + \pi_M^2(x_i)\pi_N^2(x_i)$. $\leq \frac{u_M^4(x_i)+u_N^4(x_i)}{2} + \frac{v_M^4(x_i)+v_N^4(x_i)}{2} + \frac{\pi_M^4(x_i)+\pi_N^4(x_i)}{2}$.

It is obviously that $u_\triangle^2$,$v_\triangle^2$ and $\pi_\triangle^2$ are greater than 0, where $\triangle = M, N$. Thus, $k(M,N) \in [0,1]$.

(2) Due to $C(M,N) = C(N,M)$, then $k(M,N) = k(N,M)$.

(3) If $k(M,N) = 1$, there exists $2C(M,N) = C(M,M) + C(N,N)$, then $\sum_{i=1}^{n} [u_M^2(x_i)u_N^2(x_i) + v_M^2(x_i)v_N^2(x_i) + \pi_M^2(x_i)\pi_N^2(x_i)]$. $= \sum_{i=1}^{n} [u_M^4(x_i) + v_M^4(x_i) + \pi_M^4(x_i)] + \sum_{i=1}^{n}[u_N^4(x_i) + v_N^4(x_i) + \pi_N^4(x_i)]$. Thus, $u_M(x_i) = u_N(x_i)$, $v_M(x_i) = v_N(x_i)$ and $\pi_M(x_i) = \pi_N(x_i)$.

Therefore, $M = N$.

Generally speaking, Definition 13 and Theorem 1 describe the conflict degree of agents towards issue set A.

3.2 Trisecting Agents Set

A fundamental aspect of conflict analysis is to trisect agents set, which can guide us to make informed decisions, that is, support, neutralize and oppose. Establishing a convenient and effective model to realize the trisection of agents in *PFCIS* is a key focus of this paper.

Definition 14 Assume $IS = (U, A, V, f)$ is a *PFCIS*. For correlation coefficient $k(M,N)$ and a pair of thresholds α, β with $0 \leq \beta < \alpha \leq 1$, the alliance set, neutral set and conflict set with respect to agent x are defined respectively as:

$$\begin{aligned} &(1)\, AL_{(A,\alpha,\beta)}(x) = \{y \in U | k(M,N) \geq \alpha\} \\ &(2)\, NE_{(A,\alpha,\beta)}(x) = \{y \in U | \beta < k(M,N) < \alpha\} \\ &(3)\, CO_{(A,\alpha,\beta)}(x) = \{y \in U | k(M,N) \leq \beta\} \end{aligned} \tag{17}$$

Accordingly, one can judge the conflict state of any agents x,y towards issue set A by Definition 14.

Definition 15 Assume $IS = (U, A, V, f)$ is a *PFCIS*. For correlation coefficient $k(M,N)$ and a pair of thresholds α, β with $0 \leq \beta < \alpha \leq 1$, the no-conflict set, no-neutral set and no-alliance set about the agent x are given respectively in the following formals:

(1) no-conflict:

$$\begin{aligned} CO^C_{(A,\alpha,\beta)}(x) &= \{y \in U | k(M,N) > \beta\} \\ &= AL_{(A,\alpha,\beta)}(x) \cup NE_{(A,\alpha,\beta)}(x) \end{aligned}$$

(2) no-neutral:

$$\begin{aligned} NE^C_{(A,\alpha,\beta)}(x) &= \{y \in U | k(M,N) \leq \beta \text{ or } k(M,N) \geq \alpha\} \\ &= AL_{(A,\alpha,\beta)}(x) \cup CO_{(A,\alpha,\beta)}(x) \end{aligned}$$

(3) no-alliance:

$$\begin{aligned} AL^C_{(A,\alpha,\beta)}(x) &= \{y \in U | k(M,N) < \alpha\} \\ &= NE_{(A,\alpha,\beta)}(x) \cup CO_{(A,\alpha,\beta)}(x) \end{aligned}$$

Theorem 2 *Assume* $IS = (U, A, V, f)$ *is a PFCIS. The following equalities are valid.*

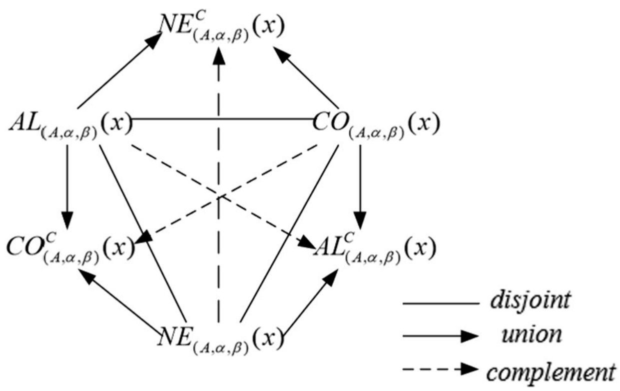

Fig. 1 Relationships among agent sets

$$\begin{aligned} &(1)\, CO^C_{(A,\alpha,\beta)}(x) \cap NE^C_{(A,\alpha,\beta)}(x) = AL_{(A,\alpha,\beta)}(x) \\ &(2)\, CO^C_{(A,\alpha,\beta)}(x) \cap AL^C_{(A,\alpha,\beta)}(x) = NE_{(A,\alpha,\beta)}(x) \\ &(3)\, AL^C_{(A,\alpha,\beta)}(x) \cap NE^C_{(A,\alpha,\beta)}(x) = CO_{(A,\alpha,\beta)}(x) \\ &(4)\, AL^C_{(A,\alpha,\beta)}(x) \cap NE^C_{(A,\alpha,\beta)}(x) \cap CO^C_{(A,\alpha,\beta)}(x) = \varnothing \end{aligned} \tag{18}$$

Proof Intuitively, from Definition 14 and Definition 15, one can easily verify the above conclusions.

Apparently, the relations of agents among Definitions 14,15 and Theorem 2 are clearly shown in Fig. 1.

As we all know, decision-making is often accompanied by risks, that is, agents will choose alliance, neutral or conflict with the loss of decision risk. Therefore, a decision-making model with risk is very practical.

Definition 16 Let $IS = (U, A, V, f)$ be a *PFCIS*. The set of actions is denoted by $E = \{a_A, a_N, a_C\}$, where a_A,a_N and a_C represent the actions for classifying agents into $AL_{(A,\alpha,\beta)}(U)$, $NE_{(A,\alpha,\beta)}(U)$ and $CO_{(A,\alpha,\beta)}(U)$, respectively, where λ_{*P} denotes the decision loss of taking action a_* for classifying agents in X into the region specified by $*$, and λ_{*N} denotes the decision loss of taking action a_* for classifying agents that not belong to X into the region specified by $*$. For $x, y \in U$, the expected loss of the decision action taken is denoted as $R(a_*|x, y)$, which is described in the following:

$$\begin{aligned} &(1)\, R(a_A|x, y) = \lambda_{AP} k(M, N) + \lambda_{AN}(1 - k(M, N)) \\ &(2)\, R(a_N|x, y) = \lambda_{NP} k(M, N) + \lambda_{NN}(1 - k(M, N)) \\ &(3)\, R(a_C|x, y) = \lambda_{CP} k(M, N) + \lambda_{CN}(1 - k(M, N)) \end{aligned} \tag{19}$$

Theorem 3 *Assume $IS = (U, A, V, f)$ is a PFCIS. For any agents $x, y \in U$, $R(a_A|x, y)$, $R(a_N|x, y)$ and $R(a_C|x, y)$ are expected losses of taking decision actions a_A,a_N and a_C, respectively. Then, there exist.*

(1) *If* $R(a_A|x, y) \le R(a_N|x, y)$ *and.*

$R(a_A|x, y) \le R(a_C|x, y)$, *then* $x, y \in AL_{(A,\alpha,\beta)}(X)$;

(2) *If* $R(a_N|x, y) \le R(a_A|x, y)$ *and.*

$R(a_N|x, y) \le R(a_C|x, y)$, *then* $x, y \in NE_{(A,\alpha,\beta)}(X)$;

(3) *If* $R(a_C|x, y) \le R(a_A|x, y)$ *and.*

$R(a_C|x, y) \le R(a_N|x, y)$, *then* $x, y \in CO_{(A,\alpha,\beta)}(X)$.

The proof of Theorem 3 is provided in the appendix.

According to the proof of Theorem 3, we can obtain the following Rule 3.

Rule 3:

(*A*) *If* $k(M, N) \ge \alpha$, *then* $x, y \in AL_{(A,\alpha,\beta)}(X)$;

(*N*) *If* $\beta < k(M, N) < \alpha$, *then* $x, y \in NE_{(A,\alpha,\beta)}(X)$;

(*C*) *If* $k(M, N) \le \beta$, *then* $x, y \in CO_{(A,\alpha,\beta)}(X)$.

To provide a clear illustration of the decision actions of agents in Theorem 3, we illustrate it through Example 4.

Table 8 The *PFCIS* of Middle East conflict [11]

	a_1	a_2	a_3	a_4	a_5
x_1	(1,0)	(0.9,0.3)	(0.8,0.2)	(0.9,0.1)	(0.9,0.2)
x_2	(0.9,0.1)	(0.5,0.5)	(0.1,0.9)	(0.3,0.8)	(0.1,0.9)
x_3	(0.1,0.9)	(0.1,0.9)	(0.2,0.8)	(0.1,0.9)	(0.5,0.5)
x_4	(0.5,0.5)	(0.1,0.9)	(0.3,0.7)	(0.5,0.5)	(0.1,0.9)
x_5	(0.9,0.2)	(0.4,0.6)	(0.1,0.9)	(0.1,0.9)	(0.3,0.9)
x_6	(0,1)	(0.9,0.1)	(0.2,0.9)	(0.5,0.5)	(0.8,0.4)

Example 4 The *PFCIS* of the Middle East conflict is shown in Table 8.

Let $\lambda_{AP} = 0.8$,$\lambda_{AN} = 3$,$\lambda_{NP} = 1.4$,$\lambda_{NN} = 1.7$,$\lambda_{CP} = 2.6$ and $\lambda_{CN} = 1.5$, thus, $\alpha = 0.68$ and $\beta = 0.45$. According to Rule 3, the decision results of Table 8 are shown in Table 9. Additionally, Table 9 also displays the comparison of decision results between Rule 3 and Lang's rule [11].

According to Table 9, we can draw a significant conclusion that our rule provides a more detailed and specific representation of conflict information. Additionally, it effectively highlights the internal causes of conflict formation.

Table 9 Comparison of decision results

Rule	Information system	$AL_{(A,\alpha,\beta)}(X)$	$NE_{(A,\alpha,\beta)}(X)$	$CO_{(A,\alpha,\beta)}(X)$
Lang's rule	The Middle East conflict	x_1	x_6	x_2, x_3, x_4, x_5
Rule 3	The Middle East conflict	x_2, x_4	x_1, x_6	x_1, x_2
		x_2, x_5	x_2, x_3	x_1, x_3
		x_3, x_4	x_2, x_6	x_1, x_4
		x_4, x_5	x_3, x_5	x_1, x_5
			x_3, x_6	
			x_4, x_6	
			x_5, x_6	

Table 10 Loss function values of different actions in different states

	X	$\sim X$
a_A	$\tilde{\lambda}_{AP} = [A^-_{AP}(\eta), A^+_{AP}(\eta)]$	$\tilde{\lambda}_{AN} = [A^-_{AN}(\eta), A^+_{AN}(\eta)]$
a_N	$\tilde{\lambda}_{NP} = [A^-_{NP}(\eta), A^+_{NP}(\eta)]$	$\tilde{\lambda}_{NN} = [A^-_{NN}(\eta), A^+_{NN}(\eta)]$
a_C	$\tilde{\lambda}_{CP} = [A^-_{CP}(\eta), A^+_{CP}(\eta)]$	$\tilde{\lambda}_{CN} = [A^-_{CN}(\eta), A^+_{CN}(\eta)]$

Table 11 Decision thresholds for optimists and pessimists

	optimists	pessimists
α	$\alpha^O = \frac{A^-_{AN}(\eta) - A^-_{NN}(\eta)}{(A^-_{AN}(\eta) - A^-_{NN}(\eta)) + (A^-_{NP}(\eta) - A^-_{AP}(\eta))}$	$\alpha^P = \frac{A^+_{AN}(\eta) - A^+_{NN}(\eta)}{(A^+_{AN}(\eta) - A^+_{NN}(\eta)) + (A^+_{NP}(\eta) - A^+_{AP}(\eta))}$
β	$\beta^O = \frac{A^-_{NN} - A^-_{CN}}{(A^-_{NN} - A^-_{CN}) + (A^-_{CP} - A^-_{NP})}$	$\beta^P = \frac{A^+_{NN}(\eta) - A^+_{CN}(\eta)}{(A^+_{NN}(\eta) - A^+_{CN}(\eta)) + (A^+_{CP}(\eta) - A^+_{NP}(\eta))}$

3.3 Trisecting Agents Set with Different Risk Preference

In particular, risk preference also has a significant impact on decision-making for agents. For example, some people prefer risk because big risk often comes with large profit. Additionally, some people avoid risk because it can result in loss. Therefore, this subsection is devoted to establishing a trisecting agents set model with different risk preferences.

Definition 17 Assume $\underset{\sim}{A} = \langle \alpha, \beta \rangle$ is a PFN. A binary group $\left\langle u_{\underset{\sim}{A}}(x), v_{\underset{\sim}{A}}(x) \right\rangle$ is a characteristic function, $\eta \in [0, 1]$, the $A_\eta = \left\{ \alpha_\eta, \beta^C_\eta \right\}$ is defined as level cut set on A, where, $\alpha_\eta = \left\{ x | u_{\underset{\sim}{A}}(x) \geq \eta \right\}$ and $\beta^C_\eta = \left\{ x | 1 - v_{\underset{\sim}{A}}(x) \geq \eta \right\}$. Suppose $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ are PFNs, the left and right points of PFNs $\underset{\sim}{P}, \underset{\sim}{Q}$ lower ideal of level cut-off are defined as $u_{\underset{\sim}{P}}(p^-_\eta)$, $u_{\underset{\sim}{P}}(p^+_\eta)$, $u_{\underset{\sim}{Q}}(q^-_\eta)$ and $u_{\underset{\sim}{Q}}(q^+_\eta)$.

Accordingly, one can acquire thresholds of decision-making by Bayes risk decision theory.

Definition 18 Let $IS = (U, A, V, f)$ be a $PFCIS$. According to Bayes risk decision theory, the state space $\Theta = \{X, \sim X\}$ is constructed to describe the state of agents belonging to the set X, which indicate that an agent is in X, and not in X, respectively. The set of actions is written by $E = \left\{ a_A, a_N, a_C \right\}$, where a_A, a_N and a_C represent the actions in classifying agents into $AL_{(A,\alpha,\beta)}(X)$, $NE_{(A,\alpha,\beta)}(X)$ and $CO_{(A,\alpha,\beta)}(X)$, respectively. The loss function values of the corresponding actions in different states are depicted PFNs. For $\eta \in [0, 1]$, $A^-_{\cdot\cdot}(\eta)$ and $A^+_{\cdot\cdot}(\eta)$ are regarded as left and right points of lower ideal set of PFNs $\tilde{\lambda}_{\cdot\cdot}$, then $A^-_{\cdot\cdot}(\eta) < A^+_{\cdot\cdot}(\eta)$, and decision loss functions are shown in Table 10.

(1) For optimists

$$R^O(a_A | x, y) = A^-_{AP}(\eta) k(M, N) + A^-_{AN}(\eta)(1 - k(M, N))$$

$$R^O(a_N | x, y) = A^-_{NP}(\eta) k(M, N) + A^-_{NN}(\eta)(1 - k(M, N))$$

$$R^O(a_C | x, y) = A^-_{CP}(\eta) k(M, N) + A^-_{CN}(\eta)(1 - k(M, N))$$

(2) For pessimists

$$R^P(a_A | x, y) = A^+_{AP}(\eta) k(M, N) + A^+_{AN}(\eta)(1 - k(M, N))$$

$$R^P(a_N | x, y) = A^+_{NP}(\eta) k(M, N) + A^+_{NN}(\eta)(1 - k(M, N))$$

$$R^P(a_C | x, y) = A^+_{CP}(\eta) k(M, N) + A^+_{CN}(\eta)(1 - k(M, N))$$

According to Bayes risk decision theory, there exist.

(1) If $R^\triangle(a_A | x, y) \leq R^\triangle(a_N | x, y)$ and $R^\triangle(a_A | x, y) \leq R^\triangle(a_C | x, y)$, then $x, y \in AL_{(A,\alpha,\beta)}(X)$;
(2) If $R^\triangle(a_N | x, y) \leq R^\triangle(a_A | x, y)$ and $R^\triangle(a_N | x, y) \leq R^\triangle(a_C | x, y)$, then $x, y \in NE_{(A,\alpha,\beta)}(X)$;
(3) If $R^\triangle(a_C | x, y) \leq R^\triangle(a_A | x, y)$ and $R^\triangle(a_C | x, y) \leq R^\triangle(a_N | x, y)$, then $x, y \in CO_{(A,\alpha,\beta)}(X)$.

where $\triangle$ denotes O and P, respectively. According to the assumption $0 \le \tilde{\lambda}_{AP} < \tilde{\lambda}_{NP} < \tilde{\lambda}_{CP}$ and $0 \le \tilde{\lambda}_{CN} < \tilde{\lambda}_{NN} < \tilde{\lambda}_{AN}$, one may easily obtain the decision thresholds of optimists and pessimists, respectively, shown in Table 11.

where the range of values of threshold α as follows

$$\alpha \in \left[\frac{A^-_{AN}(\eta) - A^+_{NN}(\eta)}{(A^+_{AN}(\eta) - A^-_{NN}(\eta)) + (A^+_{NP}(\eta) - A^-_{AP}(\eta))}, \min\left(\frac{A^+_{AN}(\eta) - A^-_{NN}(\eta)}{(A^-_{AN}(\eta) - A^+_{NN}(\eta)) + (A^-_{NP}(\eta) - A^+_{AP}(\eta))}, 1\right)\right]$$

and the range of values of threshold β in the following

$$\beta \in \left[\frac{A^-_{NN}(\eta) - A^+_{CN}(\eta)}{(A^+_{NN}(\eta) - A^-_{CN}(\eta)) + (A^+_{CP}(\eta) - A^-_{NP}(\eta))}, \min\left(\frac{A^+_{NN}(\eta) - A^-_{CN}(\eta)}{(A^-_{NN}(\eta) - A^+_{CN}(\eta)) + (A^-_{CP}(\eta) - A^+_{NP}(\eta))}, 1\right)\right]$$

Intuitively, one can get the range of values of thresholds α and β from Table 11. Therefore, decision actions with different risk preference can be obtained.

Definition 19 Assume $IS = (U, A, V, f)$ is a $PFCIS$, the M and N are the attitudes of any agents $x, y \in U$ on the issue set A. For $k(M, N)$ and $\eta \in [0, 1]$, then the optimistic alliance, the pessimistic alliance, the optimistic neutral, the pessimistic neutral, the optimistic conflict and the pessimistic conflict are defined, respectively, in the following formals:

$$\begin{aligned}
&(1)\, AL^O_{(A,\alpha,\beta)}(x) = \left\{y \in U \mid k(M,N) \ge \alpha^{\min}\right\}\\
&(2)\, AL^P_{(A,\alpha,\beta)}(x) = \{y \in U \mid k(M,N) \ge \alpha^{\max}\}\\
&(3)\, NE^O_{(A,\alpha,\beta)}(x) = \left\{y \in U \mid \beta^{\min} < k(M,N) < \alpha^{\max}\right\}\\
&(4)\, NE^P_{(A,\alpha,\beta)}(x) = \left\{y \in U \mid \beta^{\max} < k(M,N) < \alpha^{\min}\right\}\\
&(5)\, CO^O_{(A,\alpha,\beta)}(x) = \{y \in U \mid k(M,N) \le \beta^{\max}\}\\
&(6)\, CO^P_{(A,\alpha,\beta)}(x) = \left\{y \in U \mid k(M,N) \le \beta^{\min}\right\}
\end{aligned} \tag{20}$$

The relationships of agents in Definition 19 are clearly illustrated in Fig. 2. Consequently, we have the following proposition based on Definitions 18 and 19.

Proposition 1 *Assume $IS = (U, A, V, f)$ is a $PFCIS$. There exist.*

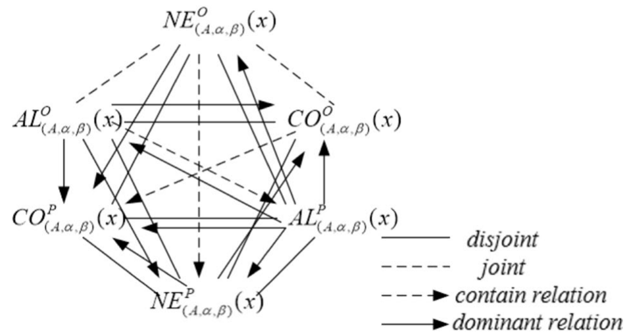

Fig. 2 Relationships among sets of agents

$$\begin{aligned}
&(1)\, AL^O_{(A,\alpha,\beta)}(x) \cap NE^O_{(A,\alpha,\beta)}(x) \subseteq AL_{(A,\alpha,\beta)}(x)\\
&(2)\, NE^O_{(A,\alpha,\beta)}(x) \cap CO^O_{(A,\alpha,\beta)}(x) \subseteq NE_{(A,\alpha,\beta)}(x)\\
&(3)\, CO^O_{(A,\alpha,\beta)}(x) \cap CO^P_{(A,\alpha,\beta)}(x) \subseteq CO_{(A,\alpha,\beta)}(x)\\
&(4)\, AL^O_{(A,\alpha,\beta)}(x) \cup AL^P_{(A,\alpha,\beta)}(x) \supseteq AL_{(A,\alpha,\beta)}(x)\\
&(5)\, NE^O_{(A,\alpha,\beta)}(x) \cup NE^P_{(A,\alpha,\beta)}(x) \supseteq NE_{(A,\alpha,\beta)}(x)\\
&(6)\, CO^O_{(A,\alpha,\beta)}(x) \cup CO^P_{(A,\alpha,\beta)}(x) \supseteq CO_{(A,\alpha,\beta)}(x)\\
&(7)\, AL^O_{(A,\alpha,\beta)}(x) \cup NE^P_{(A,\alpha,\beta)}(x) = CO^{OC}_{(A,\alpha,\beta)}(x)\\
&(8)\, AL^P_{(A,\alpha,\beta)}(x) \cup NE^O_{(A,\alpha,\beta)}(x) = CO^{PC}_{(A,\alpha,\beta)}(x)\\
&(9)\, CO^O_{(A,\alpha,\beta)}(x) \cup NE^P_{(A,\alpha,\beta)}(x) = AL^{OC}_{(A,\alpha,\beta)}(x)\\
&(10)\, CO^P_{(A,\alpha,\beta)}(x) \cup NE^O_{(A,\alpha,\beta)}(x) = AL^{PC}_{(A,\alpha,\beta)}(x)\\
&(11)\, AL^{OC}_{(A,\alpha,\beta)}(x) \cap CO^{OC}_{(A,\alpha,\beta)}(x) = NE^P_{(A,\alpha,\beta)}(x)\\
&(12)\, AL^{PC}_{(A,\alpha,\beta)}(x) \cap CO^{PC}_{(A,\alpha,\beta)}(x) = NE^O_{(A,\alpha,\beta)}(x)\\
&(13)\, AL^P_{(A,\alpha,\beta)}(x) \cup CO^P_{(A,\alpha,\beta)}(x) = NE^{OC}_{(A,\alpha,\beta)}(x)\\
&(14)\, AL^O_{(A,\alpha,\beta)}(x) \cup CO^O_{(A,\alpha,\beta)}(x) = NE^{PC}_{(A,\alpha,\beta)}(x)
\end{aligned} \tag{21}$$

Proof. From Definitions 18,19 and basic properties of set, the above conclusions can be easily proved by intuition.

Theorem 4. *Assume $IS = (U, A, V, f)$ is a $PFCIS$. The following properties hold.*

$$\begin{aligned}
&(1)\, \text{If } x, y \in AL^P_{(A,\alpha,\beta)}(x),\ \text{then } x, y \in AL^O_{(A,\alpha,\beta)}(x);\\
&(2)\, \text{If } x, y \in NE^O_{(A,\alpha,\beta)}(x),\ \text{then } x, y \in AL^P_{(A,\alpha,\beta)}(x);\\
&(3)\, \text{If } x, y \in CO^P_{(A,\alpha,\beta)}(x),\ \text{then } x, y \in CO^O_{(A,\alpha,\beta)}(x).
\end{aligned} \tag{22}$$

Proof Based on the formula (20) and basic properties of set, the above relations can be easily verified.

In decision-making process, the agents need to consider not only their individual attitudes towards each issue, but also their overall attitude towards the issue set.

Definition 20 [42]. Assume $IS = (U, A, V, f)$ is a $PFCIS$. For any agents $x, y \in U$ and $a \in A$, the degree of conflict of an issue a, which can be expressed as:

$$con(a) = \frac{card(X_a^+) \cdot card(X_a^-)}{\mathrm{int}\left(\frac{n}{2}\right) \cdot \left(n - \mathrm{int}\left(\frac{n}{2}\right)\right)} \tag{23}$$

where $X_a^+ = \{x \in U | u_a(x) \geq m^* \wedge v_a(x) \leq m_*\}$, $X_a^- = \{x \in U | u_a(x) < m^* \wedge v_a(x) > m_*\}$, thresholds $m^* \geq 0.5$ and $m_* \geq 0.5$. $card(\,)$ expresses cardinality of a set, int( ) denotes the rounding of any number, n denotes the population of agents and formula (24) denotes conflict degree of situation $S = (U, A)$.

Accordingly, the degree of conflict of an issue set A, which can be expressed as following formal:

$$con(A) = \frac{\sum_{a \in A} con(a)}{card(A)} \tag{24}$$

Definition 21 Assume $IS = (U, A, V, f)$ is a $PFCIS$. A pair of thresholds l, h with $0 \leq l < h \leq 1$. Conflict situation, defuse situation and non-conflict situation are respectively defined as:

$$CS_{PFCIS}^{(l,h)} = \{A | con(A) \geq h\}$$

$$DS_{PFCIS}^{(l,h)} = \{A | l < con(A) < h\}$$

$$NS_{PFCIS}^{(l,h)} = \{A | con(A) \leq l\} \tag{25}$$

Definition 21 reveals that the conflict degree of a situation $S = (U, A)$ can be divided as conflict situation, defuse situation and non-conflict situation through a pair of thresholds.

Definition 22. Suppose $IS = (U, A, V, f)$ is a $PFCIS$, then the strong conflict, the weak conflict, the strong alliance, the weak alliance, the strong neutral and the weak neutral are respectively defined as:

$$\begin{aligned}
&(1)\ SC_A(x) = \{y \in U | k(M,N) \leq \beta^{\min}, con(A) \geq h\} \\
&(2)\ WC_A(x) = \{y \in U | k(M,N) \leq \beta^{\max}, con(A) \geq h\} \\
&(3)\ SA_A(x) = \{y \in U | k(M,N) \geq \alpha^{\max}, con(A) \leq l\} \\
&(4)\ WA_A(x) = \{y \in U | k(M,N) \geq \alpha^{\min}, con(A) \leq l\} \\
&(5)\ SN_A(x) = \{y \in U | \beta^{\max} < k(M,N) < \alpha^{\min}, l < con(A) < h\} \\
&(6)\ WN_A(x) = \{y \in U | \beta^{\min} < k(M,N) < \alpha^{\max}, l < con(A) < h\}
\end{aligned} \tag{26}$$

According to the concepts in Definition 22, the following inclusion relations hold.

Theorem 5 *Assume $IS = (U, A, V, f)$ is a PFCIS. These following properties hold.*

Table 12 Pythagorean fuzzy conflict information system

U	a_1	a_2	a_3
x_1	(0.7, 0.2)	(0.4, 0.9)	(0.5, 0.6)
x_2	(0.1, 0.8)	(0.8, 0.2)	(0.7, 0.6)
x_3	(0.8, 0.2)	(0.1, 0.7)	(0.4, 0.4)

$$\begin{aligned}
&(1)\ SC_A(x) \subseteq WC_A(x) \\
&(2)\ SA_A(x) \subseteq WA_A(x) \\
&(3)\ SN_A(x) \subseteq WN_A(x)
\end{aligned} \tag{27}$$

Proof Taking into account Definitions 20, 21 and 22, the above conclusions are verified easily.

To clearly depict the decision actions of agents in the above theorem, we illustrate them through Example 5.

Example 5 A $PFCIS$ is shown in Table 12.

Assume that the decision loss functions of a_A, a_N and a_C are PFNs, $\eta = 0.5$, and the upper and low bound of decision loss function are as follows:

$A_{AP}^-(\eta) = 3.1$, $A_{AP}^+(\eta) = 3.3$, $A_{NP}^-(\eta) = 4.5$, $A_{NP}^+(\eta) = 5.0$, $A_{CP}^-(\eta) = 9.0$, $A_{CP}^+(\eta) = 10.3$, $A_{CN}^-(\eta) = 2.2$, $A_{CN}^+(\eta) = 2.5$, $A_{NN}^-(\eta) = 5.0$, $A_{NN}^+(\eta) = 5.2$, $A_{AN}^-(\eta) = 9.8$ and $A_{AN}^-(\eta) = 10.0$.

From Definition 18, we have $\alpha \in [0.67, 0.86]$ and $\beta \in [0.28, 0.46]$. The attitudes of agents x_1,x_2 and x_3 on issues set A are B, C and D, respectively. By Definition 13, we have $k(B,C) = 0.44$,$k(B,D) = 0.83$ and $k(C,D) = 0.38$.

For optimists, by Definition 19, there exist $CO_{(A,\alpha,\beta)}^{O}(U) = \{\{x_1, x_2\}, \{x_2, x_3\}\}$, $AL_{(A,\alpha,\beta)}^{O}(U) = \{x_1, x_3\}$.

Suppose $m^* = 0.6$,$m_* = 0.5$, according to Definition 20, there exists $con(A) = 0.67$. Assume $l = 0.3$, $h = 0.6$, by Definition 21, then $WC_A(U) = \{x_1, x_2, x_3\}$.

For pessimists, there exists $WC_A(U) = \{x_1, x_2, x_3\}$ under the same conditions.

4 Trisecting Agents Based on Multi-granulation Pythagorean Fuzzy Decision-Theoretic Rough Set

Notably, individuals may sometimes need to make their decisions based on the level of granularity in the available information. For example, let's consider a company that aims to develop a new product. They would require both fine-grained information, such as customer preferences, demographics and purchase history, as well as macro-level information like industry trends and macroeconomic indicators. This multi-granular perspective enables the company to make informed decisions by incorporating micro and macro information into their decision-making process. In this section, the conflict analysis model is extended to the multi-granulation Pythagorean fuzzy conflict rough approximations, and then the conflicts between multi-source information system and multi-dimensional information system are also discussed in detail.

4.1 Optimistic, Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

We can acquire some different rough set models by adjusting the values of thresholds. In this subsection, optimistic model and pessimistic model of multi-granulation Pythagorean fuzzy conflict rough approximations are studied. respectively. Hereinafter, we abbreviate them as *OMRS* and *PMRS*, respectively.

Definition 23 Assume $IS=(U,A,V,f)$ is a *PFCIS*. The lower approximation $\underline{\sum_{i=1}^{m} R_i{}^{O}(X)}$ and upper approximation $\overline{\sum_{i=1}^{m} R_i{}^{O}(X)}$ of the *OMRS* are defined respectively as in the following formals:

$$\underline{\sum_{i=1}^{m} R_i{}^{O}(X)} = \left\{ y \in U \mid \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha \right\}$$
$$\overline{\sum_{i=1}^{m} R_i{}^{O}(X)} = \sim \underline{\sum_{i=1}^{m} R_i{}^{O}(\sim X)} \tag{28}$$

Formally, the boundary region of the *OMRS* is defined by:

$$BN^{O}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i{}^{O}(X)} - \underline{\sum_{i=1}^{m} R_i{}^{O}(X)} \tag{29}$$

Accordingly, by combining the features Definition 23, we can state the following proposition.

Proposition 2 *Assume $IS=(U,A,V,f)$ is a PFCIS. The following relations hold.*

$$\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m} R_i{}^{O}(X)} \supseteq \underline{R_i}^{O}(X), i \leq m\\
&(2)\ \overline{\sum_{i=1}^{m} R_i{}^{O}(X)} \supseteq \overline{R_i}^{O}(X), i \leq m\\
&(3)\ \underline{\sum_{i=1}^{m} R_i{}^{O}(X)} = \bigcup_{i=1}^{m} \underline{R_i}^{O}(X), i \leq m\\
&(4)\ \overline{\sum_{i=1}^{m} R_i{}^{O}(X)} = \bigcup_{i=1}^{m} \overline{R_i}^{O}(X), i \leq m
\end{aligned} \tag{30}$$

The proof of Proposition 2 is provided in the appendix.

Analogous to the Rule 2, when $\alpha > \beta$, one can acquire Decision Rules 1 which can be expressed as follows.

Decision Rules 1. *Assume $IS=(U,A,V,f)$ is a PFCIS. For the OMRS, when $\alpha > \beta$, we acquire the following decision rules.*

(OA) If $\exists i \in \{1,2\cdots,m\}$ such that $k_i(M,N) \geq \alpha$, then $x,y \in AL^{O}_{(A,\alpha,\beta)}(X)$;

(OC) If $\forall i \in \{1,2,\cdots,m\}$ such that $k_i(M,N) \leq \beta$, then $x,y \in CO^{O}_{(A,\alpha,\beta)}(X)$;

(ON) Otherwise,$x,y \in NE^{O}_{(A,\alpha,\beta)}(X)$.

Similar to OMRS, one can investigate PMRS in the same way.

Definition 24 Assume $IS=(U,A,V,f)$ is a *PFCIS*. The lower approximation and upper approximation of the *PMRS* are given respectively as:

$$\underline{\sum_{i=1}^{m} R_i{}^{P}(X)} = \left\{ y \in U \mid \bigwedge_{i=1}^{m} k_i(M,N) \geq \alpha \right\}$$
$$\overline{\sum_{i=1}^{m} R_i{}^{P}(X)} = \sim \underline{\sum_{i=1}^{m} R_i{}^{P}(\sim X)} \tag{31}$$

Correspondingly, the boundary region of the *PMRS* is defined by:

$$BN^{P}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i{}^{P}(X)} - \underline{\sum_{i=1}^{m} R_i{}^{P}(X)} \tag{32}$$

Accordingly, by Definition 24, one can require some conclusions of the lower and upper approximations of the *PMRS*.

Proposition 3 *Assume* $IS = (U, A, V, f)$ *is a* $PFCIS$. *These conclusions hold in the following.*

$$
\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m} R_i}^{P}(X) \subseteq \underline{R_i}^{P}(X), i \le m\\
&(2)\ \overline{\sum_{i=1}^{m} R_i}^{P}(X) \supseteq \overline{R_i}^{P}(X), i \le m\\
&(3)\ \underline{\sum_{i=1}^{m} R_i}^{P}(X) = \bigcap_{i=1}^{m} \underline{R_i}^{P}(X), i \le m\\
&(4)\ \overline{\sum_{i=1}^{m} R_i}^{P}(X) = \bigcap_{i=1}^{m} \overline{R_i}^{P}(X), i \le m
\end{aligned}
\tag{33}
$$

Proof Analogous to Proposition 2, one can verify these properties.

Similar to the Rule 2, when $\alpha > \beta$, one can obtain Decision Rules 2 which can be expressed in the following.

Decision Rules 2. Assume $IS = (U, A, V, f)$ is a $PFCIS$. For the $PMRS$, when $\alpha > \beta$, we acquire the following decision rules:

(PA) If $\forall i \in \{1, 2, \cdots, m\}$ such that $k_i(M, N) \ge \alpha$, then $x, y \in AL^{P}_{(A,\alpha,\beta)}(X)$;

(PC) If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M, N) \le \beta$, then $x, y \in CO^{P}_{(A,\alpha,\beta)}(X)$;

(PN) Otherwise, $x, y \in NE^{P}_{(A,\alpha,\beta)}(X)$.

According to Definition 18, the decision loss functions of agents are PFNs, then the range of values of thresholds α, β in $PFCIS$ can be denoted, respectively, in the following:

$$
\alpha \in \left[\frac{A^{-}_{AN}(\eta) - A^{+}_{NN}(\eta)}{(A^{+}_{AN}(\eta) - A^{-}_{NN}(\eta)) + (A^{+}_{NP}(\eta) - A^{-}_{AP}(\eta))}, \min\left(\frac{A^{+}_{AN}(\eta) - A^{-}_{NN}(\eta)}{(A^{-}_{AN}(\eta) - A^{+}_{NN}(\eta)) + (A^{-}_{NP}(\eta) - A^{+}_{AP}(\eta))}, 1 \right) \right]
$$

$$
\beta \in \left[\frac{A^{-}_{NN}(\eta) - A^{+}_{CN}(\eta)}{(A^{+}_{NN}(\eta) - A^{-}_{CN}(\eta)) + (A^{+}_{CP}(\eta) - A^{-}_{NP}(\eta))}, \min\left(\frac{A^{+}_{NN}(\eta) - A^{-}_{CN}(\eta)}{(A^{-}_{NN}(\eta) - A^{+}_{CN}(\eta)) + (A^{-}_{CP}(\eta) - A^{+}_{NP}(\eta))}, 1 \right) \right]
$$

Therefore, the agents need to consider the range of thresholds when making a decision of allied, neutral or conflict action in the multi-granulation Pythagorean fuzzy conflict information system.

According to different combinations of some special values (extreme values) of α and β, we can also construct some specific rough set models of Pythagorean fuzzy conflict decision-theoretic rough set. We will discuss them in Sects. 4.2 to 4.9 in detail.

4.2 Supper Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 25 Assume $IS = (U, A, V, f)$ is a $PFCIS$. The lower approximation $\underline{\sum_{i=1}^{m} R_i}^{SO}(X)$ and upper approximation $\overline{\sum_{i=1}^{m} R_i}^{SO}(X)$ of the Supper Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (for short, $SOMRS$) are given respectively by:

$$
\begin{aligned}
&\underline{\sum_{i=1}^{m} R_i}^{SO}(X) = \left\{ y \in U \mid \bigvee_{i=1}^{m} k_i(M, N) \ge \alpha_i^{\min} \right\}\\
&\overline{\sum_{i=1}^{m} R_i}^{SO}(X) = U - \left\{ y \in U \mid \bigwedge_{i=1}^{m} k_i(M, N) \le \beta_i^{\max} \right\}
\end{aligned}
\tag{34}
$$

In this case, the boundary region of the $SOMRS$ is expressed as follows:

$$
BN^{SO}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i}^{SO}(X) - \underline{\sum_{i=1}^{m} R_i}^{SO}(X) \tag{35}
$$

Intuitively, from Definition 25, we can obtain some properties of the lower and upper approximations of the $SOMRS$ as follows.

Proposition 4 *Assume* $IS = (U, A, V, f)$ *is a* $PFCIS$, *then.*

$$
\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m} R_i}^{SO}(X) \supseteq \underline{R_i}^{SO}(X), i \le m\\
&(2)\ \overline{\sum_{i=1}^{m} R_i}^{SO}(X) \supseteq \overline{R_i}^{SO}(X), i \le m\\
&(3)\ \underline{\sum_{i=1}^{m} R_i}^{SO}(X) = \bigcup_{i=1}^{m} \underline{R_i}^{SO}(X), i \le m\\
&(4)\ \overline{\sum_{i=1}^{m} R_i}^{SO}(X) = \bigcup_{i=1}^{m} \overline{R_i}^{SO}(X), i \le m
\end{aligned}
\tag{36}
$$

Proof Similar to Proposition 2, these properties can be easily verified.

Theorem 6. *Suppose* $IS = (U, A, V, f)$ *is a* $PFCIS$, $0 \le \beta_i < \alpha_i \le 1$, $X, Y \subseteq U$. *The following relations hold.*

$$
\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m} R_i}^{SO}(X) \subseteq \overline{\sum_{i=1}^{m} R_i}^{SO}(X)\\
&(2)\ \underline{\sum_{i=1}^{m} R_i}^{SO}(\varnothing) = \overline{\sum_{i=1}^{m} R_i}^{SO}(\varnothing) = \varnothing,
\end{aligned}
$$

$$\underline{\sum_{i=1}^{m}} R_i{}^{SO}(U) = \overline{\sum_{i=1}^{m}} R_i{}^{SO}(U) = U$$

(3)If $\alpha_1 \le \alpha_2$, $\beta_1 \le \beta_2$, then $\underline{\sum_{i=1}^{m} R_{i\alpha_2}}{}^{SO}(X) \subseteq \underline{\sum_{i=1}^{m} R_{i\alpha_1}}{}^{SO}(X)$ *and* $\overline{\sum_{i=1}^{m} R_{i\beta_2}}{}^{SO}(X) \subseteq \overline{\sum_{i=1}^{m} R_{i\beta_1}}{}^{SO}(X)$.

The proof of Theorem 6 is provided in the appendix.

Correspondingly, analogous to the OMRS, one can obtain the Decision Rules 3 which can be described in the following.

Decision Rules 3. *Assume $IS = (U, A, V, f)$ is a PFCIS. When $\alpha_i^{\min} > \beta_i^{\max}(i = 1, 2, \cdots, m)$, we can acquire the following decision rules:*

(SOA) If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M, N) \ge \alpha_i^{\min}$, then $x, y \in AL^{SO}_{(A,\alpha,\beta)}(X)$;

(SOC) If $\forall i \in \{1, 2, \cdots, m\}$ such that $k_i(M, N) \le \beta_i^{\max}$, then $x, y \in CO^{SO}_{(A,\alpha,\beta)}(X)$;

(SON) Otherwise $x, y \in NE^{SO}_{(A,\alpha,\beta)}(X)$.

4.3 Weakly Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 26 Suppose $IS = (U, A, V, f)$ is a $PFCIS$. For any agent $x, y \in U$, the lower approximation $\underline{\sum_{i=1}^{m}} R_i{}^{O'}(X)$ and upper approximation $\overline{\sum_{i=1}^{m}} R_i{}^{O'}(X)$ of the Weakly Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (namely,*WOMRS*) are defined respectively in the following formals:

$$\underline{\sum_{i=1}^{m}} R_i{}^{O'}(X) = \left\{ y \in U | \bigvee_{i=1}^{m} k_i(M, N) \ge \alpha_i^{\max} \right\}$$

$$\overline{\sum_{i=1}^{m}} R_i{}^{O'}(X) = U - \left\{ y \in U | \bigwedge_{i=1}^{m} k_i(M, N) \le \beta_i^{\min} \right\} \tag{37}$$

Accordingly, the boundary region of the *WOMRS* is given by:

$$BN^{O'}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m}} R_i{}^{O'}(X) - \underline{\sum_{i=1}^{m}} R_i{}^{O'}(X) \tag{38}$$

Obviously, by Definition 26, one can obtain some conclusions of the lower and upper approximations of the *WOMRS* as follows.

Theorem 7 *Assume $IS = (U, A, V, f)$ is a PFCIS. The following relationships hold.*

$$\begin{aligned} &\underline{\sum_{i=1}^{m}} R_i{}^{O'}(X) \subseteq \underline{\sum_{i=1}^{m}} R_i{}^{SO}(X), \\ &\overline{\sum_{i=1}^{m}} R_i{}^{O'}(X) \subseteq \overline{\sum_{i=1}^{m}} R_i{}^{SO}(X). \end{aligned} \tag{39}$$

The proof of Theorem 7 is provided in the appendix.

Accordingly, one can get the following Decision Rules.

Decision Rules 4. *Suppose $IS = (U, A, V, f)$ is a PFCIS. When $\alpha_i^{\min} > \beta_i^{\max}(i = 1, 2, \cdots, m)$, we get the following decision rules of the WOMRS:*

(WOA) If $\exists\, i \in \{1, 2 \cdots, m\}$ such that $k_i(M, N) \ge \alpha_i^{\max}$, then $x, y \in AL^{O'}_{(A,\alpha,\beta)}(X)$;

(WOC) If $\forall i \in \{1, 2, \cdots, m\}$ such that $k_i(M, N) \le \beta_i^{\min}$, then $x, y \in CO^{O'}_{(A,\alpha,\beta)}(X)$;

(WON) Otherwise, $x, y \in NE^{O'}_{(A,\alpha,\beta)}(X)$.

4.4 Supper Pessimistic Multi-granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 27 Let $IS = (U, A, V, f)$ be a $PFCIS$, the lower approximation $\underline{\sum_{i=1}^{m}} R_i{}^{SP}(X)$ and upper approximation $\overline{\sum_{i=1}^{m}} R_i{}^{SP}(X)$ of the Supper Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (noted as *SPMRS*) are described as follows:

$$\underline{\sum_{i=1}^{m}} R_i{}^{SP}(X) = \left\{ y \in U | \bigwedge_{i=1}^{m} k_i(M, N) \ge \alpha_i^{\max} \right\}$$

$$\overline{\sum_{i=1}^{m}} R_i^{SP}(X) = U - \left\{ y \in U \middle| \bigvee_{i=1}^{m} k_i(M,N) \le \beta_i^{\min} \right\} \tag{40}$$

Correspondingly, the boundary region is written as:

$$BN^{SP}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m}} R_i^{SP}(X) - \underline{\sum_{i=1}^{m}} R_i^{SP}(X) \tag{41}$$

Accordingly, by Definition 27, one can require these following relations of the lower and upper approximations of the *SPMRS*.

Proposition 5 *Assume* $IS = (U, A, V, f)$ *is a PFCIS, then.*

$$\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m}} R_i^{SP}(X) \subseteq \underline{R_i}^{SP}(X), i \le m \\
&(2)\ \overline{\sum_{i=1}^{m}} R_i^{SP}(X) \supseteq \overline{R_i}^{SP}(X), i \le m \\
&(3)\ \underline{\sum_{i=1}^{m}} R_i^{SP}(X) = \bigcap_{i=1}^{m} \underline{R_i}^{SP}(X), i \le m \\
&(4)\ \overline{\sum_{i=1}^{m}} R_i^{SP}(X) = \bigcap_{i=1}^{m} \overline{R_i}^{SP}(X), i \le m
\end{aligned} \tag{42}$$

Proof Intuitively, analogous to Proposition 2, we can verify these above conclusions.

Theorem 8 *Assume* $IS = (U, A, V, f)$ *is a PFCIS.* $R_1, R_2, \cdots, R_m$ *are m granular structures,* $0 \le \beta_i < \alpha_i \le 1$, $X, Y \subseteq U$, *there exist.*

(1) $\underline{\sum_{i=1}^{m}} R_i^{SP}(\emptyset) = \overline{\sum_{i=1}^{m}} R_i^{SP}(\emptyset) = \emptyset$,

$\underline{\sum_{i=1}^{m}} R_i^{SP}(U) = \overline{\sum_{i=1}^{m}} R_i^{SP}(U) = U$;

(2) $\underline{\sum_{i=1}^{m}} R_i^{SP}(X) \subseteq \overline{\sum_{i=1}^{m}} R_i^{SP}(X)$;

(3) *If* $\alpha_1 \le \alpha_2, \beta_1 \le \beta_2$, *then* $\underline{\sum_{i=1}^{m}} R_{i\alpha_2}{}^{SP}(X) \subseteq \underline{\sum_{i=1}^{m}} R_{i\alpha_1}{}^{SP}(X)$.

and $\overline{\sum_{i=1}^{m}} R_{i\beta_2}{}^{SP}(X) \subseteq \overline{\sum_{i=1}^{m}} R_{i\beta_1}{}^{SP}(X)$.

Proof These conclusions can be proven based on the intuition provided by Theorem 6.

Similar to the *PMRS*, one can acquire the following Decision Rules.

Decision Rules 5. Assume $IS = (U, A, V, f)$ is a *PFCIS*. When $\alpha_i^{\min} > \beta_i^{\max} (i = 1, 2, \cdots, m)$, we get the following decision rules of the *SPMRS*:

(*SPA*) If $\forall i \in \{1, 2, \cdots, m\}$ such that $k_i(M,N) \ge \alpha_i^{\max}$, then $x, y \in AL^{SP}_{(A,\alpha,\beta)}(X)$;

(*SPC*) If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \le \beta_i^{\min}$, then $x, y \in CO^{SP}_{(A,\alpha,\beta)}(X)$;

(*SPN*) Otherwise, $x, y \in NE^{SP}_{(A,\alpha,\beta)}(X)$.

4.5 Weakly Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 28. Let $IS = (U, A, V, f)$ be a *PFCIS*, the lower approximation $\underline{\sum_{i=1}^{m}} R_i^{P'}(X)$ and upper approximation $\overline{\sum_{i=1}^{m}} R_i^{P'}(X)$ of the Weakly Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (abbreviated as *WPMRS*) are defined respectively in the following formals:

$$\begin{aligned}
&\underline{\sum_{i=1}^{m}} R_i^{P'}(X) = \left\{ y \in U \middle| \bigwedge_{i=1}^{m} k_i(M,N) \ge \alpha_i^{\min} \right\} \\
&\overline{\sum_{i=1}^{m}} R_i^{P'}(X) = U - \left\{ y \in U \middle| \bigvee_{i=1}^{m} k_i(M,N) \le \beta_i^{\max} \right\}
\end{aligned} \tag{43}$$

In this sense, the boundary region of the *WPMRS* is described as:

$$BN^{P'}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m}} R_i^{P'}(X) - \underline{\sum_{i=1}^{m}} R_i^{P'}(X) \tag{44}$$

From Definition 28, one can obtain the following properties of the *WPMRS*.

Theorem 9 *Assume* $IS = (U, A, V, f)$ *is a PFCIS. These properties hold, which can be expressed as follows:*

$$\begin{aligned}
&\underline{\sum_{i=1}^{m}} R_i^{SP}(X) \subseteq \underline{\sum_{i=1}^{m}} R_i^{P'}(X), \\
&\overline{\sum_{i=1}^{m}} R_i^{SP}(X) \subseteq \overline{\sum_{i=1}^{m}} R_i^{P'}(X)
\end{aligned} \tag{45}$$

Proof Similar to Theorem 7, one may easily verify these properties.

In particular, analogous to the *PMRS*, one can also acquire Decision Rules 6 of the *WPMRS*.

Decision Rules 6. Suppose $IS = (U, A, V, f)$ is a *PFCIS*. The decision rules of the *WPMRS* are given as follows:

(WPA) If $\forall i \in \{1, 2, \cdots, m\}$ such that $k_i(M,N) \geq \alpha_i^{\min}$, then $x, y \in AL^{P'}_{(A,\alpha,\beta)}(X)$;

(WPC) If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \leq \beta_i^{\max}$, then $x, y \in CO^{P'}_{(A,\alpha,\beta)}(X)$;

(WPN) Otherwise, $x, y \in NE^{P'}_{(A,\alpha,\beta)}(X)$.

4.6 Optimistic-Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 29 Suppose $IS = (U, A, V, f)$ is a $PFCIS$. The lower approximation $\underline{\sum_{i=1}^{m} R_i}^{OP}(X)$ and upper approximation $\overline{\sum_{i=1}^{m} R_i}^{OP}(X)$ of the Optimistic-Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (for short, $OPMRS$) are described as follows:

$$\underline{\sum_{i=1}^{m} R_i}^{OP}(X) = \left\{ y \in U \mid \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha_i^{\min} \right\}$$

$$\overline{\sum_{i=1}^{m} R_i}^{OP}(X) = U - \left\{ y \in U \mid \bigvee_{i=1}^{m} k_i(M,N) \leq \beta_i^{\min} \right\} \tag{46}$$

Accordingly, the boundary region of the $OPMRS$ is given by:

$$BN^{OP}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i}^{OP}(X) - \underline{\sum_{i=1}^{m} R_i}^{OP}(X) \tag{47}$$

Analogously, from Definition 29, one can obtain the following properties of the $OPMRS$.

Proposition 6 *Let $IS = (U, A, V, f)$ be a $PFCIS$, these following properties hold.*

$$\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m} R_i}^{OP}(X) \supseteq \underline{R_i}^{OP}(X), i \leq m\\
&(2)\ \overline{\sum_{i=1}^{m} R_i}^{OP}(X) \supseteq \overline{R_i}^{OP}(X), i \leq m\\
&(3)\ \underline{\sum_{i=1}^{m} R_i}^{OP}(X) = \bigcup_{i=1}^{m} \underline{R_i}^{OP}(X), i \leq m\\
&(4)\ \overline{\sum_{i=1}^{m} R_i}^{OP}(X) = \bigcap_{i=1}^{m} \overline{R_i}^{OP}(X), i \leq m
\end{aligned} \tag{48}$$

Proof Particularly, analogous to Proposition 2, one can prove easily these above relations.

Theorem 10 *Assume $IS = (U, A, V, f)$ is a $PFCIS$, then.*

$$\begin{aligned}
&(1)\ \underline{\sum_{i=1}^{m} R_i}^{OP}(\varnothing) = \overline{\sum_{i=1}^{m} R_i}^{OP}(\varnothing) = \varnothing,\\
&\underline{\sum_{i=1}^{m} R_i}^{OP}(U) = \overline{\sum_{i=1}^{m} R_i}^{OP}(U) = U\\
&(2)\ \underline{\sum_{i=1}^{m} R_i}^{OP}(X) \subseteq \overline{\sum_{i=1}^{m} R_i}^{OP}(X)\\
&(3)\ \text{If } \alpha_1 < \alpha_2, \beta_1 < \beta_2,\\
&\text{then } \underline{\sum_{i=1}^{m} R_{i\alpha_2}}^{OP}(X) \subseteq \underline{\sum_{i=1}^{m} R_{i\alpha_1}}^{OP}(X) \text{ and}\\
&\overline{\sum_{i=1}^{m} R_{i\beta_2}}^{OP}(X) \subseteq \overline{\sum_{i=1}^{m} R_{i\beta_1}}^{OP}(X).
\end{aligned} \tag{49}$$

Proof Analogous to Theorem 6, one can easily verify these properties.

Theorem 10 implies that increasing the threshold value in each granular structure will result in smaller lower and upper approximations across all granularities.

Consequently, we can gain the decision rules of the $OPMRS$.

Decision Rules 7. Assume that $IS = (U, A, V, f)$ is a $PFCIS$, then the $OPMRS$ has the following decision rules:

(OPA) If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \geq \alpha_i^{\min}$, then $x, y \in AL^{OP}_{(A,\alpha,\beta)}(X)$;

(OPC) If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \leq \beta_i^{\min}$, then $x, y \in CO^{OP}_{(A,\alpha,\beta)}(X)$;

(OPN) Otherwise, $x, y \in NE^{OP}_{(A,\alpha,\beta)}(X)$.

4.7 Weakly Optimistic-Pessimistic Multi-granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 30 Let $IS = (U, A, V, f)$ be a $PFCIS$. The lower approximation $\underline{\sum_{i=1}^{m} R_i}^{OP'}(X)$ and upper approximation $\overline{\sum_{i=1}^{m} R_i}^{OP'}(X)$ of the Weakly Optimistic-Pessimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (namely, $WOPMRS$) are written respectively in the following formals:

$$\underline{\sum_{i=1}^{m} R_i}^{OP'}(X) = \left\{ y \in U | \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha_i^{\max} \right\}$$
$$\overline{\sum_{i=1}^{m} R_i}^{OP'}(X) = U - \left\{ y \in U | \bigvee_{i=1}^{m} k_i(M,N) \leq \beta_i^{\max} \right\} \tag{50}$$

Correspondingly, the boundary region of the *WOPMRS* is described by:

$$BN^{OP'}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i}^{OP'}(X) - \underline{\sum_{i=1}^{m} R_i}^{OP'}(X) \tag{51}$$

From Definition 30, one can acquire the following properties of the *WOPMRS*.

Theorem 11 *Assume* $IS = (U, A, V, f)$ *is a PFCIS. The following properties hold*:

$$\underline{\sum_{i=1}^{m} R_i}^{OP'}(X) \subseteq \underline{\sum_{i=1}^{m} R_i}^{OP}(X),$$
$$\overline{\sum_{i=1}^{m} R_i}^{OP}(X) \subseteq \overline{\sum_{i=1}^{m} R_i}^{OP'}(X) \tag{52}$$

Proof Intuitively, similar to Theorem 7, one can verify these above properties.

Decision rules 8. Suppose $IS = (U, A, V, f)$ is a *PFCIS*. When $\alpha_i^{\min} > \beta_i^{\max} (i = 1, 2, \cdots, m)$. The decision rules of the *WOPMRS* are given as follows:

$(WOPA)$ If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \geq \alpha_i^{\max}$, then $x, y \in AL^{OP'}_{(A,\alpha,\beta)}(X)$;

$(WOPC)$ If $\exists i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \leq \beta_i^{\max}$,then.

$x, y \in CO^{OP'}_{(A,\alpha,\beta)}(X)$;

$(WOPN)$ Otherwise, $x, y \in NE^{OP'}_{(A,\alpha,\beta)}(X)$.

4.8 Pessimistic-Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 31 Suppose $IS = (U, A, V, f)$ is a *PFCIS*. The lower approximation $\underline{\sum_{i=1}^{m} R_i}^{PO}(X)$ and upper approximation $\overline{\sum_{i=1}^{m} R_i}^{PO}(X)$ of the Pessimistic-Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (noted as *POMRS*) are denoted respectively in the following:

$$\underline{\sum_{i=1}^{m} R_i}^{PO}(X) = \left\{ y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \geq \alpha_i^{\max} \right\}$$
$$\overline{\sum_{i=1}^{m} R_i}^{PO}(X) = U - \left\{ y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \leq \beta_i^{\max} \right\} \tag{53}$$

The boundary region of the *POMRS* is described as:

$$BN^{PO}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i}^{PO}(X) - \underline{\sum_{i=1}^{m} R_i}^{PO}(X) \tag{54}$$

Analogously, by Definition 31, one can get the following properties of the *POMRS*.

Proposition 7 *Let* $IS = (U, A, V, f)$ *be a PFCIS, then.*

$$(1)\ \underline{\sum_{i=1}^{m} R_i}^{PO}(X) \subseteq \underline{R_i}^{PO}(X), i \leq m$$
$$(2)\ \overline{\sum_{i=1}^{m} R_i}^{PO}(X) \supseteq \overline{R_i}^{PO}(X), i \leq m$$
$$(3)\ \underline{\sum_{i=1}^{m} R_i}^{PO}(X) = \bigcap_{i=1}^{m} \underline{R_i}^{PO}(X), i \leq m$$
$$(4)\ \overline{\sum_{i=1}^{m} R_i}^{PO}(X) = \bigcup_{i=1}^{m} \overline{R_i}^{PO}(X), i \leq m \tag{55}$$

Proof Analogous to Proposition 2, one can verify these properties.

Theorem 12 *Suppose* $IS = (U, A, V, f)$ *is a PFCIS. These following relations hold.*

$$(1)\ \underline{\sum_{i=1}^{m} R_i}^{PO}(\emptyset) = \overline{\sum_{i=1}^{m} R_i}^{PO}(\emptyset) = \emptyset,$$
$$\underline{\sum_{i=1}^{m} R_i}^{PO}(U) = \overline{\sum_{i=1}^{m} R_i}^{PO}(U) = U$$
$$(2)\ \underline{\sum_{i=1}^{m} R_i}^{PO}(X) \subseteq \overline{\sum_{i=1}^{m} R_i}^{PO}(X)$$
$$(3)\ \text{If } \alpha_1 \leq \alpha_2, \beta_1 \leq \beta_2, \text{then } \underline{\sum_{i=1}^{m} R_{i\alpha_2}}^{PO}(X) \subseteq \underline{\sum_{i=1}^{m} R_{i\alpha_1}}^{PO}(X) \text{ and}$$
$$\overline{\sum_{i=1}^{m} R_{i\beta_2}}^{PO}(X) \subseteq \overline{\sum_{i=1}^{m} R_{i\beta_1}}^{PO}(X) \tag{56}$$

Proof Intuitively, the proof is similar to Theorem 6.

Decision Rules 9. Assume that $IS = (U, A, V, f)$ is a *PFCIS*.

The decision rules of the *POMRS* are given as follows:

(POA) If $\forall i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \geq \alpha_i^{\max}$, then $x, y \in AL^{PO}_{(A,\alpha,\beta)}(X)$;

(POC) If $\forall i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \leq \beta_i^{\max}$, then $x, y \in CO^{PO}_{(A,\alpha,\beta)}(X)$;

(PON) Otherwise, $x, y \in NE^{PO}_{(A,\alpha,\beta)}(X)$.

4.9 Weakly Pessimistic-Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Definition 32 Let $IS = (U, A, V, f)$ be a *PFCIS*, then the lower approximation $\underline{\sum_{i=1}^{m} R_i}^{PO'}(X)$ and upper approximation $\overline{\sum_{i=1}^{m} R_i}^{PO'}(X)$ of the Weakly Pessimistic-Optimistic Multi-Granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set (abbreviated as *WPOMRS*) are denoted respectively by:

$$\underline{\sum_{i=1}^{m} R_i}^{PO'}(X) = \left\{ y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \geq \alpha_i^{\min} \right\}$$

$$\overline{\sum_{i=1}^{m} R_i}^{PO'}(X) = U - \left\{ y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \leq \beta_i^{\min} \right\} \tag{57}$$

In this case, the boundary region of the *WPOMRS* is described as:

$$BN^{PO'}_{\sum_{i=1}^{m} R_i}(X) = \overline{\sum_{i=1}^{m} R_i}^{PO'}(X) - \underline{\sum_{i=1}^{m} R_i}^{PO'}(X) \tag{58}$$

Accordingly, from Definition 32, one can get the following properties of the *WOPMRS*.

Theorem 13 *Assume $IS = (U, A, V, f)$ is a PFCIS. These relations hold, which can be expressed in the following*:

$$\underline{\sum_{i=1}^{m} R_i}^{PO'}(X) \subseteq \underline{\sum_{i=1}^{m} R_i}^{PO}(X) \text{ and } \overline{\sum_{i=1}^{m} R_i}^{PO}(X) \subseteq \overline{\sum_{i=1}^{m} R_i}^{PO'}(X) \tag{59}$$

Proof Intuitively, similar to Theorem 7, one may verify these properties.

Analogously, one can obtain the following Decision Rules.

Table 13 Pythagorean fuzzy conflict information system

U	a_1	a_2	a_3
x_1	(0.8, 0.4)	(0.7, 0.1)	(0.1, 0.8)
x_2	(0.4, 0.6)	(0.2, 0.8)	(0.8, 0.4)
x_3	(0.1, 0.7)	(0.2, 0.8)	(0.6, 0.3)

Table 14 Decision results of the *SOMRS*

$AL^{SO}_{(A,\alpha,\beta)}(U)$	$NE^{SO}_{(A,\alpha,\beta)}(U)$	$CO^{SO}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{x_1, x_2\}$	$\{x_1, x_3\}$

Table 15 Decision results of the *WOMRS*

$AL^{O'}_{(A,\alpha,\beta)}(U)$	$NE^{O'}_{(A,\alpha,\beta)}(U)$	$CO^{O'}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{x_1, x_2\}, \{x_1, x_3\}$	$\{\emptyset\}$

Table 16 Decision results of the *SPMRS*

$AL^{SP}_{(A,\alpha,\beta)}(U)$	$NE^{SP}_{(A,\alpha,\beta)}(U)$	$CO^{SP}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{x_1, x_2\}, \{x_1, x_3\}$	$\{\emptyset\}$

Table 17 Decision results of the *WPMRS*

$AL^{P'}_{(A,\alpha,\beta)}(U)$	$NE^{P'}_{(A,\alpha,\beta)}(U)$	$CO^{P'}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{\emptyset\}$	$\{x_1, x_2\}, \{x_1, x_3\}$

Table 18 Decision results of the *OPMRS*

$AL^{OP}_{(A,\alpha,\beta)}(U)$	$NE^{OP}_{(A,\alpha,\beta)}(U)$	$CO^{OP}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{x_1, x_2\}, \{x_1, x_3\}$	$\{\emptyset\}$

Decision Rules 10. Assume that $IS = (U, A, V, f)$ is a *PFCIS*. The decision rules of the *WPOMRS* are given as follows:

($WPOA$) If $\forall i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \geq \alpha_i^{\min}$, then $x, y \in AL^{PO'}_{(A,\alpha,\beta)}(X)$;

($WPOC$) If $\forall i \in \{1, 2 \cdots, m\}$ such that $k_i(M,N) \leq \beta_i^{\min}$, then $x, y \in CO^{PO'}_{(A,\alpha,\beta)}(X)$;

($WPON$) Otherwise, $x, y \in NE^{PO'}_{(A,\alpha,\beta)}(X)$.

4.10 Three-Way Conflict Analysis Based on Multi-granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

Three-way decision theory offers an important research idea for solving uncertain issues. In order to further explore the conflict mechanism of agent in different decision-making cases, this subsection discusses the construction of three-way conflict analysis model of multi-granulation Pythagorean fuzzy rough approximations.

Definition 33 Assume $IS = (U, A, V, f)$ is a $PFCIS$, $R_1, R_2, \cdots, R_m$ are m granular structures, then the global conflict is defined as:

$$S(A) = \sum_{i=1}^{m} con(A_i) \tag{60}$$

where $A_i(i = 1, 2, \cdots, m)$ and $con(A_i)$ are the issue set and conflict degree under R_i, respectively. Formula (60) shows that the conflict degree under each granular structure constitutes the global conflict degree, and provides a precondition for describing global conflict, global moderation, and no-global conflict.

Table 20 Decision results of the *POMRS*

$AL^{PO}_{(A,\alpha,\beta)}(U)$	$NE^{PO}_{(A,\alpha,\beta)}(U)$	$CO^{PO}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{x_1, x_2\}$	$\{x_1, x_3\}$

Definition 34 Suppose $IS = (U, A, V, f)$ is a $PFCIS$. Given a pair of thresholds (c_*, c^*), and global conflict, global moderation, and no-global conflict are defined respectively as:

$$\begin{aligned} &GS_A(U) = \{A | S(A) \geq c^*\} \\ &GM_A(U) = \left\{A | c_* < S(A) < c^*\right\} \\ &GN_A(U) = \left\{A | S(A) \leq c_*\right\} \end{aligned} \tag{61}$$

In particular, based on Definitions 20, 21, 33 and 34, we can know that if $S(A) < c^*$ and $con(A_i) \geq h_i$, then this situation is a local condition conflict.

Definition 35 Let $IS = (U, A, V, f)$ be a $PFCIS$, then the global strong conflict, the global weak conflict, the global strong alliance, the global weak alliance, the global strong neutral, the global weak neutral, the local strong conflict, the local weak conflict, the local strong alliance, the local weak alliance, the local strong neutral and the local weak neutral are defined respectively as:

$$\begin{aligned}
&(1)\, GSC_A(x) = \left\{y \in U | k(M,N) \leq \beta^{\min}, S(A) \geq c^*\right\} \\
&(2)\, GWC_A(x) = \{y \in U | k(M,N) \leq \beta^{\max}, S(A) \geq c^*\} \\
&(3)\, GSA_A(x) = \left\{y \in U | k(M,N) \geq \alpha^{\max}, S(A) \leq c_*\right\} \\
&(4)\, GWA_A(x) = \left\{y \in U | k(M,N) \geq \alpha^{\min}, S(A) \leq c_*\right\} \\
&(5)\, GSN_A(x) = \left\{y \in U | \beta^{\max} < k(M,N) < \alpha^{\min}, c_* < S(A) < c^*\right\} \\
&(6)\, GWN_A(x) = \left\{y \in U | \beta^{\min} < k(M,N) < \alpha^{\max}, c_* < S(A) < c^*\right\} \\
&(7)\, LSC_{A_i}(x) = \left\{y \in U | k_i(M,N) \leq \beta_i^{\min}, S(A) < c^*, con(A_i) \geq h_i\right\} \\
&(8)\, LWC_{A_i}(x) = \left\{y \in U | k_i(M,N) \leq \beta_i^{\max}, S(A) < c^*, con(A_i) \geq h_i\right\} \\
&(9)\, LSA_{A_i}(x) = \left\{y \in U | k_i(M,N) \geq \alpha_i^{\max}, S(A) < c^*, con(A_i) \leq l_i\right\} \\
&(10)\, LWA_{A_i}(x) = \left\{y \in U | k_i(M,N) \geq \alpha_i^{\min}, S(A) < c^*, con(A_i) \leq l_i\right\} \\
&(11)\, LSN_{A_i}(x) = \{y \in U | \beta_i^{\max} < k_i(M,N) < \alpha_i^{\min}, S(A) < c^*, l_i < con(A_i) < h_i\} \\
&(12)\, LWN_{A_i}(x) = \{y \in U | \beta_i^{\min} < k_i(M,N) < \alpha_i^{\max}, S(A) < c^*, l_i < con(A_i) < h_i\}
\end{aligned} \tag{62}$$

Table 19 Decision results of the *WOPMRS*

$AL^{OP'}_{(A,\alpha,\beta)}(U)$	$NE^{OP'}_{(A,\alpha,\beta)}(U)$	$CO^{OP'}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{\varnothing\}$	$\{x_1, x_2\}, \{x_1, x_3\}$

where α_i and β_i are a pair of thresholds under R_i, l_i and h_i are a pair of conflict degree thresholds under R_i, c_* and c^* are given as global conflict thresholds.

In particular, with formulas (61) and (62), one can acquire the following properties of the Multi-granulation Pythagorean fuzzy conflict decision-theoretic rough set.

Table 21 Decision results of the *WPOMRS*

$AL^{PO'}_{(A,\alpha,\beta)}(U)$	$NE^{PO'}_{(A,\alpha,\beta)}(U)$	$CO^{PO'}_{(A,\alpha,\beta)}(U)$
$\{x_2, x_3\}$	$\{x_1, x_2\}, \{x_1, x_3\}$	$\{\varnothing\}$

Theorem 14. *Assume $IS = (U, A, V, f)$ is a PFCIS. Given a pair of thresholds (c_*, c^*), then these relations hold as follows.*

$$\begin{aligned}
&(1)\ \text{If}\ x, y \in GSC_A(x),\ \text{then}\ x, y \in GWC_A(x);\\
&(2)\ \text{If}\ x, y \in GSA_A(x),\ \text{then}\ x, y \in GWA_A(x);\\
&(3)\ \text{If}\ x, y \in GSN_A(x),\ \text{then}\ x, y \in GWN_A(x);\\
&(4)\ \text{If}\ x, y \in LSC_{A_i}(x),\ \text{then}\ x, y \in LWC_{A_i}(x);\\
&(5)\ \text{If}\ x, y \in LSA_{A_i}(x),\ \text{then}\ x, y \in LWA_{A_i}(x);\\
&(6)\ \text{If}\ x, y \in LSN_{A_i}(x),\ \text{then}\ x, y \in LWN_{A_i}(x).
\end{aligned} \tag{63}$$

Proof Intuitively, from formula (62) and basic properties of set, the above conclusions can be easily proved.

In order to gain a deeper understanding of the rough set models above-mentioned in this article, we illustrate the process of acquiring decision rules for each rough set model through the Example 6.

Example 6 A *PFCIS* is shown in Table 13.

Assume $m = 2$, $R_1 = \{a_1, a_2\}$, $R_2 = \{a_1, a_2, a_3\}$, decision loss functions are PFNs, and $\eta = 0.5$. The lower and upper bounds of six decision loss functions for R_1 are given as:

$A^-_{AP_1}(\eta) = 2.8$, $A^+_{AP_1}(\eta) = 3.1$, $A^-_{NP_1}(\eta) = 4.4$, $A^+_{NP_1}(\eta) = 4.7$,

$A^-_{CP_1}(\eta) = 9.0$, $A^+_{CP_1}(\eta) = 10.0$, $A^-_{CN_1}(\eta) = 2.0$, $A^+_{CN_1}(\eta) = 2.6$, $A^-_{NN_1}(\eta) = 4.8$, $A^+_{NN_1}(\eta) = 5.0$, $A^-_{AN_1}(\eta) = 9.6$, $A^+_{AN_1}(\eta) = 9.8$.

According to Definition 18, there exist $\alpha_1 \in [0.67, 0.81]$, $\beta_1 \in [0.26, 0.46]$.

The lower and upper bounds of six decision loss functions for R_2 are defined as:

$A^-_{AP_2}(\eta) = 2.0$, $A^+_{AP_2}(\eta) = 3.0$, $A^-_{NP_2}(\eta) = 4.4$, $A^+_{NP_2}(\eta) = 4.8$, $A^-_{CP_2}(\eta) = 9.6$, $A^+_{CP_2}(\eta) = 10.2$, $A^-_{CN_2}(\eta) = 1.7$, $A^+_{CN_2}(\eta) = 2.6$,

$A^-_{NN_2}(\eta) = 4.7$, $A^+_{NN_2}(\eta) = 5.0$, $A^-_{AN_2}(\eta) = 9.8$, $A^+_{AN_2}(\eta) = 10.0$.

According to Definition 18, there exist $\alpha_2 \in [0.56, 0.85]$, $\beta_2 \in [0.23, 0.48]$.

Suppose the attitudes of x_1, x_2 and x_3 on the issue set A are B, C and D, respectively. According to Definition 13, we have $k_1(B, C) = 0.47$, $k_1(B, D) = 0.38$, $k_1(C, D) = 0.98$, $k_2(B, C) = 0.43$, $k_2(B, D) = 0.42$, and $k_2(C, D) = 0.91$. Due to Decision Rules 3–10, decision results are expressed in Table 14, 15, 16, 17, 18, 19, 20, 21.

Given $m^* = 0.8$, $m_* = 0.5$, according to Definition 20, we have $con(A_1) = 0.5$, $con(A_2) = 0.5$. Thus, global conflict degree $S(A) = 1$. Given $c_* = 0.4$, $c^* = 0.8$, due to $S(A) > c^*$, therefore, it is global conflict. Decision thresholds $\alpha = [0.7, 0.8]$, $\beta = [0.35, 0.45]$ are given under the global conflict and decision thresholds $l_1 = 0.3$, $h_1 = 0.6$ under R_1 and decision thresholds $l_2 = 0.4$, $h_2 = 0.8$ under R_2 are given. Due to $con(A_1) < h_1$, $con(A_2) < h_2$, thus, no conflict in local situation. According to Definition 35, global strong conflict and global weak conflict are $\{\varnothing\}$ and $\{x_1, x_2, x_3\}$, respectively.

5 Measures in Multi-granulation Pythagorean Fuzzy Conflict Decision-Theoretic Rough Set

The existence of boundary region of rough set results in uncertainty of target set. In order to measure this uncertainty, we develop accuracy measure methods of the models presented in this article by referring to the method in [42].

Definition 36 Assume $IS = (U, A, V, f)$ is a *PFCIS*, $R_1, R_2, \cdots, R_m$ are m granular structures, for any $X \subseteq U$, the accuracy measure of X in *OPMRS* is described as:

$$\sigma_{OP}(X) = \frac{\underline{\underline{\sum_{i=1}^{m} R_i^{OP}(X)}}}{\overline{\sum_{i=1}^{m} R_i^{OP}(X)}} \tag{64}$$

Definition 37 Suppose $IS = (U, A, V, f)$ is a *PFCIS*. The accuracy measure of X in *WOPMRS* is given by:

$$\sigma_{OP'}(X) = \frac{\underline{\underline{\sum_{i=1}^{m} R_i^{OP'}(X)}}}{\overline{\sum_{i=1}^{m} R_i^{OP'}(X)}} \tag{65}$$

Accordingly, from the formulas (64) and (65), one can obtain the following properties.

Theorem 15 *Assume* $IS = (U, A, V, f)$ *is a PFCIS, then.*

$$\sigma_{OP}(X) \leq \sigma_{OP'}(X) \tag{66}$$

Proof Intuitively, from Definition 29 and Definition 30, there exist.

$\underline{\sum_{i=1}^{m} R_i^{OP'}(X)} \supseteq \underline{\sum_{i=1}^{m} R_i^{OP}(X)}$, $\underline{\sum_{i=1}^{m} R_i^{PO'}(X)} \subseteq \overline{\sum_{i=1}^{m} R_i^{OP}(X)}$,

then $\frac{\underline{\sum_{i=1}^{m} R_i^{OP}(X)}}{\overline{\sum_{i=1}^{m} R_i^{OP}(X)}} \leq \frac{\underline{\sum_{i=1}^{m} R_i^{OP'}(X)}}{\overline{\sum_{i=1}^{m} R_i^{OP'}(X)}}$.

Thus, $\sigma_{OP}(X) \leq \sigma_{OP'}(X)$.

More especially, Theorem 15 reveals that the accuracy measure of a set increases as the thresholds increases.

Definition 38 Suppose $IS = (U, A, V, f)$ is a *PFCIS*. For any $X \subseteq U$, the accuracy measure of X in *POMRS* is described as:

$$\sigma_{PO}(X) = \frac{\underline{\sum_{i=1}^{m} R_i^{PO}(X)}}{\overline{\sum_{i=1}^{m} R_i^{PO}(X)}} \tag{67}$$

Definition 39 Assume $IS = (U, A, V, f)$ is a *PFCIS*. For any $X \subseteq U$, the accuracy measure of X in *WPOMRS* is given by:

$$\sigma_{PO'}(X) = \frac{\underline{\sum_{i=1}^{m} R_i^{PO'}(X)}}{\overline{\sum_{i=1}^{m} R_i^{PO'}(X)}} \tag{68}$$

According to formulas (67) and (68), one can gain the following important conclusions.

Theorem 16 *Assume* $IS = (U, A, V, f)$ *is a PFCIS. The following relations hold.*

$$\sigma_{PO'}(X) \leq \sigma_{PO}(X) \tag{69}$$

Proof From Definition 31 and Definition 32, there exist
$\underline{\sum_{i=1}^{m} R_i^{PO}(X)} \supseteq \underline{\sum_{i=1}^{m} R_i^{PO'}(X)}$, $\overline{\sum_{i=1}^{m} R_i^{PO}(X)} \subseteq \overline{\sum_{i=1}^{m} R_i^{PO'}(X)}$,

then $\frac{\underline{\sum_{i=1}^{m} R_i^{PO}(X)}}{\overline{\sum_{i=1}^{m} R_i^{PO}(X)}} \geq \frac{\underline{\sum_{i=1}^{m} R_i^{PO'}(X)}}{\overline{\sum_{i=1}^{m} R_i^{PO'}(X)}}$.

Thus, $\sigma_{PO'}(X) \leq \sigma_{PO}(X)$.

In particular, Theorem 16 further reveals that the accuracy measure of a set increases with the increase of the thresholds.

6 Conclusions and Future Work

In order to describe more specifically and accurately the attitudes of the agents towards the issue set, the Pythagorean fuzzy numbers are used to express the attitudes of agents on issues, and the agents are trisected with a pair of thresholds by combining Bayes risk theory. In particular, a three-way conflict analysis model building on multi-granulation Pythagorean fuzzy rough approximations is also established by combining conflict analysis with multi-granulation rough approximations, which produces a novel model of trisecting agents set in high-dimensional data.

In this paper, we establish Pythagorean fuzzy three-way conflict models from a theoretical perspective. In future work, we will explore the improvement of our conflict model by incorporating other correlation coefficients, like the Pythagorean fuzzy correlation coefficient proposed by Thao [45]. It is important to note that conflict analysis is closely linked to the process of consensus building in group decision making. Methods such as the threshold-based value-driven method [46] and the method based on stochastic multi-criteria acceptability analysis [47] will serve as valuable references for studying the consensus strategy conflict model. Furthermore, our objective is to thoroughly investigate the multi-scale decision-making problem within the conflict information system, aiming to achieve optimal decisions aligned with the interests of agents under the influence of multiple factors. On the other hand, from an application perspective, we will apply conflict analysis model in industrial scenarios and other fields to address some specific applications, such as the conflict issues arising from group decision-making in the context of Shipping Industry 4.0 [48].

In particular, it is worth mentioning that in decision-making, decision actions of agents are often affected by various factors, including the uncertain degree of agents, personal preferences and linguistic information. Notable research results include using some special Sugeno-like operators to handle the involved preference and uncertainty in both input vector and fuzzy measures [49], as well as a proportional interval T2 hesitant fuzzy TOPSIS approach to address language decision making under uncertainty [50]. These findings will also serve as important references for our future work on constructing a general conflict model.

Appendix

1. Proof of Theorem 3

Proof (A) $R(a_A|x,y) \le R(a_N|x,y)$

$\Leftrightarrow \lambda_{AP}k(M,N) + \lambda_{AN}(1-k(M,N)) \le \lambda_{NP}k(M,N) + \lambda_{NN}(1-k(M,N))$.

Thus, $k(M,N) \ge \frac{\lambda_{AN}-\lambda_{NN}}{(\lambda_{AN}-\lambda_{NN})+(\lambda_{NP}-\lambda_{AP})}$.

$R(a_A|x,y) \le R(a_C|x,y)$

$\Leftrightarrow \lambda_{AP}k(M,N) + \lambda_{AN}(1-k(M,N)) \le \lambda_{CP}k(M,N) + \lambda_{CN}(1-k(M,N))$. Therefore, $k(M,N) \ge \frac{\lambda_{AN}-\lambda_{CN}}{(\lambda_{AN}-\lambda_{CN})+(\lambda_{CP}-\lambda_{AP})}$.

(N) $R(a_N|x,y) \le R(a_A|x,y)$

$\Leftrightarrow \lambda_{NP}k(M,N) + \lambda_{NN}(1-k(M,N)) \le \lambda_{AP}k(M,N) + \lambda_{AN}(1-k(M,N))$

$R(a_N|x,y) \le R(a_C|x,y)$

$\Leftrightarrow \lambda_{NP}k(M,N) + \lambda_{NN}(1-k(M,N)) \le \lambda_{CP}k(M,N) + \lambda_{CN}(1-k(M,N))$

Then, there exists

$$\frac{\lambda_{NN}-\lambda_{CN}}{(\lambda_{NN}-\lambda_{CN})+(\lambda_{CP}-\lambda_{NP})} \le k(M,N) \le \frac{\lambda_{AN}-\lambda_{NN}}{(\lambda_{AN}-\lambda_{NN})+(\lambda_{NP}-\lambda_{AP})}$$

(C) $R(a_C|x,y) \le R(a_A|x,y)$

$\Leftrightarrow \lambda_{CP}k(M,N) + \lambda_{CN}(1-k(M,N)) \le \lambda_{AP}k(M,N) + \lambda_{AN}(1-k(M,N))$

Thus, $k(M,N) \le \frac{\lambda_{AN}-\lambda_{CN}}{(\lambda_{AN}-\lambda_{CN})+(\lambda_{CP}-\lambda_{AP})}$.

$R(a_C|x,y) \le R(a_N|x,y)$

$\Leftrightarrow \lambda_{CP}k(M,N) + \lambda_{CN}(1-k(M,N)) \le \lambda_{NP}k(M,N) + \lambda_{NN}(1-k(M,N))$

Hence, $k(M,N) \le \frac{\lambda_{NN}-\lambda_{CN}}{(\lambda_{NN}-\lambda_{CN})+(\lambda_{CP}-\lambda_{NP})}$,

where

$$\alpha = \frac{\lambda_{AN}-\lambda_{NN}}{(\lambda_{AN}-\lambda_{NN})+(\lambda_{NP}-\lambda_{AP})}$$

$$\beta = \frac{\lambda_{AN}-\lambda_{CN}}{(\lambda_{AN}-\lambda_{CN})+(\lambda_{CP}-\lambda_{AP})}$$

$$\gamma = \frac{\lambda_{NN}-\lambda_{CN}}{(\lambda_{NN}-\lambda_{CN})+(\lambda_{CP}-\lambda_{NP})}$$

2. Proof of Proposition 2

Proof For any $x \in \underline{R_i}^O(X)$,

$x \in \underline{R_1}^O(X) \vee \underline{R_2}^O(X) \vee \cdots \vee \underline{R_m}^O(X)$, then

$x \in \{y \in U | k_1(M,N) \ge \alpha\} \vee \{y \in U | k_2(M,N) \ge \alpha\} \vee \cdots \vee \{y \in U | k_m(M,N) \ge \alpha\}$

Equivalently,

$x \in \{y \in U | k_1(M,N) \ge \alpha \vee k_2(M,N) \ge \alpha \vee \cdots \vee k_m(M,N) \ge \alpha\}$.

Thus,$x \in \underline{\sum_{i=1}^{m} R_i}^O(X)$.

(2) For any $x \in \overline{R_i}^O(X)$, then.

$x \in \overline{R_1}^O(X) \vee \overline{R_2}^O(X) \vee \cdots \vee \overline{R_m}^O(X)$ and then we have.

$x \in U - \{y \in U | k_1(M,N) \le \beta\} \vee x \in U - \{y \in U | k_2(M,N) \le \beta\} \vee \cdots \vee x \in U - \{y \in U | k_m(M,N) \le \beta\}$.

Equivalently,

$x \in \{y \in U | k_1(M,N) > \beta\} \wedge x \in \{y \in U | k_2(M,N) > \beta\} \wedge \cdots \wedge x \in \{y \in U | k_m(M,N) > \beta\}$.

Hence, $x \in \overline{\sum_{i=1}^{m} R_i}^O(X)$.

(3) For $\forall x \in \underline{\sum_{i=1}^{m} R_i}^O(X), i = 1, 2, \cdots, m$

$\Leftrightarrow x \in \{y \in U | k_1(M,N) \ge \alpha \vee \cdots \vee k_m(M,N) \ge \alpha\}$

$\Leftrightarrow x \in \{y \in U | k_1(M,N) \ge \alpha\} \vee \cdots \vee x \in \{y \in U | k_m(M,N) \ge \alpha\}$

$\Leftrightarrow x \in \underline{R_1}^O(X) \vee x \in \underline{R_2}^O(X) \vee \cdots \vee x \in \underline{R_m}^O(X)$

$\Leftrightarrow x \in \bigcup_{i=1}^{m} \underline{R_i}^O(X)$.

(4) For $\forall x \in \overline{\sum_{i=1}^{m} R_i}^O(X)$

$\Leftrightarrow x \in U - \left\{y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \le \beta\right\}$

$\Leftrightarrow x \in U - \{y \in U | k_1(M,N) \le \beta \wedge k_2(M,N) \le \beta \wedge \cdots \wedge k_m(M,N) \le \beta\}$

$\Leftrightarrow x \in \{y \in U | k_1(M,N) > \beta \vee k_2(M,N) > \beta \vee \cdots \vee k_m(M,N) > \beta\}$

$\Leftrightarrow x \in \{y \in U | k_1(M,N) > \beta\} \vee x \in \{y \in U | k_2(M,N) > \beta\} \vee \cdots \vee x \in \{y \in U | k_m(M,N) > \beta\}$

$\Leftrightarrow x \in \bigcup_{i=1}^{m} \overline{R_i}^O(X)$.

3. Proof of Theorem 6

Proof (1) For any $x \in \underline{\sum_{i=1}^{m} R_i}^{SO}(X)$, by Definition 24, there exist.

$x \in \left\{y \in U | k_1(M,N) \geq \alpha_1^{\min}\right\}$

$\vee x \in \left\{y \in U | k_2(M,N) \geq \alpha_2^{\min}\right\}$

$\vee \cdots \vee x \in \left\{y \in U | k_m(M,N) \geq \alpha_m^{\min}\right\}$.
Equivalently,

$x \in \{y \in U | k_1(M,N) \geq \alpha_1^{\min} \vee k_2(M,N) \geq \alpha_2^{\min}$

$\vee \cdots \vee k_m(M,N) \geq \alpha_m^{\min}\}$.
Then we have

$x \in \{y \in U | k_1(M,N) \geq \beta_1^{\min} \vee k_2(M,N) \geq \beta_2^{\min}$

$\vee \cdots \vee k_m(M,N) \geq \beta_m^{\min}\}$.
Obviously,

$x \in U - \left\{y \in U | k_1(M,N) \leq \beta_1^{\max}\right\}$

$\vee x \in U - \left\{y \in U | k_2(M,N) \leq \beta_2^{\max}\right\}$

$\vee \cdots \vee x \in U - \left\{y \in U | k_m(M,N) \leq \beta_m^{\max}\right\}$.

Hence, $x \in \overline{\sum_{i=1}^{m} R_i}^{SO}(X)$.
(2) (2a) From (1), there exist.
$\underline{\sum_{i=1}^{m} R_i}^{SO}(X) \subseteq \emptyset$ and $\emptyset \subseteq \underline{\sum_{i=1}^{m} R_i}^{SO}(X)$.

Hence, $\underline{\sum_{i=1}^{m} R_i}^{SO}(X) = \emptyset$.

(2b) We try to disproof it. Assume $\overline{\sum_{i=1}^{m} R_i}^{SO}(X) \neq \emptyset$, then, $\exists x \in \overline{\sum_{i=1}^{m} R_i}^{SO}(\emptyset)$. Hence, $\left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) > \beta_i^{\min}\right\} \cap \emptyset \neq \emptyset$.
Therefore, the assumption is not true.

Thus, $\overline{\sum_{i=1}^{m} R_i}^{SO}(X) = \emptyset$.
(3) (3a) If $\alpha_1 < \alpha_2$ and $x \in \underline{\sum_{i=1}^{m} R_{i\alpha_2}}^{SO}(X)$, then there exists

$x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha_{i2}^{\min}\right\}$, and then.

$x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha_{i1}^{\min}\right\}$.

Thus, $\underline{\sum_{i=1}^{m} R_{i\alpha_2}}^{SO}(X) \subseteq \underline{\sum_{i=1}^{m} R_{i\alpha_1}}^{SO}(X)$.

(3b) If $\beta_1 \leq \beta_2$ and $x \in \overline{\sum_{i=1}^{m} R_{i\beta_2}}^{SO}(X)$, there exists.

$x \in U - \left\{y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \leq \beta_i^{\max}\right\}$, and then.

$x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) > \beta_{i2}^{\min}\right\}$.

Thus, $x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) \geq \beta_{i1}^{\min}\right\}$.

Hence, $\overline{\sum_{i=1}^{m} R_{i\beta_2}}^{SO}(X) \subseteq \overline{\sum_{i=1}^{m} R_{i\beta_1}}^{SO}(X)$.

4. Proof of Theorem 7

Proof (a) For any $x \in \underline{\sum_{i=1}^{m} R_i}^{O'}(X)$, there exists.

$x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha_i^{\max}\right\}$, then.

$x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) \geq \alpha_i^{\min}\right\}$.

Hence, $\underline{\sum_{i=1}^{m} R_i}^{O'}(X) \subseteq \underline{\sum_{i=1}^{m} R_i}^{SO}(X)$.

(b) For any $x \in \overline{\sum_{i=1}^{m} R_i}^{O'}(X)$, then.

$x \in U - \left\{y \in U | \bigwedge_{i=1}^{m} k_i(M,N) \leq \beta_i^{\min}\right\}$, then.

$x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) > \beta_i^{\max}\right\}$,

and then, $x \in \left\{y \in U | \bigvee_{i=1}^{m} k_i(M,N) > \beta_i^{\min}\right\}$.

Thus, $x \in \overline{\sum_{i=1}^{m} R_i}^{SO}(X)$.

Hence, $\overline{\sum_{i=1}^{m} R_i}^{O'}(X) \subseteq \overline{\sum_{i=1}^{m} R_i}^{SO}(X)$.

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Data Availability The data used in this paper are reasonable to the conclusion.

Declarations

Conflict of Interest The authors declare that they have no conflicts of interest.

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