Hierarchical Sequential Three-Way Multi-Attribute Decision-Making Method Based on Regret Theory in Multi-Scale Fuzzy Decision Systems

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***Abstract*—Most of the existing multi-attribute decision-making models under multi-scale decision information systems are established by selecting the optimal scale or fusing multi-scale information into a single scale. These models will lose part of the decision information, resulting in inaccurate decision results. However, sequential three-way decision can not only process information hierarchically, but also provide delayed decision between acceptance and rejection. In addition, the irrational behavior of decision-makers will have a certain impact on the decision-making results. To this end, for multi-scale and diversity decision-making problems, this paper proposes a hierarchical sequential three-way multi-attribute decision-making method based on regret theory. Specifically, to represent this diversity, the multi-scale evaluation information table is converted into a digital evaluation value table through a fuzzy membership function. Second, based on the regret-rejoicing function of regret theory, the regret-rejoicing relation of alternatives in multi-scale information systems is established, which can be used to calculate conditional probability. Third, the relative loss functions based on regret theory are proposed by considering the psychological behaviors of decision-makers. Finally, the hierarchical sequential three-way multi-attribute decision-making method for solving the multi-scale decision-making problem is proposed. The stability and effectiveness of the proposed method are verified by the corresponding experiments and the comparative analysis of practical cases. The proposed method solves the fusion problem of multi-scale decision information and obtains flexible ranking results according to the risk factor.**

***Index Terms*—hierarchical sequential three-way decision, multi-scale fuzzy decision systems, regret-rejoicing function, regret theory, multi-attribute decision-making.**

I. Introduction

MULTI-attribute decision-making (MADM) is a process of integrated evaluation and decision-making by using appropriate methods or models and considering multiple evaluation criteria when facing multiple possible alternatives. With the advent of the era of big data, the decision environment and decision data have become more complex, diverse, and uncertain, and the traditional single-scale decision information systems (SSDISs) are difficult to meet the requirements of the modern decision environment. In this context, multi-scale decision information systems (MSDISs) emerge as necessary, storing extensive and precise data and thereby offering valuable support for decision-making. They have become an essential component of modern decision-making science.

For multi-scale decision problem, two core issues need to be addressed. One is how to represent and process this complex multi-scale information quantitatively, and the other is how to aggregate multi-scale decision results into a whole, including classification results and ranking results. In addition, uncertainty and fuzziness run through the whole process of MADM, because the decision result is easily influenced by subjective factors, such as the knowledge structure, experience level and personal preference of the decision-maker. Thus, the complicity and uncertainty of MADM problems is still a challenging task.

In a MSDIS, each object can take on different tagged values under the same attribute depending on different scales or granularity of the observation, and there is a full projection of the granularity transformation from the fine-scale attribute value domain to the coarse-scale attribute value domain. Multi-scale decision tables are currently handled by two main methods: (1) selecting the optimal combination of scales, and (2) fusing multiple scales into a single scale. In recent years, these two methods have become the main research direction for multi-scale data analysis, and numerous research results have been achieved. For example, Wu and Leung [1] investigated the problem of optimal scale selection for various requirements in multi-scale decision tables from the perspective of granular computing. Chen et al. [2], [3] studied the optimal scale selection problem based on sequential three-way decision (S3WD) in dynamic MSDISs, which solves the problem of more difficult selection of the optimal scale due to the dynamic growth of data. Zhang et al. [4] investigated the optimal scale selection problem with the reduced cost and uncertainty under S3WD theory by considering cost sensitivity. On the other hand, Deng et al. [5] fused multiple scales into a single scale by assigning different weights to different scales and performing weighted aggregation, then combined it with three-

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way decision (3WD) theory to conduct the MADM study.

Both of these approaches will cause a certain degree of loss of original information during processing, and the chosen multi-scale fusion and optimal scale selection are different, and the ranking result and the best solution may also be different. Therefore, this paper adopts a new method, the hierarchical sequential three-way decision (HS3WD) method, to ensure that multi-scale information can participate in decision-making, rather than be part of the decision-making.

3WD [6], [7] based on the decision-theoretic rough sets theoretical model and further combined with decision scenarios. The basic idea is to divide the whole into three disjoint regions and to adopt different decision-making behaviors or partitioning strategies for different regions, thus liberating the decision outcome from the two-sidedness of the traditional decision-making problem. 3WD has been successfully applied to a variety of fields, such as data mining [8], medical decision [9], conflict analysis [10]. Considering the cost of the decision process and decision outcome in real life, Yao and Deng [11] proposed a S3WD model from the multi-granulation perspective, in which the more important sets of conditional attributes are sequentially selected to form a multi-granularity space for multi-step decision. Because its decision-making strategy is close to human thinking patterns, it has attracted many scholars to study extended models of S3WD in different scenarios from different perspectives. Qian et al. [12] argued that a more reasonable decision may be obtained by using a multi-view granular structure for solving complex problems, and proposed a multi-granularity S3WD. Later, Qian et al. [13], [14] generalized the concept of conditional attributes based on the concept hierarchy tree, designed a multi-level S3WD model with multiple levels of granularity, and then proposed a generalized hierarchical multi-level S3WD model based on hierarchical decision attributes. Besides, there are many studies on 3WD in fuzzy environments [15]–[17].

Classical MADM methods [18]–[22] only provide results for ranking objects. However, many real-life decision-making scenarios require objects to be classified, and the three divisions idea of 3WD allows objects to be classified into three mutually disjoint regions. In recent years, many scholars have extended and improved the 3WD theory and introduced it into MADM [23]–[26]. All the above studies are rational decisions under the premise of perfect decision-making methods. However, in the actual decision-making process, decision-makers are often not perfectly rational but finitely rational, because they have different mental states when facing risks.

Two important scientific theories describe the psychological behavior of decision-makers, prospect theory [27] and regret theory [28], both of which are theories that take into account other factors in decision-making rather than just the expected benefits. Based on the different psychological behaviors of decision-makers when making decisions, current research on psychological-behavior decision-making can be broadly classified into three categories: behavior decision-making based on regret theory [29]–[31], behavior decision-making based on prospect theory [32], and behavior decision-making based on prospect-regret theory [33]. Huang and Zhan [30] proposed a new method for calculating conditional probability and relative utility functions by defining a dominance relation based on the regret-rejoicing value of objects under each attribute. Deng et al. [31] combined the generalized 3WD with regret theory under incomplete multi-scale information systems to remedy the existing MADM from the perspective of utility. Huang et al. [33] fully considered the loss and utility in the decision-making process, and proposed a three-way classification ranking method to solve multi-scale information systems problems. Zhan et al. [34] and Deng et al. [36] studied a three-way multi-attribute decision-making model based on triangular fuzzy number and interval fuzzy number respectively under incomplete multi-scale information systems. Xiao et al. [38], [39] proposed a new 3WD method and a group consensus method under intuitionistic fuzzy number and interval multi-scale data respectively.

According to the above analysis, most of the existing multi-attribute decision models [30]–[37] in multi-scale decision information systems are established by selecting the optimal scale or fusing multi-scale information into a single scale. These models will lose some decision information, resulting in inaccurate decision results. One of the basic assumptions of traditional decision models is that decision-makers are completely rational, that is, they follow the principle of maximizing benefits. In fact, people's decisions are influenced by their emotions, and it is difficult for decision-makers to remain completely rational. However, using existing methods can not effectively solve similar problems mentioned above. For multi-scale data, we can use the developed 3WD method to complete the single-scale decision first, to obtain the ranking and classification results for each scale. Therefore, how to summarize the ranking and classification results of all scales becomes a problem.

Although there is a large amount of 3WD research based on behavioral decision theory, most 3WD models and methods [40]–[42] are proposed based on single-scale information systems in complete environments. Considering that the information of decision problems in real life may be multi-scale and diverse, the sequential three-way decision paradigm can solve the problem of information loss in multi-scale decision-making. However, from existing 3WD research, we can see that there are two main components: loss function and conditional probability. At present, there are also some deficiencies, either considering behavioral decisions in the loss function or considering behavioral decisions in conditional probability, lacking the case of considering both parts simultaneously.

To sum up, we summarize the motivation of this study as follows. On the one hand, in the existing MADM studies, new features such as multi-scale and diversity of data are not considered at the same time. In order to adapt to realistic decision scenarios, we need to develop new methods that match the actual data features. On the other hand, in order to overcome the above shortcomings of the existing MADM method, it is necessary to explore decision theory and methods for solving multi-scale decision problems.

Based on the above motives and considerations, a new hierarchical sequential three-way multi-attribute decision method based on regret theory is proposed to deal with a class of multi-scale and diversity MADM problems. Firstly, we introduce a fuzzy membership function to convert multi-scale and digital information, transforming the multi-scale evaluation information table into a digital evaluation value table. Secondly, we utilize the regret-rejoicing function to establish a regret-rejoicing relationship, and use the loss function and conditional probability to obtain the classification and ranking results of all objects. Thirdly, we propose a new fusion method for ranking results, which takes into account the decision-makers' tolerance for risk, to achieve optimal ranking and classification results. Finally, we prove the feasibility, validity and stability of this method through empirical analysis of application cases.

The main contributions of our study are concluded as follows.

(1) In this paper, a regret-rejoicing dominance relationship is proposed from the psychological behavioral perspective of the decision-maker, which can be effectively differentiated according to the dominance of the object. Based on this, a new method for calculating conditional probability is proposed. The method can effectively represent the regret-rejoicing relationship of each alternative.

(2) Considering the influence of the irrational behavior of the decision maker on the decision and enhancing the rationality of the decision-making process, this paper designs a new method for calculating the relative loss function based on regret theory. It is worth noting that the construction process reduces the risk of the subjective loss function as well as considers the risk attitude of the decision-maker.

(3) For the ranking of alternatives, this paper obtains the rejoicing matrix and regret matrix based on the regret-rejoicing function, which is used to calculate the relative advantageous relationship between objects. Unlike the general relationship based on the distance between objects, this paper takes into account the influence of behavioral decisions and is therefore more practical.

The remainder of the paper is organized as follows. Section II introduces some basic knowledge and concepts of MSDISs, regret theory, and 3WD. Section III explores a new dominance relation, a relative loss function based on regret theory and a new three-way MADM method. Section IV uses the proposed method to solve a practical case. The validity, feasibility, and stability of the regret theory-based three-way MADM method are verified using real data sets in Section V. In Section VII, the full paper is summarized and future research directions are proposed.

II. Preliminaries

To facilitate the introduction of subsequent content, this section briefly reviews some basic concepts of MSDISs, regret theory, and HS3WD.

A. Multi-Scale Decision Information Systems

As an extension of SSDISs, MSDISs can help humans better deal with multi-level and multi-granulation decision problems and provide information closer to the real world, allowing us to better reflect the complexity of real-world problems. The specific concepts are described below.

Definition 1: ([43]) Let $S = (U, C)$ be a single-scale information system, where $U = \{x_1, x_2, ..., x_l\}$ is a finite non-empty set of objects, and $C = \{c_1, c_2, ..., c_m\}$ is a finite non-empty set of conditional attributes. A multi-scale information system can be written as $S = (U, C = \{c_j^i | j = 1, 2, ..., m, i = 1, 2, ..., s\})$, where $c_j^i(x)$ is the attribute value of the object x at the scale i under the attribute c_j , and i is the scale number from coarse to fine.

A multi-scale information system is a merging of several single-scale information systems. Let $V_{c_j^i}$ be the domain of values of an attribute c_j at scale i, for any $i \in \{1, 2, ..., s-1\}$, there exists a full projection mapping $g_j^{i,i+1} : V_{c_j^{i+1}} \to V_{c_j^i}$, such that for any $x \in U$, there exists $c_j^i(x) = g_j^{i+1,i}(c_j^{i+1}(x))$, where $g_j^{i,i+1}$ is a composite function called the information granule transformation function from scale i to scale $i+1$.

Definition 2: ([43]) Given a MSDIS $S = (U, C \cup D)$, where $U = \{x_1, x_2, ..., x_l\}$ is a finite set of non-empty objects, $C = \{c_1, c_2, ..., c_m\}$ and D are a non-empty finite set of conditional and decision attributes of S, respectively. A MSDIS can be represented as $S = (U, C \cup D) = (U, \{c_j^i | j = 1, 2, ..., m, i = 1, 2, ..., s\} \cup D)$.

B. Regret Theory

Regret theory describes regret as the emotion that arises when comparing outcomes or states of affairs for a given event. Specifically, it refers to the psychological state of regret that investors often experience during the investment process.

The perceived utility consists of both the utility of the chosen alternative and the regret-rejoicing value resulting from the comparison with other alternatives. Let a_1, a_2 be the outcome of the alternatives x_1, x_2 , then the decision maker's perceived utility of the alternative x_1 is computed by the following formula:

$$u_1 = v(a_1) + R(v(a_1) - v(a_2)) \tag{1}$$

where $v(a_1)$ is the utility gained by the decision-maker from the outcome a_1, and $R(v(a_1) - v(a_2))$ is the regret or rejoicing of the decision-maker for choosing the alternative x_1 that x_2 is not chosen. Specifically, when $R(v(a_1) - v(a_2)) > 0$, $R(v(a_1) - v(a_2))$ represents the rejoicing value, otherwise it is a regret value.

Bell [28] gave the corresponding utility function, which can reflect the degree of risk preference of the decision-maker through the parameters. The utility value $v(a)$ is computed by the following formula:

$$v(a) = \frac{1 - e^{-\theta a}}{\theta} \tag{2}$$

where $\theta \in (0, 1)$ denotes the risk aversion coefficient. The greater θ, the greater the risk aversion of the decision-maker, the smaller the utility value.

In addition, Bell gave a specific form of regret-rejoicing function in the form of

$$R(\Delta v) = 1 - e^{-\partial \Delta v} \tag{3}$$

where $\Delta v = v(a_1) - v(a_2)$, ∂ denotes the risk preference of the decision-maker and satisfies $\partial \in [0, +\infty)$. The greater ∂, the stronger the decision maker's regret.

C. Hierarchical Sequential Three-way Decision

The idea of HS3WD was proposed by Qian et al. [13], [14]. It is based on S3WD and aims to create a comprehensive HS3WD model.

Definition 3: ([6]) Given a decision table $S = (U, C \cup D)$, where $C = \{c_1, c_2, ..., c_m\}$ is a set of conditional attributes, D is the set of decision attributes, the partition of the universe domain U with respect to D is $\pi_D = \{D_1, D_2, ..., D_r\}$, where $k \in \{1, 2, ..., r\}$, the conditional probability function is defined as

$$P(D_k|[x]_C) = \frac{|[x]_C \cap D_k|}{|[x]_C|}. \tag{4}$$

Based on the idea of 3WD, the decision-theoretic rough set uses two state sets and three action sets to describe the decision process. The set of states is assumed to be $\Omega = \{X, \neg X\}$, indicating that the object belongs X or does not belong to X, and the set $\Lambda = \{a_P, a_B, a_N\}$ indicates the decision actions that divide the objects into three regions, namely acceptance, delayed decision and rejection.

When the object $x \in U$ belongs to X, $\lambda_{PP}, \lambda_{BP}$ and λ_{NP} indicate the loss of the three strategies a_P, a_B and a_N, respectively. When the object $x \in U$ belongs to $\neg X$, $\lambda_{PN}, \lambda_{BN}$ and λ_{NN} indicate the loss of the three strategies a_P, a_B and a_N, respectively. At the same time, it assumes that $\lambda_{PP} \leq \lambda_{BP} \leq \lambda_{NP}$ and $\lambda_{PN} \geq \lambda_{BN} \geq \lambda_{NN}$. Thus, the expected loss $\Phi(a_\diamond|[x])(\diamond = P, B, N)$ for each object $x \in U$ are calculated as follows:

$$\Phi(a_\diamond|[x]) = \lambda_{\diamond P} P(X|[x]) + \lambda_{\diamond N} P(\neg X|[x]). \tag{5}$$

Definition 4: ([13]) Given a multilevel decision table $S^i (i = 1, 2, ..., s)$, a multilevel granular structure $G = \{G_1, G_2, ..., G_n\}$ based on a sequence of sequential sets of attributes $C_1 \subset C_2 ... \subset C_n \subseteq C$, with corresponding sequential threshold pairs $(\alpha, \beta) = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2), ..., (\alpha_n, \beta_n)\}$. For $x \in U_q$, the equivalence class containing the object x is denoted as $[x]_{C_q} = \{y \in U | (x, y) \in C_q\}$. For the q-level granular structure $G_q (q \in \{1, 2, ..., n\})$, three regions are defined as follows:

$$POS_{G_q}^{(\alpha_q,\beta_q)}(D_{t,i}) = \bigcup_{i=1}^{s} \underline{apr}_{C_q^i}^{(\alpha_q^i,\beta_q^i)}(D_{q,k}^i); \tag{6}$$

$$NEG_{G_q}^{(\alpha_q,\beta_q)}(D_{q,k}) = \bigcup_{i=1}^{s} (U_q^i - \overline{apr}_{C_q^i}^{(\alpha_t^i,\beta_t^i)}(D_{q,k}^i)); \tag{7}$$

$$BND_{G_q}^{(\alpha_q,\beta_q)}(D_{q,k}) = U_q - POS_{G_q}^{(\alpha_q,\beta_q)}(D_{q,k}) - NEG_{G_q}^{(\alpha_q,\beta_q)}(D_{q,k}). \tag{8}$$

where $\overline{apr}_{C_q^i}^{(\alpha_q^i,\beta_q^i)}(D_{q,k}^i)$ and $\underline{apr}_{C_q^i}^{(\alpha_q^i,\beta_q^i)}(D_{q,k}^i)$ are the upper and lower approximations of $D_{q,k}^i$, respectively.

III. A Hierarchical Sequential Three-Way Multi-Attribute Decision Making Method Based on Regret Theory

Most existing methods can only solve decision problems under a single scale. Therefore, we define in this section a regret theory-HS3WD method to solve the MADM problems under multi-scale.

A. Conditional Probability

Considering the influence of the irrational behavior of decision-makers on decision-making, this paper designs a new method for calculating conditional probability and relative loss functions based on regret theory. Specifically, a detailed explanation is provided on how to construct the regret-rejoicing dominance relation and class, including the relevant definitions and propositions. Conditional probability is calculated using this approach.

Definition 5: Given an information system $S = (U, C)$, for any $x_t, x_u \in U$, $t, u \in \{1, 2, ..., l\}$, the dominance of regret-rejoicing of x_t, x_u with respect to the set of attributes C are calculated as follows:

$$\begin{aligned} R^+ &= \sum_{j=1}^{m} w_j R(v_j(x_t) - v_j(x_u)), \\ R^- &= \sum_{j=1}^{m} w_j R(v_j(x_u) - v_j(x_t)). \end{aligned} \tag{9}$$

where w_j is the weight of the j^{th} attribute, $j \in \{1, 2, ..., m\}$, $0 \leq w_j \leq 1$ and $\sum_{j=1}^{m} w_j = 1$.

The absolute regret-rejoicing dominance relationship R of x_t, x_u under the set of attributes C are expressed as:

$$\begin{aligned} R_C^{\uparrow} &= \{(x_t, x_u) \in U \times U | R^+ \geq 0 \ and \ R^- \leq 0\}, \\ R_C^{\downarrow} &= \{(x_t, x_u) \in U \times U | R^+ < 0 \ and \ R^- > 0\}. \end{aligned} \tag{10}$$

where $R_C^{\uparrow}$ and $R_C^{\downarrow}$ are the absolute rejoicing dominance set and absolute regret dominance set, respectively. For the special case of $R^+(x_t, x_u) = 0, R^-(x_t, x_u) = 0$, namely $x_t = x_u$, we assume that x_t, x_u has an absolute rejoicing dominance relationship. For the case of $R^+(x_t, x_u) > 0, R^-(x_t, x_u) > 0$ or $R^+(x_t, x_u) < 0, R^-(x_t, x_u) < 0$, it is impossible to determine whether x_t, x_u is an absolute rejoicing dominance relationship or an absolute regret dominance relationship, and will not be considered.

In addition, the absolute regret-rejoicing dominance class of x_t is obtained based on the conditional attributes set C and expressed as follows:

$$[x_t]_C^R = \{x_u \in U | (x_u, x_t) \in R_C^{\uparrow} and (x_u, x_t) \notin R_C^{\downarrow}, t = 1, 2, ..., l\} \tag{11}$$

Remark 1: The interpretation of Eq. (11) is that if $(x_u, x_t) \in R_C^{\uparrow}$ and $(x_u, x_t) \notin R_C^{\downarrow}$, then $x_u \in [x_t]_C^R$, i.e. x_u is superior to x_t. From the point of view of profit and advantage, the greater the advantage, the higher the profit, which is the optimal decision effect.

Proposition 1: For the absolute regret-rejoicing dominance relationship R, we conclude that it has the following property:

(1) Self-reflexivity: for a set $[x_u]_C^R$ and a relation R based on $[x_u]_C^R$, for any element x_u in $[x_u]_C^R$, there $< x_u, x_u >$ is an element of R.

Proof: As can be seen from Definition 5, the above (1) are easily proved. ■

Definition 6: Given a decision information system $IS = (U, C \cup D)$, where $C = \{c_1, c_2, ..., c_m\}$ is a set of conditional attributes, D is a set of decision attributes. The decision attributes classify U into two categories: X and $\neg X$, the conditional probability that object x_t belongs to X or $\neg X$ under attribute set C is as follows:

$$P(X|[x_t]_C^R) = \frac{|[x_t]_C^R \cap X|}{|[x_t]_C^R|}. \quad (12)$$

Proposition 2: Given a decision information system $IS = (U, C \cup D)$, $P(X|[x_t]_C^R)$ and $P(\neg X|[x_t]_C^R)$ represent the conditional probability that object x_t belongs to X and $\neg X$ under attribute set C, respectively, then we have $P(X|[x_t]_C^R) + P(\neg X|[x_t]_C^R) = 1$.

Proof: $P(X|[x_t]_C^R) + P(\neg X|[x_t]_C^R) = \frac{|[x_t]_C^R \cap X|}{|[x_t]_C^R|} + \frac{|[x_t]_C^R \cap \neg X|}{|[x_t]_C^R|} = \frac{|[x_t]_C^R \cap U|}{|[x_t]_C^R|} = 1$.

The proof of Proposition 2 is thus complete. ■

B. Relative Aggregation Loss Functions

The loss functions serve as a description of the loss of different behaviors taken by an object in a certain state, reflecting the decision maker's attitude. Considering the influence of the irrational behavior of decision-makers on decision-making, this paper will define the value of loss for taking different actions in different states based on regret theory and propose the relative loss functions.

First, in a decision information system $IS = (U, C \cup D)$, for any $x_t \in U$, the utility value of x_t for attribute set C is computed to obtain a utility matrix. The calculation is shown below:

$$v(x) = [v_j(x_t)]_{l \times m} \quad (13)$$

where the utility function is $v_j(x_t) = [(1 - e^{-\theta x_{tj}})/\theta]$, θ is the risk aversion coefficient and ranges from $\theta \in (0, 1)$.

Then, the rejoicing and regret values between each object x_t and the optimal and inferior objects are calculated to obtain the corresponding rejoicing and regret matrices, which are calculated as follows:

$$\begin{aligned} D^+ &= [R(v_j(x_t) - \min(v_j(x)))]_{l \times m}, \\ D^- &= [R(v_j(x_t) - \max(v_j(x)))]_{l \times m}. \end{aligned} \quad (14)$$

where $\min(v_j(x))$ is the utility value of the inferior object under the j^{th} conditional attribute, and $\max(v_j(x))$ is the utility value of the optimal object under the j^{th} conditional attribute. Each row of the matrix D^+ represents the rejoicing index between each object and the inferior object under each condition attribute, and each row of the matrix D^- represents the regret index between each object and the optimal object under each condition attribute.

Afterward, the rejoicing and regret matrices are summed to obtain the integrated preference matrix, which is calculated as follows:

$$D_{total} = D^+ + D^- \quad (15)$$

where each row of the D_{total} matrix represents the difference between the distance of an object from the optimal object and the inferior object under each conditional attribute, the larger the value, the better the object.

Finally, the integrated preference matrix is normalized and calculated as follows:

$$\overline{D_{total}} = \frac{D_{total}}{\max(|D_{total}|)}, j = 1, 2, ..., m \quad (16)$$

After introducing regret theory into the decision information system, the concept and metric of relative aggregation loss functions are proposed from a loss perspective.

The basic loss functions are constructed based on Jia and Liu [25], defining Γ and $\neg\Gamma$ as two different sets of states, a_P, a_B and a_N as the three adopted strategies, namely acceptance, delayed decision and rejection. Thus, for any object $x_t \in U$, the relative aggregation loss functions are shown in TABLE I.

TABLE I
RELATIVE AGGREGATION LOSS FUNCTIONS FOR x_t

	Γ	$\neg\Gamma$
a_P	0	$\max - \sum_{j=1}^m w_j \overline{D_{total,j}}$
a_B	$\sum_{j=1}^m \vartheta_j w_j (\overline{D_{total,j}} - \min)$	$\sum_{j=1}^m \vartheta_j w_j (\max - \overline{D_{total,j}})$
a_N	$\sum_{j=1}^m w_j \overline{D_{total,j}} - \min$	0

In TABLE I, $\max = \max(\overline{D_{total}})$, $\min = \min(\overline{D_{total}})$. $0, \sum_{j=1}^m \vartheta_j w_j (\overline{D_{total,j}} - \min)$ and $\sum_{j=1}^m w_j \overline{D_{total,j}} - \min$ are the losses if actions a_P, a_B and a_N are taken when $x_t \in \Gamma$, respectively, while $\max - \sum_{j=1}^m w_j \overline{D_{total,j}}$, $\sum_{j=1}^m \vartheta_j w_j (\max - \overline{D_{total,j}})$ and 0 are the losses if actions a_P, a_B and a_N are taken when $x_t \in \neg\Gamma$, respectively. w_j is the weight of the j^{th} conditional attribute, ϑ_j is the risk aversion factor for the j^{th} conditional attribute and $\vartheta_j \in [0, 0.5)$. Clearly, the relative aggregation loss functions based on regret theory satisfy $\lambda_{PP} \le \lambda_{BP} \le \lambda_{NP}$ and $\lambda_{PN} \ge \lambda_{BN} \ge \lambda_{NN}$.

Once the relative loss functions have been established, the thresholds α and β for object x_t are calculated as follows:

$$\begin{aligned} \alpha &= \frac{\sum_{j=1}^m w_j (1 - \vartheta_j)(\max - \overline{D_{total,j}})}{\sum_{j=1}^m w_j (1 - \vartheta_j)(\max - \overline{D_{total,j}}) + \sum_{j=1}^m w_j \vartheta_j (\overline{D_{total,j}} - \min)}, \\ \beta &= \frac{\sum_{j=1}^m w_j \vartheta_j (\max - \overline{D_{total,j}})}{\sum_{j=1}^m w_j \vartheta_j (\max - \overline{D_{total,j}}) + \sum_{j=1}^m w_j (1 - \vartheta_j)(\overline{D_{total,j}} - \min)}. \end{aligned} \quad (17)$$

Theorem 1: Considering α and β as functions of ϑ_j, and the risk aversion coefficient $\vartheta \in [0, 0.5)$, then for any $x_t \in U$, the following conclusion holds:

(1) The value of α decreases as the value of ϑ_j increases;

(2) The value of β increases as the value of ϑ_j increases.

Proof: For (1), calculate the derivative of α:

$$\begin{aligned} \alpha' = & \frac{-\sum_{j=1}^m w_j (\max - \overline{D_{total,j}}) \sum_{j=1}^m w_j \vartheta_j (\overline{D_{total,j}} - \min)}{\left(\sum_{j=1}^m w_j (1 - \vartheta_j)(\max - \overline{D_{total,j}}) + \sum_{j=1}^m w_j \vartheta_j (\overline{D_{total,j}} - \min)\right)^2} \\ & - \frac{-\sum_{j=1}^m w_j (\overline{D_{total,j}} - \min) \sum_{j=1}^m w_j (1 - \vartheta_j)(\max - \overline{D_{total,j}})}{\left(\sum_{j=1}^m w_j (1 - \vartheta_j)(\max - \overline{D_{total,j}}) + \sum_{j=1}^m w_j \vartheta_j (\overline{D_{total,j}} - \min)\right)^2}. \end{aligned}$$

Due to $\overline{D_{total,j}} \in (\min, \max)$, then $\max - \overline{D_{total,j}} \ge 0$, and $\overline{D_{total,j}} - \min \ge 0$, hence $(\max - \overline{D_{total,j}})(\overline{D_{total,j}} - \min) > 0$ can be obtained. Therefore, the numerator of α' is less than 0, and the denominator of α' is greater than 0, then $\alpha' < 0$. The proof of (1) is complete.

(2) is proved as (1). ■

Theorem 2: The thresholds α and β satisfy $0 \le \beta < \alpha \le 1$ when $0 \le \vartheta < 0.5$.

Proof: $\alpha > \beta$

$$\Leftrightarrow \frac{\sum_{j=1}^{m} w_j(1-\vartheta_j)(\max - \overline{D_{total,j}})}{\sum_{j=1}^{m} w_j(1-\vartheta_j)(\max - \overline{D_{total,j}}) + \sum_{j=1}^{m} w_j\vartheta_j(\overline{D_{total,j}} - \min)}$$
$$> \frac{\sum_{j=1}^{m} w_j\vartheta_j(\max - \overline{D_{total,j}})}{\sum_{j=1}^{m} w_j\vartheta_j(\max - \overline{D_{total,j}}) + \sum_{j=1}^{m} w_j(1-\vartheta_j)(\overline{D_{total,j}} - \min)}$$
$$\Leftrightarrow (1-\vartheta_j)^2 > \vartheta_j^2 \Leftrightarrow 1 - 2\vartheta_j > 0 \Leftrightarrow 0 \leq \vartheta_j < 0.5.$$

Therefore, it is proved that $\alpha > \beta$. According to the calculation formula of α and β, it is clear that $0 \leq \alpha, \beta \leq 1$. Based on the above reasoning, it is proved that Theorem 2 holds. ∎

According to Theorem 2, the following three-way classification rule can be obtained:

(P) If $P(\Gamma|[x_t]_C^R) \geq \alpha_t$, then $x_t \in POS(\Gamma)$;
(B) If $\beta_t < P(\Gamma|[x_t]_C^R) < \alpha_t$, then $x_t \in BND(\Gamma)$;
(N) If $P(\Gamma|[x_t]_C^R) \leq \beta_t$, then $x_t \in NEG(\Gamma)$.

C. Hierarchical Sequential Three-Way Multi-Attribute Decision Making Method

In what follows, we introduce the HS3WD into the MSDIS to address the issue of multi-scale information. It processes the multi-scale information hierarchically and sequentially, resulting in the final classification result.

Compared with the classical S3WD model, HS3WD model has less classification and shorter rule length. It is also easy to understand the specific semantics of the rules, which is in line with the cognition of ordinary people. Because multi-scale data has hierarchy, general granular computing tools can't solve this kind of problem, and the hierarchical sequential computing paradigm can fit this kind of problem well, so we adopt the hierarchical sequential in multi-scale classification decision.

For a MSDIS $MIS = (U, \{c_j^i | j = 1, 2, ..., m, i = 1, 2, ..., s\} \cup \{d\})$, the SSDIS at the i^{th} scale is $S^i = (U, \{c_1^i, c_2^i, ..., c_m^i\} \cup \{d\})$, and given a sequential set of attributes $C_1 \subset C_2 ... \subset C_q ... \subset C_n \subseteq C$, its induced multilevel granular structure is $G = \{G_1, G_2, ..., G_q, ..., G_n\}$. Under each granular structure, there are s SSDISs.

Starting from the granular structure G_1, the 3WD is made from the coarsest scale S^1 according to the principle of coarse-to-fine. And the conditional probability $P(\Gamma^i|[x_t]_{G_1}^i)$ and loss function as well as the thresholds α_t^i and β_t^i are calculated based on subsections III-A and III-B, and the universe U is divided into positive region, negative region and boundary region. The boundary region $BND_{G_1}^1$ is used as the universe $U_{G_1}^2$ of the next scale S^2, the above process is repeated up to the finest scale S^s. The boundary region BND_1 under this granular structure is used as the universe $U_{G_2}^1$ of the next granular structure G_2 and the process is repeated until the universe is empty or the last granular structure G_n is finished. The specific sequential process is shown in Fig. 1, where the three ellipses represent the positive, negative and boundary regions after the decision on granular structure G_q is completed.

The positive, negative and boundary regions after all granular structure decisions are respectively

$$POS_G = \bigcup_{q=1}^{n} POS_{G_q} = \bigcup_{q=1}^{n} \{x | P(\Gamma^i|[x_t]_{G_q}^i) \geq \alpha_t^i, x \in U_{G_q}^i\}; \tag{18}$$

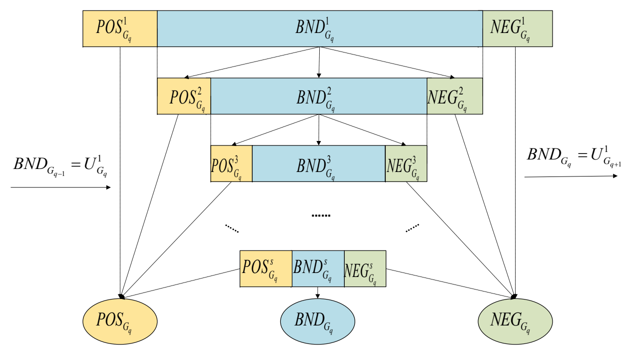

Fig. 1. Sequential process between the different levels in G.

$$NEG_G = \bigcup_{q=1}^{n} NEG_{G_q} = \bigcup_{q=1}^{n} \{x | P(\Gamma^i|[x_t]_{G_q}^i) \leq \beta_t^i, x \in U_{G_q}^i\}; \tag{19}$$

$$BND_G = \bigcap_{q=1}^{n} BND_{G_q} = \bigcap_{q=1}^{n} \{x | \beta_t^i < P(\Gamma^i|[x_t]_{G_q}^i) < \alpha_t^i, x \in U_{G_q}^i\}. \tag{20}$$

For the ranking of objects, two methods are proposed in this paper. The data processing part before getting the ranking result is as follows.

Firstly, the comprehensive utility value of each scale is calculated by

$$H^i = \sum\nolimits_{j=1}^{m} w_j D_j^{i,+} + \sum\nolimits_{j=1}^{m} w_j D_j^{i,-}. \tag{21}$$

Then, the comprehensive utility value of each scale is normalized by

$$\overline{H^i} = \frac{H^i}{|\max(H^i)|}. \tag{22}$$

Method 1: The coarse ranking is carried out under each scale, and then the fusion is carried out according to the principle that the finer the scale is, the larger the weight is, and finally the comprehensive ranking result is obtained.

The comprehensive utility value of each scale is fused by

$$H = \frac{i}{1 + ... + i + ... + s} H^i. \tag{23}$$

Method 2: The ranking results on the coarsest and finest scales are selected and comprehensively ranked according to the optimist-pessimistic fusion strategy. The specific formula is as follows:

$$H = \eta H^1 + (1-\eta) H^s \tag{24}$$

where the magnitude of the parameter $\eta \in [0, 1]$, η indicates the risk attitude of the decision maker.

D. Specific Steps in the Hierarchical Sequential Three-Way Multi-Attribute Decision Making Method

The discussion in the previous three subsections mainly deals with the calculation of conditional probability and loss functions, and the construction of the HS3WMADM method.

Algorithm 1: three-way MADM method in a MSDI.

Input: A multi-scale decision table IS, attribute weights w and four parameters $\theta, \partial, \vartheta, \eta$.

Output: Classification and ranking of all objects.

1 Normalize the multi-scale decision table according to Eq. (25);
2 Initialize POS_G, BND_G and NEG_G;
3 **for** q =*1 to* n **do**
4   **for** i =*1 to* s **do**
5   Calculate the utility value of each object according to Eqs. (13)-(16);
6   Calculate conditional probability of each object according to Definitions 5 and 6;
7   Calculate α, β of each object according to Eq. (17);
8   (P) If $P(\Gamma|[x_t]_C^E) \geq \alpha_t^i$, then $x_t \in POS_{G_q}^i(\Gamma)$;
9   (B) If $\beta_t^i < P(\Gamma|[x_t]_B^E) < \alpha_t^i$, then $x_t \in BND_{G_q}^i(\Gamma)$;
10   (N) If $P(\Gamma|[x_t]_B^E) \leq \beta_t^i$, then $x_t \in NEG_{G_q}^i(\Gamma)$;
11   **end**
12 **end**
13 Obtain POS_G, BND_G and NEG_G according to Eqs. (18)-(20);
14 **for** i =*1 to* s **do**
15   Calculate the comprehensive utility value of each object according to Eqs. (21) and (22);
16 **end**
17 The first ranking result is obtained by fusing the comprehensive utility value according to Eq. (23);
18 The optimistic-pessimistic fusion of the combined utility value is performed according to Eq. (24) to obtain the second ranking result;

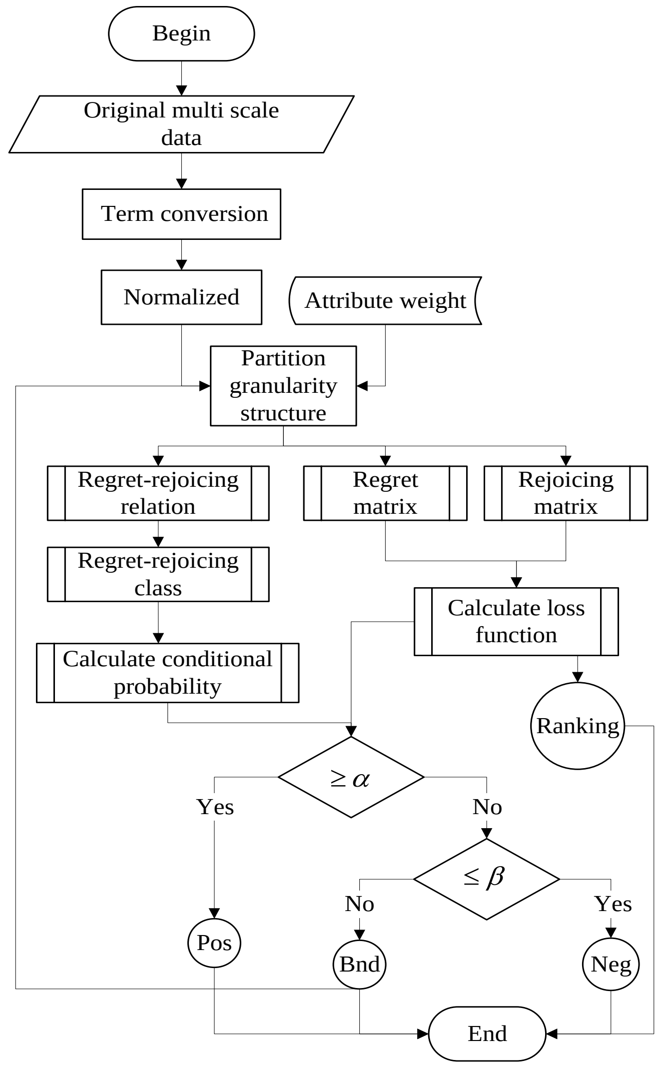

Fig. 2. Flowchart of this model.

The above processes are summarized in a corresponding algorithm, as shown in Algorithm 1.

Remark 2: In Algorithm 1, the time complexity of steps 1-2 is $O(|U|)$. The time complexity of steps 3-7 is $O(|n| \times |s| \times |U|)$. The time complexity of step 8 is $O(|U|)$. The time complexity of step 9 is $O(|s|)$, and the time complexity of steps 10-11 is $O(|U|)$. Therefore, the maximum time complexity of Algorithm 1 is $O(|n| \times |s| \times |U|)$. Obviously, the space complexity is $O(|U|^2)$.

The above processes are summarized in a corresponding flowchart, as shown in Fig. 2.

IV. An Application Case

A. Selected Case of Outstanding Employee of the Year

TABLE II is a primitive multi-scale decision table describing whether an employee in a particular company will be awarded Employee of the Year. $U = \{x_1, ..., x_{16}\}$ is the employee in the company; $C = \{a_1, a_2, a_3\}$ is the conditional attributes set, indicating education, usual performance, and job title; d is the decision attribute, indicating whether or not the employee is an employee of the year. Each conditional attribute has three scales, $V_{a_1^1}$ ={H, L}, representing "High" and "Low" respectively; $V_{a_1^2} = \{$B, M, S$\}$, representing "High grade", "Medium grade" and "Low grade " respectively; $V_{a_1^3} = \{$G, U, K, O$\}$, representing "Graduate", "Undergraduate", "high school student" and "Other" respectively; $V_{a_2^1} = \{$P, N$\}$, representing "Pass" and "Fail" respectively; $V_{a_2^2} = \{$I, II, III, IV$\}$, representing "Excellent", "Good", "Average" and "Poor" respectively; $V_{a_2^3} = \{3, 4, 5, 6, 7, 8, 9, 10\}$, representing the employee's rating in daily performance; $V_{a_3^1} = \{$A, C, J$\}$, representing "Senior", "Intermediate" and "Junior" respectively; $V_{a_3^2} = \{$J1, J2, J3, J4$\}$, representing "Senior", "Engineer", "Assistant" and "Technician" respectively; $V_{a_3^3} = \{$P1, P2, P3, P4, P5$\}$, representing "Senior engineer", "Associate senior engineer", "Engineer" and "Technician" respectively $V_d = \{0, 1\}$, representing "No" and "Yes" respectively.

The processing method of Deng et al. [31] is used to convert linguistic terms into hierarchical terms. a_2 as an example, term conversion is shown in Fig. 3. a_1 and a_3 are converted using the same method.

The transformed multi-scale decision table is shown in TABLE III.

The multi-scale hierarchy is transformed into values in the

TABLE II
MULTI-SCALE DECISION TABLE

U	a_1^1	a_1^2	a_1^3	a_2^1	a_2^2	a_2^3	a_3^1	a_3^2	a_3^3	d
x_1	L	S	O	N	IV	3	J	J4	P5	0
x_2	L	S	O	N	IV	5	J	J4	P4	0
x_3	L	S	K	N	IV	4	J	J4	P4	0
x_4	L	S	K	P	III	6	J	J3	P5	0
x_5	H	B	U	N	IV	5	J	J4	P4	0
x_6	L	S	K	P	III	6	J	J3	P4	0
x_7	H	B	U	P	III	6	J	J4	P4	0
x_8	H	B	U	N	IV	4	C	J2	P3	0
x_9	H	B	U	N	IV	5	C	J2	P3	0
x_{10}	H	B	G	N	IV	3	J	J4	P4	0
x_{11}	H	B	G	N	IV	3	J	J4	P5	0
x_{12}	L	S	O	P	I	9	C	J2	P3	0
x_{13}	H	B	U	P	II	8	A	J1	P2	1
x_{14}	H	B	U	P	II	8	A	J1	P1	1
x_{15}	L	M	K	P	I	10	A	J1	P1	1
x_{16}	H	B	G	P	III	7	A	J1	P1	1

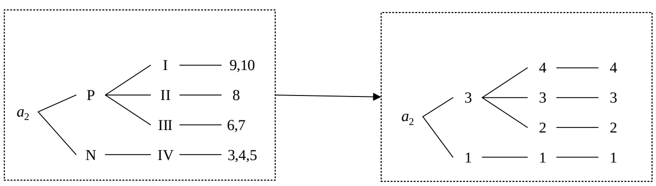

Fig. 3. Term conversion.

TABLE III
MULTI-SCALE HIERARCHICAL DECISION TABLE

U	a_1^1	a_1^2	a_1^3	a_2^1	a_2^2	a_2^3	a_3^1	a_3^2	a_3^3	d
x_1	1.5	1.5	1	1	1	1	1	1.5	1.5	0
x_2	1.5	1.5	1	1	1	1	1	1.5	1.5	0
x_3	1.5	1.5	2	1	1	1	1	1.5	1.5	0
x_4	1.5	1.5	2	3	2	2	2	1.5	1.5	0
x_5	3.5	3.5	3	1	1	1	1	1.5	1.5	0
x_6	1.5	1.5	2	3	2	2	2	1.5	1.5	0
x_7	3.5	3.5	3	3	2	2	2	1.5	1.5	0
x_8	3.5	3.5	3	1	1	1	1	3	3	0
x_9	3.5	3.5	3	1	1	1	1	3	3	0
x_{10}	3.5	3.5	4	1	1	1	1	1.5	1.5	0
x_{11}	3.5	3.5	4	1	1	1	1	1.5	1.5	0
x_{12}	1.5	1.5	1	3	4	4	4	3	3	0
x_{13}	3.5	3.5	3	3	3	3	3	4.5	4.5	1
x_{14}	3.5	3.5	3	3	3	3	3	4.5	4.5	1
x_{15}	1.5	1.5	2	3	4	4	4	4.5	4.5	1
x_{16}	3.5	3.5	4	3	2	2	2	4.5	4.5	1

interval [0, 1], and the Cauchy distribution and logarithmic distribution are used to describe the satisfaction of decision-makers with the evaluation values. A low-level evaluation value corresponds to a low degree of satisfaction, and a high-level evaluation value corresponds to a high degree of satisfaction. The quantitative functions were selected based on the range of values in TABLE III as follows:

TABLE IV
MULTI-SCALE NUMERICAL DECISION TABLE

U	a_1^1	a_1^2	a_1^3	a_2^1	a_2^2	a_2^3	a_3^1	a_3^2	a_3^3	d
x_1	0.2053	0.2053	0.0100	0.0100	0.0100	0.0100	0.2053	0.2053	0.0100	0
x_2	0.2053	0.2053	0.0100	0.0100	0.0100	0.0100	0.2053	0.2053	0.4563	0
x_3	0.2053	0.2053	0.4563	0.0100	0.0100	0.0100	0.2053	0.2053	0.4563	0
x_4	0.2053	0.2053	0.4563	0.7500	0.4563	0.4563	0.2053	0.2053	0.0100	0
x_5	0.8224	0.8224	0.7500	0.0100	0.0100	0.0100	0.2053	0.2053	0.4563	0
x_6	0.2053	0.2053	0.4563	0.7500	0.4563	0.4563	0.2053	0.2053	0.4563	0
x_7	0.8224	0.8224	0.7500	0.7500	0.4563	0.4563	0.2053	0.2053	0.4563	0
x_8	0.8224	0.8224	0.7500	0.0100	0.0100	0.0100	0.7500	0.7500	0.7500	0
x_9	0.8224	0.8224	0.7500	0.0100	0.0100	0.0100	0.7500	0.7500	0.7500	0
x_{10}	0.8224	0.8224	0.8852	0.0100	0.0100	0.0100	0.2053	0.2053	0.4563	0
x_{11}	0.8224	0.8224	0.8852	0.0100	0.0100	0.0100	0.2053	0.2053	0.0100	0
x_{12}	0.2053	0.2053	0.0100	0.7500	0.8852	0.8852	0.7500	0.7500	0.7500	0
x_{13}	0.8224	0.8224	0.7500	0.7500	0.7500	0.7500	0.9405	0.9405	0.8852	1
x_{14}	0.8224	0.8224	0.7500	0.7500	0.7500	0.7500	0.9405	0.9405	0.9900	1
x_{15}	0.2053	0.2053	0.4563	0.7500	0.8852	0.8852	0.9405	0.9405	0.9900	1
x_{16}	0.8224	0.8224	0.8852	0.7500	0.4563	0.4563	0.9405	0.9405	0.9900	1

$$f(x) = \begin{cases} \left[1 + k(x-\lambda)^{-2}\right]^{-1}, & 1 \leq x < 3 \\ \mu \ln(x) + v, & 3 \leq x \leq 5 \end{cases}. \tag{25}$$

To solve for this quantification function, assume that "very satisfied", "satisfied" and "very poor" are 0.99, 0.75 and 0.01, respectively, thus, $f(1) = 0.01$, $f(3) = 0.75$ and $f(5) = 0.99$ are obtained. The undetermined coefficient method is used to solve the coefficients, and to obtain $k = 1.503, \lambda = 0.8768, \mu = 0.4698, v = 0.2339$, after substituting into Eq. (25), TABLE IV is obtained.

B. Analysis of Experimental Results

We select five methods for ranking and classification comparison with our method, including TOPSIS (M-I) [44], TODIM (M-II) [21], Jia and liu's method (M-III) [25], Deng et al.'s method (M-IV) [31] and Huang et al.'s method (M-V) [33].

The weights $w = (0.5, 0.3, 0.2)$ are given by the experts, assuming $\theta = 0.3, \partial = 0.6$ and $\vartheta = 0.1$, the granular structure constructed from the sequential attribute set is $G = \{G_1, G_2\}$, where $G_1 = (a_1, a_2)$ and $G_2 = (a_1, a_2, a_3)$. The classification results obtained according to Algorithm 1 are shown in Fig. 4 and the ranking results are shown in TABLE V.

Fig. 4 illustrates the specific process of three-way decision sequencing, it can be seen that $\{x_{13}, x_{14}, x_{15}, x_{16}\}$ are partitioned in the positive region, $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$ are partitioned in the negative region and the boundary region is empty. The classification results are consistent with the actual classification, thus verifying the validity of our proposed method.

The results of the proposed ranking method 1 and 2 with $\eta = 0.1, \eta = 0.5$ and $\eta = 0.9$ are shown in TABLE V. From the ranking results, the optimal object is consistently x_{14}, and the ranking results are almost identical, which is also consistent with the ranking of objects in the positive region

TABLE V
RANKING RESULTS OF THE PROPOSED METHOD

Methods	Ranking	Optimal object
Method 1	$x_{14} \succ x_{13} \succ x_{16} \succ x_7 \succ x_{15} \succ x_8 = x_9 \succ x_{12} \succ x_6 \succ x_{10} \succ x_5 \succ x_4 \succ x_{11} \succ x_3 \succ x_2 \succ x_1$	x_{14}
Method 2 ($\eta = 0.1$)	$x_{14} \succ x_{13} \succ x_{16} \succ x_{15} \succ x_7 \succ x_8 = x_9 \succ x_6 \succ x_{12} \succ x_{10} \succ x_5 \succ x_{11} \succ x_4 \succ x_3 \succ x_2 \succ x_1$	x_{14}
Method 2 ($\eta = 0.5$)	$x_{14} \succ x_{13} \succ x_{16} \succ x_7 \succ x_{15} \succ x_8 = x_9 \succ x_6 \succ x_{12} \succ x_{10} \succ x_5 \succ x_4 \succ x_{11} \succ x_3 \succ x_2 \succ x_1$	x_{14}
Method 2 ($\eta = 0.9$)	$x_{14} \succ x_{13} \succ x_{16} \succ x_{15} \succ x_7 \succ x_8 = x_9 \succ x_{10} \succ x_6 \succ x_{12} \succ x_5 \succ x_{11} \succ x_4 \succ x_3 \succ x_2 \succ x_1$	x_{14}

TABLE VI
CLASSIFICATION RESULTS OF THE PROPOSED METHOD

Methods	Pos	Bnd	Neg
Our method	$\{x_{13}, x_{14}, x_{15}, x_{16}\}$	$\varnothing$	$\{x_1, \ldots\ldots, x_{12}\}$
Our method (non-hierarchical)	$\{x_4, x_6, x_7, x_8, x_9, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}\}$	$\varnothing$	$\{x_1, x_2, x_3, x_5, x_{10}, x_{11}\}$
M-III [25]	$\{x_{13}, x_{14}, x_{15}, x_{16}\}$	$\varnothing$	$\{x_1, \ldots\ldots, x_{12}\}$
M-IV [31]	$\varnothing$	U	$\varnothing$
M-V [33]	$\{x_5\}$	$\{x_1, x_2, x_3, x_4, x_6, \ldots\ldots, x_{16}\}$	$\varnothing$

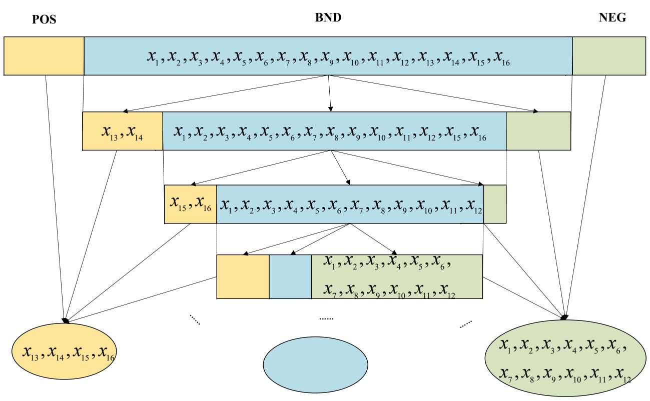

Fig. 4. Classification results of the proposed method.

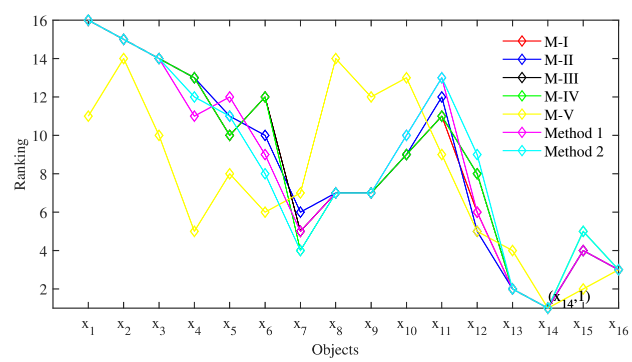

Fig. 5. Comparison of different ranking methods.

before objects in the boundary region and negative region objects.

To better test the proposed method in this paper, classical methods and recent methods are selected to compare with our proposed method. The methods are M-I [44], M-II [21], M-III [25], M-IV [31] and M-V [33].

The comparison results are shown in Fig. 5. It can be found that our method has almost the same ranking results as those of other methods, and the optimal object is x_{14}, which verifies the effectiveness of our method. Compared with the methods of M-I [44], M-II [21] and M-III [25], our method takes into account the psychological changes of decision makers. Compared with the method of M-IV [31] and M-V [33], our method takes into account the importance of different scales, which makes the decision results more scientific and reasonable.

To further investigate the consistency of our method with existing methods, this paper used the Spearman correlation coefficient (SCC) to indicate the correlation between the ranking results of different methods. In general, when the SCC is greater than 0.6, there is a significant correlation between the two methods. TABLE VII gives the SCC of the ranking

TABLE VII
SCC FOR DIFFERENT RANKING METHODS

	Method 1	Method 2	M-I [44]	M-II [21]	M-III [25]	M-IV [31]	M-V [33]
Method 1	1.0000						
Method 2	0.9764	1.0000					
M-I [44]	0.9676	0.9470	1.0000				
M-II [21]	0.9853	0.9558	0.9882	1.0000			
M-III [25]	0.9588	0.9617	0.9912	0.9735	1.0000		
M-IV [31]	0.9558	0.9647	0.9882	0.9676	0.9971	1.0000	
M-V [33]	0.6979	0.6139	0.6389	0.6610	0.5873	0.5696	1.0000

results of different methods, from which it can be seen that our method has a strong correlation with existing methods, which indicates that our method has good validity and high reliability.

The classification results are shown in TABLE VI. Our method is the same as that obtained in M-III [25] and is consistent with reality. Without hierarchical sequence, it is inappropriate to classify $\{x_4, x_6, x_7, x_8, x_9, x_{12}\}$ into positive region. M-IV [31] has only boundary region, which is obviously unreasonable. M-V [33] divides $\{x_{13}, x_{14}, x_{15}, x_{16}\}$ into boundary region and x_5 into positive region, and x_5 ranks

TABLE VIII
SCC FOR DIFFERENT RANKING METHODS

Data set	Objects	Conditional attributes	Scales	Decision attributes	Classes
Car Evaluation (CE)	1728	5	3	1	2
Liver Disorders (LD)	345	5	3	1	2
Haberman's Survival (HS)	306	3	3	1	2

TABLE IX
SCC FOR DIFFERENT METHODS UNDER THE DATASET CE

	Our method	M-I [44]	M-II [21]	M-III [25]	M-IV [31]	M-V [33]
Our method	1.0000					
M-I [44]	0.9751	1.0000				
M-II [21]	0.9545	0.8741	1.0000			
M-III [25]	0.9998	0.9737	0.9565	1.0000		
M-IV [31]	0.9945	0.9745	0.9510	0.9945	1.0000	
M-V [33]	-0.5951	-0.5549	-0.6075	-0.5967	-0.5847	1.0000

behind $\{x_{13}, x_{14}, x_{15}, x_{16}\}$ in the ranking result, which is unreasonable. Overall, the classification results obtained by our method are valid and in line with reality.

V. EXPERIMENTS AND ANALYSIS

In this section, this paper downloads some real-world datasets from the UCI machine learning repository to validate the verify and feasibility of the proposed method. Given that these datasets are single-scale, we will convert them into multi-scale datasets using the method [31]. The relevant information on the processed datasets is shown in TABLE VIII.

A. Ranking Comparison and Analysis

From the ranking results of the case in the previous section, it is clear that the ranking results of the proposed ranking methods 1 and 2 ($\eta = 0.5$) are highly consistent. Therefore, this section only compares the proposed ranking method 1. The ranking results of our proposed method are compared with those methods of M-I [44], M-II [21], M-III [25], M-IV [31] and M-V [33] to further verify that our decision results are not obtained by chance.

As can be noticed from Fig. 6, the ranking results of the five ranking methods under the datasets CE and LD follow the same trend, with a high degree of consistency in the object rankings. The results in TABLEs IX and X show that our method has a high degree of similarity with the ranking results obtained by the other four methods, with the SCC reaching a maximum value of 0.9. M-V [33] does not find the optimal scale in dataset CE and LD, which leads to low similarity between its ranking results and those of other methods. This fully demonstrates the effectiveness and feasibility of the proposed method in this paper.

B. Classification Comparison and Analysis

To verify the validity, the classification results of our method are compared with those of M-III [25],M-IV [31] and M-V [33] under the datasets CE and HS, respectively. Four commonly used classification indicators are selected and calculated as follows:

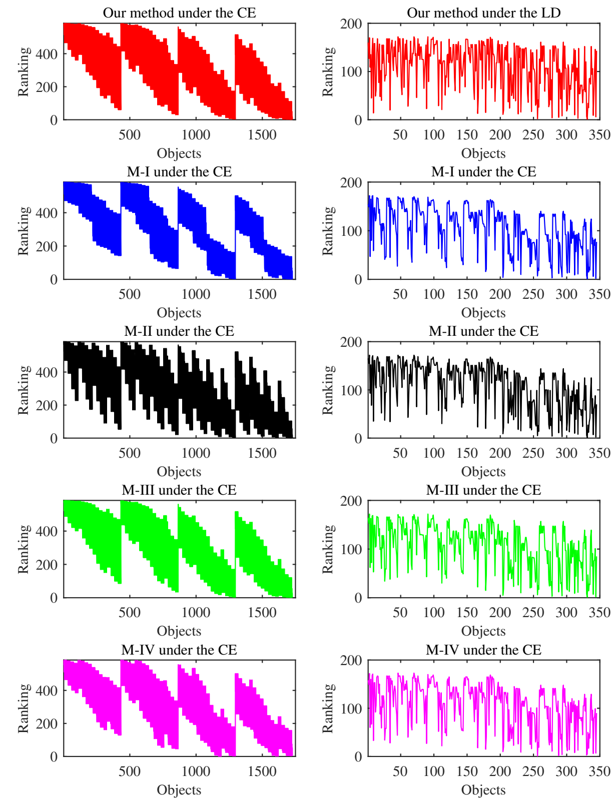

Fig. 6. Comparison of different ranking methods under the two datasets.

TABLE X
SCC FOR DIFFERENT METHODS UNDER THE DATASET LD

	Our method	M-I [44]	M-II [21]	M-III [25]	M-IV [31]	M-V [33]
Our method	1.0000					
M-I [44]	0.7490	1.0000				
M-II [21]	0.8830	0.9316	1.0000			
M-III [25]	0.8395	0.9706	0.9462	1.0000		
M-IV [31]	0.8469	0.9767	0.9641	0.9877	1.0000	
M-V [33]	0.1090	0.1548	0.1142	0.1648	0.1577	1.0000

$$Error\ rate = \frac{t_{X\to NEG} + t_{\neg X\to POS}}{|U|}; \quad (26)$$

$$Precision = \frac{t_{X\to POS} + t_{\neg X\to NEG}}{|POS| + |NEG|}; \quad (27)$$

$$Recall = \frac{t_{X\to POS}}{t_{X\to POS} + t_{X\to NEG}}; \quad (28)$$

$$F1 = \frac{2 * Precision * Recall}{Precision + Recall}. \quad (29)$$

As can be noticed from Fig. 7, the classification results of our method are more reasonable than those of the other two methods. M-IV [31] has too large boundary region and the

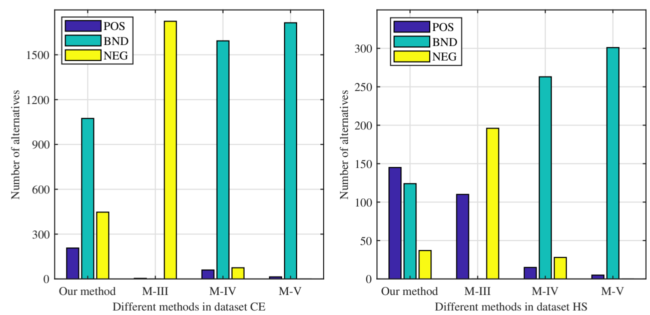

Fig. 7. Comparison of classification results by different methods.

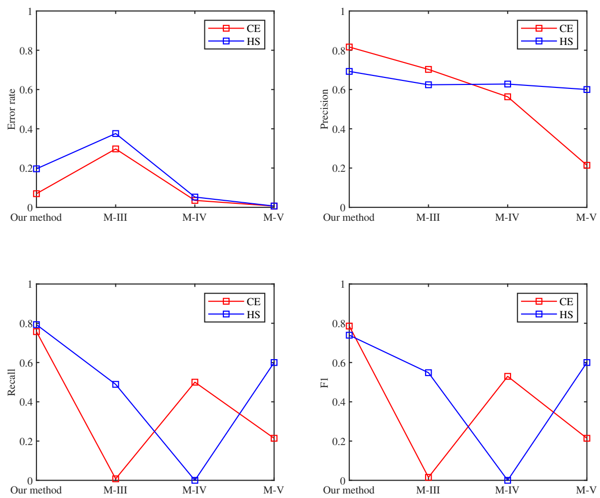

Fig. 8. Comparison of four classification indicators for different methods.

model is not stable enough. M-V [33] has a greater boundary region, and there is no negative region, while data set CE and HS has labeled, therefore, this is not compliant with the actual situation. The method of M-III [25] has only a negative region under the dataset CE and no boundary region under the dataset HS, which is obviously unreasonable. Our method has objects in all three regions and the different number of objects in the three regions is more reasonable. As can be found from Fig. 8, our method performs well under the four metrics. In particular, our method has a lower classification error rate than that of M-III [25], and higher precision, recall and F1 than the other methods. This indicates that our method has stronger classification ability and performance, and outperforms the other methods.

C. Parametric Sensitivity Analysis

The parameters related to the ranking are θ and ∂. Therefore the value of one parameter is fixed, and the effect of the change of the other parameter on the ranking result is analyzed. Fixed $\partial = 30.2$ in TABLE XI and $\theta = 0.5$ in TABLE XII. The results in TABLES XI and XII show that the changes in parameter ∂ and θ have little effect on the ranking results. It shows that our method has good stability.

TABLE XI
SCC OF THE RANKING RESULTS FOR DIFFERENT VALUES OF θ UNDER THE DATASET HS

	$\theta = 0.1$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.7$	$\theta = 0.9$
$\theta = 0.1$	1.0000				
$\theta = 0.3$	0.9999	1.0000			
$\theta = 0.5$	0.9997	0.9998	1.0000		
$\theta = 0.7$	0.9993	0.9994	0.9996	1.0000	
$\theta = 0.9$	0.9992	0.9993	0.9995	1.0000	1.0000

TABLE XII
SCC OF THE RANKING RESULTS FOR DIFFERENT VALUES OF ∂ UNDER THE DATASET HS

	$\partial = 0.2$	$\partial = 1.2$	$\partial = 10.2$	$\partial = 30.2$	$\partial = 50.2$
$\partial = 0.2$	1.0000				
$\partial = 1.2$	1.0000	1.0000			
$\partial = 10.2$	0.9999	0.9999	1.0000		
$\partial = 30.2$	0.9986	0.9987	0.9993	1.0000	
$\partial = 50.2$	0.9959	0.9960	0.9970	0.9991	1.0000

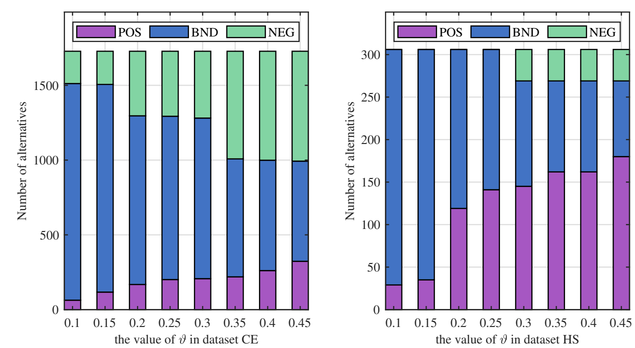

Fig. 9. Classification results for different values of ϑ under the two datasets.

The parameters associated with the classification are θ, ∂ and ϑ. When $\theta = 0.3$ and $\partial = 0.2$, the parameter ϑ increases gradually from 0.1 to 0.45, and the step size is 0.05. From Fig. 9, it can be noticed that with the increase of parameter ϑ, the positive and negative regions increase continuously, while the boundary region gradually decreases. This result conforms to the description of Theorem 1. As can be seen from Fig. 10, the larger the ϑ, the higher the classification error rate, but the lower the overall rate; the larger the ϑ, the lower the precision, recall and F1, but the higher the overall rate. In general, ϑ has little effect on the classification performance of the method, and the classification performance is good, which shows the stability of our method.

Fixed $\vartheta = 0.35$ and $\partial = 15.2$, the parameter θ is gradually increased from 0.1 to 0.9 in steps of 0.2. From Figs. 11 and 12, it can be noticed that with the increase of parameter θ, the classification results and classification performance change only slightly, so θ has little effect on the classification ability and classification performance of the method. Fixed $\vartheta = 0.35$ and $\theta = 0.3$, the parameter ∂ gradually increases from 0.2 to 50.2. From Fig. 11, it can be found that as the parameter ∂ increases, the positive and the negative regions increase continuously, and the boundary region decreases continuously.

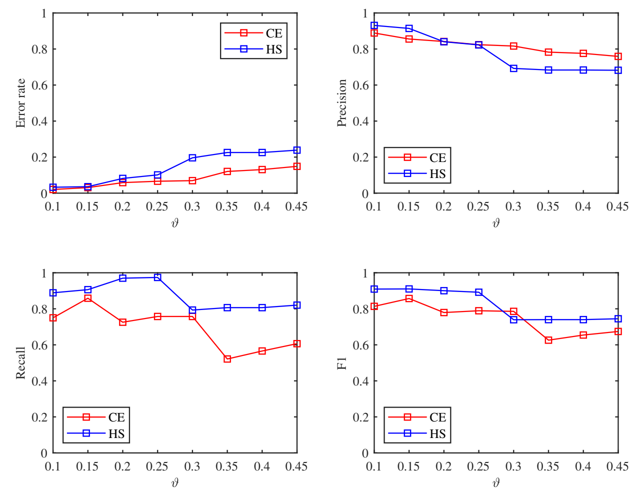

Fig. 10. Variation of the four indicators for different values of ϑ under the two datasets.

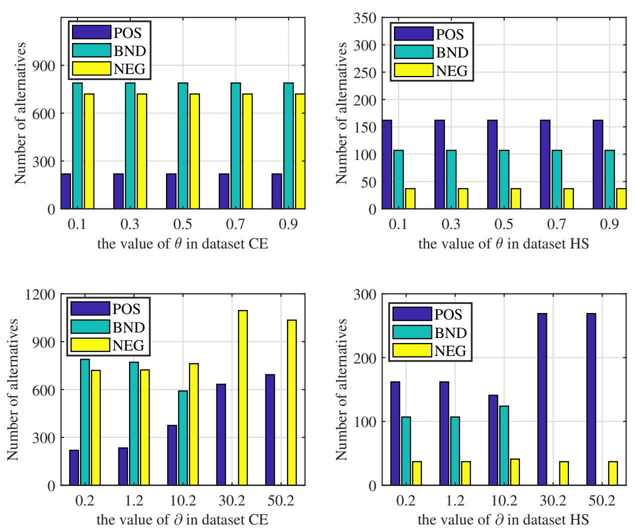

Fig. 11. Classification results for different values of θ and ∂ under the two datasets.

As can be seen from Fig. 12, the larger the ∂, the higher the classification error rate, but the overall lower; the larger the ∂, the almost constant precision and the overall higher; the larger the ∂, the higher the recall and F1, and the overall higher. In general, ∂ has little effect on the classification performance of our method, and the classification performance is good, which indicates the stability of our method.

D. Discussion

After the above quantitative analysis, the validity and stability of our proposed method have been proved. Below, we compare our method with existing decision methods from a qualitative perspective, as shown in TABLE XIII

According to TABLE XIII, our method differs from some existing methods as follows.

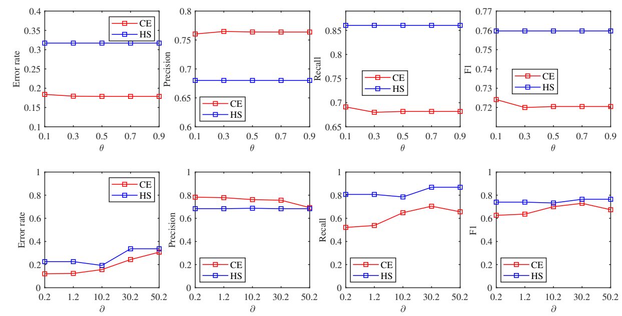

Fig. 12. Variation of the four indicators for different values of θ and ∂ under the two datasets.

(1) Among the chosen comparison methods, only M-IV [31] and M-V [33] are explored using multi-scale information. M-IV [31] results in information loss, while M-V [33] encounters difficulties in finding the optimal scale. In contrast, our method does not suffer from these issues.

(2) Among the comparison methods selected, only M-III [25] and M-V [33] considered decision attributes, and none of the others. However, in this paper, we combine subjective decision attributes with objective conditional attributes for knowledge and data-driven, to get more scientific and realistic decisions.

(3) Similar to the comparison method chosen, our method also considers attribute weights. The difference, however, is that the attribute weights in our method are adaptively adjusted according to the granular structure.

(4) For psychological behavior, other alternative methods are not considered except M-III [25] and M-V [33]. In our method, we consider both the acquisition of conditional probabilities and the acquisition of loss functions.

(5) In terms of ranking, the selected method can achieve ranking of all objects.

(6) For classification, all decision methods are available except methods M-1 [44] and M-II [21]. In addition, our method and the selected classification method are different, the specific difference is that other methods only make one classification decision, while our method makes multiple classification decisions, which is more consistent with practical decision-making.

The feasibility, validity and stability of this method are verified by the above analysis. Furthermore, we can see that our method is an extension of the existing method. More importantly, we are able to point out the advantages of our approach as follows:

(1) Our method can overcome some shortcomings of existing decision-making methods. On the one hand, compared to methods based on single scales, our method can solve more complex problems by considering multi-scale attributes. On the other hand, according to the risk factor, the ranking results are flexible, and the decision-maker can choose the appropriate parameter values according to the actual situation and needs.

(2) Our method is more practical and more in line with realistic decision scenarios. Our method can reconcile the inconsistency between conditional attribute information and

TABLE XIII
THE DIFFERENCES OF DIFFERENT METHODS

Methods	Multi-scale information	Decision attribute	Weight vector	Psychological behavior	Ranking	Classification
Our method	√	√	√	√	√	√
M-I [44]	×	×	√	×	√	×
M-II [21]	×	×	√	×	√	×
M-III [25]	×	√	√	×	√	√
M-IV [31]	√	×	√	√	√	√
M-V [33]	√	√	√	√	√	√

decision attribute information, so that we can test experts' decision experience and make scientific decisions. On the other hand, 3WD is common in reality. However, the existing classification decisions are made only once. On this basis, we combine HS3WD with regret theory and introduce conditional probability into behavior decision for the first time.

Be that as it may, there are still two limitations to our study. Firstly, according to the composite data, attribute weights are given by experts in this paper, and the influence of scale weights on ranking results is not considered. Secondly, many real-life decision problems are more complex. Some data characteristics, such as dynamic data and unbalanced data are not considered in this paper.

VI. CONCLUSION

This paper considers the complex MADM problems with multi-scale and establishes a new MADM method by using the 3WD theory and regret theory. The main work of this paper is as follows. First, according to multi-scale data, we introduce HS3WD into MADM problems under MSDISs to ensure the integrity of the original decision information. Second, we introduce regret theory into 3WD and construct a loss function based on regret theory, which takes into account the psychological behavior of the decision-maker and makes the decision more objective. Third, we establish a new MADM method based on MSDISs and give an illustration example to show the decision-making steps. Finally, through comparative analysis and sensitivity analysis, the stability, effectiveness, feasibility, and superiority of the method are discussed.

In the future, research should consider both attribute and scale weights, address dynamic and unbalanced data, and explore decision methods for more complex problems, such as financial risk evaluation in the new energy automobile industry. Additionally, further investigation into defining a state set for decision attribute value domains with more than two elements would be valuable.

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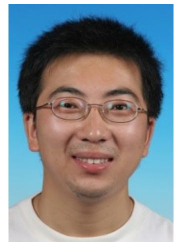

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